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Benny, Barbara, and the Ethics of EdTech

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Abstract

Erlwanger [18] shook the mathematics education world when he introduced Benny, a student who successfully worked through a behavioristic curriculum. Erlwanger showed how far removed Benny's understanding of mathematics was from expectations. Erlwanger's legacy is the basis for this comparative case study which explores students' actions in the modern, in-class computer-centered emporium classroom. Many striking similarities are found between Pearson's My-MathLabs (MML) and Benny's Individually Prescribed Instruction curriculum. In this case study we meet Barbara, a student who succeeds in MML but shows little understanding of mathematical concepts and demonstrates that the legacy of Benny is his continued appearance in our current students. However, what differentiates Benny and Barbara is more than time; it's the inequities resulting from imposing a pedagogy with well-known problematic characteristics to developmental mathematics students. Most of these developmental students are members of marginalized communities. As such, the social justice and ethical implications of using such a course structure are explored.

Keywords: computer-centered learning, equity & social justice, developmental mathematics, community college, behaviorism, educational technology, ethics

Introduction

To most mathematics educators, a reference to Benny [18] does not need explanation or elaboration. Erlwanger's case of an elementary student's (mis)understanding of mathematics is a seminal work as demonstrated by its inclusion in the book *Classics in Mathematical Education Research* [10].

An analysis of articles which cite [18] demonstrates that its influence goes beyond citations as Erlwanger's work is a part of the mathematics education cannon [25]. Over forty years later, this article is highly regarded by its appearance as a "must read" paper among mathematics educators.

In my opinion, the value of Erlwanger's work goes far beyond the findings outlined in [18]. Readers are likely to have an emotional reaction to this piece. Graduate students may shake their heads and tut tut at how Benny "fell through the cracks" when they see how far his mathematical understanding deviates from the curriculum. These students may silently promise themselves they will not allow their student to do the same. Faculty can use this as an example of how a given mathematics curriculum may not manifest as intended. This piece is a cautionary tale to reinforce the ethical duty evaluators and administrators have to thoroughly evaluate and examine the latest trends in education research for unintended effects; behaviorist-based curriculums were trendy at the time.¹ Erlwanger's shocking research on curriculum-gone-wrong implores mathematics educators to improve students' experiences learning mathematics.

Although readers may have a variety of emotional reactions to Benny, they should not consider the case as a quaint echo of the past nor should readers self-righteously dismiss Benny as a window to a bygone era. Like trends in fashion, politics, and culture, what once was old is new again. In this article, I explore similarities between Benny's Individually Prescribed Instruction (IPI) curriculum in mathematics and a recent, but similar, manifestation, Pearson's MyMathLabs (MML) emporium classroom. A previous comparison between in-class, computer-centered (I3C) mathematics classes and the IPI program with Benny was made [42], which highlights how MML is similar to IPI but does not explore the ethical and social justice significance of these similarities. As will be shown, it is not a question of HOW these courses compare; it's a question of WHO is enrolling in such a course and the implications of continuing to offer such classes to this vulnerable population of students.

¹ Behaviorism focuses on an individual's performance and incorporates practice to help the individual perform as desired. The individual is expected to react to a given stimulus and provide the appropriate response. In this case, a student is given a question and the correct answer is the appropriate response. When that response is not given, more opportunities for practice are provided. For more details, see [20].

This exploration highlights the case of Barbara, whose mathematical thinking bears a striking resemblance to Benny. Like Benny, Barbara was successful in her mathematics class because she was able to produce correct answers in the program. Benny, a sixth grade student, differed from the adult Barbara in that he developed mathematically invalid practices. However, each relied on recognizing patterns in correct answers to advance. Both students' practices of pattern finding arose out of their course structure, which emphasized correct answers over understanding. This paper culminates in outlining the ethical dilemma resulting from using a program such as MML in the modern classroom. The ethical implications of implementing a modern manifestation to what Erlwanger observed may be even more damaging to current students since they are disproportionately members of marginalized populations. These students are victimized by an educational format which has been ineffective for over forty years.

1. Computer-Centered Learning

This study is set in an I3C mathematics course, sometimes called an emporium model class, in a community college. I3C classes, a current trend to support students in developmental mathematics, are held in computer labs. Students receive their primary instruction and assessments from an online computer program that incorporates text or video lessons with questions to test students' knowledge. Unlike flipped or online classes, these computer-centered classes are held on campus during a fixed time. Students work through the content by watching lectures, reading lessons, and solving problems on their own. Instructors, tutors, or course supervisors are present to assist by answering students' questions. These courses, intended to personalize the classroom experience while efficiently teaching a large number of students, combine student individualization and flexibility with instructor support.

1.1. *Computer-Centered Learning*

Remote and independent learning is not a new phenomenon. Students have used correspondence courses for distance learning for well over a hundred years [39]. As technologies have changed, so have the options for independent learning: radio, audiotaped lectures, videotaped lessons, and now online

lectures and interactive courses. The ability to learn remotely has been helpful for those who cannot travel or dedicate preset times in their schedule to attend classes. The potential of remote, independent learning is even embedded in American legend; Abraham Lincoln used remote learning to educate himself and succeed in life.

Coupling the ability to support students with little effort by live instructors, and the need to support large numbers of students who are mathematically underprepared for college-level courses, I3C courses were proposed as a method to maximize student outcomes. Indeed, such course formats have been deemed a “silver bullet” in their ability to help students advance while reducing costs [38]. Such a lofty statement should be rigorously supported through careful evaluation since technology advances faster than our social consciousness, at times to our detriment. Here we see the ethical challenge. When new technology or the incorporation of new technology is proposed, speedy implementation and extensive research must be balanced to ensure new technologies are implemented in a manner which advances, rather than regresses, our society and development. New techniques in education should have sound pedagogical theory associated with it and these techniques should be incorporated carefully in the classroom, especially with developmental mathematics students who are often paying for zero-credit classes and prevented from starting their intended program until they complete these prerequisites.

Of all students who entered a public two-year postsecondary institution in 2003–2004, 68% took at least one developmental course, most commonly mathematics [14]. Passing rates in developmental classes are low and generally disfavor students of color, the majority of students enrolled in such classes [4, 5]. Furthermore, forty percent of students enrolled in postsecondary institutions and who required remediation did not receive a Certificate, Associates, or Bachelor’s within six years [21]. Students who need support in developmental courses are not an anomaly; those who succeed in them are. Enrolling in developmental courses reduces the likelihood that a student will achieve their academic goal.

As of January 2020,² half of Pearson’s MML curriculum available in the United States was made up of developmental mathematics courses, whereas more advanced courses such as calculus and linear algebra courses compose 11.5 percent of available courses; see Table 1 (<https://pearson.com>).

Table 1: Number and percent of total of MML courses available by course level

<u>Course level</u>	<i>n</i>	%
Calculus and Linear Algebra	29	11.5
Precalculus and Intermediate Algebra	70	27.89
Developmental Mathematics	126	50.20
Other (math for liberal arts, finite math, math methods, etc.)	26	10.36

As a result, MML students are less likely to succeed. If developmental courses are supposed to achieve educational parity, then evaluating a learning modality which has been implemented disproportionately to this population is an ethical question.

Research on the effective use of using computers to teach mathematics dates back to the late 1960s [3, 35]. The majority of such studies are quantitative and compare students’ performance using software with a different form of learning, usually lecture-based, instructor-centered learning. There is no consensus on the effects of various types of computer-assisted instruction, including I3C courses, on student learning. Study results vary widely: some show computer-assisted learning leads to improved mathematical understanding or is more effective than a traditional face-to-face class [8, 11, 13, 31, 36, 40, 43]. Meanwhile, there is a conflicting set of research showing traditional classes outperform computer-assisted student achievement [41, 44]. Others contradict these finding especially when accounting for variations in instructor grading [37]. Other findings show no statistical differences between learning modes [9, 23, 24, 41, 45].

Overall, many comparative studies focused on comparing the media, or the mode of delivery, rather than on the pedagogical structure of each course [33].

² These statistics were recorded before the COVID-19 pandemic forced instructors, including the author, to rely on MML and other forms of EdTech to support student learning at a time when there were no alternatives. A discussion of ethical implementation must be considered in full context of all circumstances.

Oblinger and Hawkins [30] warn of the consequences of adding technology without altering pedagogy. What is new in the modern computer-centered age? Unless the implicit design or implementation of the specific software is deliberate and novel, there is little to distinguish the underlying pedagogical structure of many computer-centered classes from traditional ones. This includes MML which, as will be shown, is a technological descendent of IPI. The ethical implications of utilizing this same pedagogical approach must therefore be considered.

2. Methods

This case study explored an I3C, developmental, mathematics classroom in a community college. This study used an interpretive perspective to collect and analyze data and present findings [17]. An interpretive frame recognizes reality as socially constructed and multifarious [29]; multiple perspectives and goals come together and interact in various ways. Instructors have a duty to help their students complete the class as quickly as possible. Students have multiple priorities including responsibilities outside of school, their intended degree, and overcoming any prior challenges they had learning this material. These various stories must be taken into account when considering this classroom space.

2.1. *MyMathLab in Mountainscape*

The site of this study was a developmental mathematics class in a community college which we will call Mountainscape Community College (Mountainscape) in the Southwestern United States. The course MATH075, Foundations in Mathematics (a pseudonym for the course) was designated developmental because its content focused on precollege-level mathematics. Mountainscape was a large community college system and designated a Hispanic Serving Institution (HSI). Data were collected in one class in Spring 2014.

The structure of MATH075 was similar to other Pearson “emporium” courses [1, 15, 38]. MATH075 was divided into 35 modules and personalized by Pearson to align with Mountainscape’s standards. A module was similar in scope to a chapter in a mathematics textbook; each module was composed of several units. All MATH075 students began with Module 1 and had to master

the topics before advancing to the next module. Students were expected to complete the course over three semesters. Mastery of modules was demonstrated in one of two ways. When a student began a module, they could attempt a pre-test covering the material. If they scored at least 80% on the pre-test (this cutoff was designated by Mountainscape), they advanced to the next module. If not or if the student did not choose this option, they worked through the module completing assignments for each unit. Each unit had three learning components. Videos presented a lesson. “Concept Checks” tested vocabulary and mathematical procedures. Finally, “Homework” was given in each unit. All problems were provided by MyMathLabs. Students wrote out solutions in their notebooks but entered their final answers in the computer, which recorded all grades.

Students were required to attend MATH075 to work through the software and earn attendance points. Barbara’s class was held twice a week for 1 hour, 15 minutes each. Since the software was accessed via the internet, students could make progress outside class.

Grading in the course was based on three factors: module grades, the number of modules completed, and a student’s participation / attendance. Students were required to answer “Concept Checks” with 75% accuracy and “Homework” with 90% accuracy before they could take the corresponding exam and score at least 80% on exams to advance. Cumulative exams were given every five modules, requiring a score of 70% to pass. All assessments were proctored and given through MyMathLabs. Students were expected to write submit their solutions to the instructor. If a student did not pass the exam MyMathLabs provided extra practice problems associated with the concept(s) the student answered incorrectly. If the student did not pass an exam three times, the instructor was required to intervene, providing support and supplemental problems until the student achieved the requisite grade.

The software also contained features to help students. Relevant features will be described in the forthcoming sections.

Two course instructors (instructors of record), the lab supervisor and up to three staff-tutors were present during class to answer students’ questions. Course instructors calculated students’ grades and reviewed previous exams with them, either at the instructor’s discretion or after a student failed an exam three times.

2.2. Data Collection & Analysis

All necessary permissions to collect data were obtained by participating institutions. Data were collected from five sources. The first was a cluster of data which recorded what Barbara did during class. This included video recordings of Barbara's actions in class, recordings of her computer screen, audio of her interactions, and photographs of her written work. Barbara was recorded for seven days (approximately two weeks out of a fifteen week semester) during and outside class time and was observed for 6 hours, 52 minutes. At times, when I arrived to collect data for Barbara, she was already seated at a desk, working. Interactions in this data cluster included informal conversations with Barbara clarifying her actions. The second source of data were field notes which recorded context to recordings or "observer comments", researcher thoughts elicited while gathering data [29, page 172]. Two interviews with Barbara comprised the third and fourth sources. The first interview occurred on the third day of observation and gained insight into Barbara's personal and mathematical learning experiences. The second interview occurred after observation and analyzing some data and gauged Barbara's assessments of her progress in class and included personalized questions to address specific incidents or actions during data collection which required clarification or elaboration. The fifth source was an interview with the instructor which established his perspectives on teaching and expectations for the course.

The qualitative analysis software ATLAS.TI assisted with coding and analysis. Interviews were open coded [34]. Videos of Barbara working were synchronized with the corresponding computer-recorded screenshots. Her actions were coded in two passes. In the first, codes corresponded to Barbara's actions; for example, watching a video, answering questions, actions taken to answer the question. In the second, the video was open coded. Conversations were transcribed and cross-referenced with the videos and open coded. Photographs clarified written work. The code co-occurrence command in ATLAS.TI helped themes emerge by counting the number of times any two codes occurred simultaneously.

During and after coding, I wrote analytic memos to help make sense of the data. Analytic memos, in line with an interpretivist perspective, allowed me to reflect on the coding process, the inquiry process overall, and thoughts on

emergent patterns and themes [34]. I wrote a second type of analytic memo after Barbara's data were coded. These analytic memos asked questions of the data such as "What did Barbara do after answering a question incorrectly?" Analytic memos were also written to summarize data using as many codes as possible to make sense of Barbara's process.

Since multiple types of data were recorded, findings and interpretations were triangulated. Long-term and repeated observations collected over multiple classes further supported the findings. Findings were also shared with Barbara. This conversation was not recorded because it occurred by happenstance. In this conversation, Barbara clarified some occurrences and verified my general findings.

Three major themes that emerged from observing Barbara's use of MML will be discussed in this paper. The first theme is "pattern recognition," Barbara used patterns in solving problems or in reading solutions to determine correct answers. The software contained multiple tools which Barbara used to determine answers to the questions without using mathematics. Memorization became another theme of Barbara's progress through MML. Barbara relied heavily on memorizing instructions and procedures to successfully advance through the software. The final, and central, theme of this study explores the strength of the similarities between Barbara and Benny. This theme will be explored in the discussion section (§4).

3. Findings

MyMathLabs assesses students' mathematical knowledge by presenting a set of problems for students to solve. Like questions in Benny's IPI course, these problems are rote and not conceptual and do not involve higher-level thinking. A student does not need to understand the underlying concepts to answer the questions; they can just execute the correct procedure for any given problem or find a way to the correct answer. Consequently, if a student learned how to solve the canon of MyMathLabs problems, they could be successful without understanding the associated mathematical ideas.

3.1. Looking for patterns

Barbara looked for patterns in the problems to help answer her questions. The help feature “View an Example” was one way to see patterns in the problems. This feature decomposes a problem similar to the one the student is working on into its procedural steps without student participation. This is similar to a modeled problem in a textbook. Using this feature did not always further Barbara’s understanding or practice of certain concepts, but helped her advance by learning the answer pattern. At times, Barbara calculated and entered her answer, then opened “View an Example” to check her work. If the answer(s) were of the same form, for example if a complicated expression simplified to 1, she would submit her correct answer. Other times, after a problem was marked incorrect, she read “View an Example” and entered the final answer from the modeled problem into her given question. Knowing the problems were patterned helped her skip working on the problem a second time which saved time, or gave her the opportunity to check her work or bypass doing the problem by using the answer from “View an Example”. These actions did not require her to have a fundamental understanding of the given concept.

Pattern recognition to determine correct answers without calculations was not limited to various help features. In one example, Barbara was required to simplify the rational expression:

$$\frac{24w^2 - 40w}{3w - 5}$$

Barbara entered an incorrect response. She asked for help and Julius, a tutor, helped her work through the problem. As he proofread her solution, he did not notice an error in her work and when they entered the answer it was marked incorrect. They redid the problem a third time and did not factor the numerator completely. Thus when Barbara entered the answer, it was once again incorrect. Because this was her third incorrect error the answer was given and a new question provided. Barbara now had to simplify a similar expression:

$$\frac{36z^2 - 30z}{6z - 5}$$

This was a lighthearted point of contention with Julius. “Julius! Now I have to do the problem all over again. Ok this one I’ll know”. Julius again reviewed the concept with her “You know the drill though; you want to factor out the highest —”. In the time Julian said those words, Barbara entered the correct answer, $6z$, without working out the problem. Barbara realized the greatest common factor in the numerator was the answer, following the pattern from the previous problem. Julius even noticed how quickly she entered the answer “Ok so you figured out that this system is [LAUGHTER]”. Rather than rework the problem, Barbara understood that similar problems were patterned and did not need to be solved if the pattern can be discerned.

A similar type of pattern recognition helped Barbara understand when -1 appeared when simplifying rational expressions. Several problems involved simplifying or dividing rational expressions so students learn to recognize the following form

$$\frac{a - b}{b - a} = \frac{-(b - a)}{b - a} = -1$$

When Barbara encountered such expressions, she learned certain expressions simplified to -1 and certain problems did not. “I had to memorize and write down was that I don’t have to put a negative if I’m simplifying but I do if I’m dividing. So I had to write this little note on my test”. Rather than understand the aforementioned simplification, Barbara noted the prompt in the question (simplify or divide) would let her know whether the expression simplified to -1 .

To illustrate her point, Barbara showed the following problem, which simplified to -1 .

$$\frac{36x^2 + 12x + 1}{1 - 36x^2}$$

Barbara factored and simplified the expression correctly:

$$\frac{(6x + 1)(6x + 1)}{(1 - 6x)(1 + 6x)} = \frac{6x + 1}{1 - 6x}$$

However, Barbara made a note to the left: “Don’t have to put -1 when simplifying”. Here is how she explained her reasoning:

“Weren’t you supposed to multiply by -1? I think, see that’s why I got confused because you’re supposed to multiply -1 here when you’re dividing. So it’s negative you put a negative sign before.”

To try to understand Barbara’s reasoning, I asked her to compare this answer with another problem and answer in the same problem set:

$$\frac{x^2 + 5x + 6}{5 - 5x} \div \frac{x^2 + x + 30}{4x + 16} = \frac{(x + 6)(x - 1)}{(1 - x)5} \cdot \frac{4(x + 4)}{(x + 6)(x - 5)} = -\frac{4(x + 4)}{5(x - 5)}$$

Here is her response:

“I don’t know the like — the math terms but I had to memorize — because if I’m multiplying or dividing, I can put a negative if it’s, if the problem is like one minus or if the problem’s like five minus five x [$5 - 5x$] then I can put a negative. But if I’m simplifying and the problem’s like that one minus thirty-six x [$1 - 36x$], I can’t put a negative I don’t know why.”

Barbara was clearly adept at factoring and simplifying; however, she did not understand the nuance of seeing terms which differed by a factor of -1. Rather, she determined that when directions said to multiply or divide the rational expressions, similar looking factors yield a -1, but if she was asked to “simplify”, they did not. She learned the patterns in the software without understanding or using the associated mathematical concepts.

The evidence suggests that Barbara worked expeditiously whenever she could, bypassing further practice. These opportunities were unnecessary to Barbara, who saw and learned patterns to the correct answers rather than practicing techniques. By the end of the semester Barbara completed her modules and passed the class, an example of success in MATH075. Based on her actions and interactions, however, it is reasonable to question Barbara’s understanding of the mathematics covered in the modules. Many of Barbara’s statements and actions left doubt as to the extent of her knowledge of mathematics. The likelihood that a student can achieve a passing grade in a course without the commensurate understanding is an ethical dilemma which all instructors must grapple. However it seems that pattern recognition in MML provides a larger opportunity for students to receive credits without gaining the respective knowledge.

3.2. *Learning Mathematics = Memorizing*

To Barbara, learning mathematics meant memorizing how to solve specific problems and recognizing patterns to answer questions. When asked how she saw herself as a mathematics student, Barbara stated she found mathematics confusing and explained an inability to memorize.

“Memorizing formulas [is confusing]. And I think like I can sit in the classroom and the teacher can talk to me and I’ll take notes. I’m okay. I get it. I understand it. And then I leave class and I’ll come back the next day and I’m like oh [I] just forget everything. I think it’s just maybe when I leave class and before I get to class I just have to study and memorize the material more. I think that’s just my biggest problem too.”

Barbara believed that success in mathematics comes from memorizing. Rather than reason through problems or make connections between ideas, she memorized procedures.

For example, she described her challenges in making sense of how to simplify an expression using the least common denominator (LCD). A complex rational expression (a fraction within a fraction) may be simplified in several ways; one of which is to multiply and divide the entire expression by the LCD of both internal fractions. Barbara was unsure of why or when to use the LCD. She stated,

“I didn’t know when to multiply the LCD by the numerators. So now I have to memorize when the question says simplify by multiplying, then I’ll remember to multiply the LCD by the numerators.”

Barbara did not see how multiplying a rational expression by the LCD divided by the LCD related to simplifying the expression. Rather, she learned to associate the process with the directions. Her blanket statement that she would learn to associate multiplying the LCD when she saw the directions “simplify by multiplying” would work for complex fractions, but not for other problems with similar directions.

Barbara discussed several times that she memorized and associated appropriate procedures and answers with problems to succeed. Her understanding of the mathematical concepts at hand was not central to her mission, but rather she used number sense, pattern recognition, and associated clues to determine which procedure to use for a given question. For example, Barbara explained how to know whether she should factor a quadratic expression or solve a given quadratic equation for an unknown variable.

There's a question with the same one [a quadratic expression or equation] that goes with no value and undefined, or write the answers, or use a comma to separate the answers. And that gives me a hint to set the value to zero. So I'll get like a negative or positive number, or two negatives or two positive numbers.

In working with problems with quadratics, Barbara used the answer prompt to determine what procedure to follow. If the instructions told the student to use a comma ("Use a comma to separate answers as needed"), Barbara knew to set the equation to zero and find the solution set. Figure 1 below, as well as Figures 2 and 3 on the following page, are examples of various exercises using quadratics and the instructions for entering solutions which Barbara used as prompts. These prompts did not require Barbara to understand the concepts underlying the problem. Rather the prompts helped Barbara associate a procedure with a question. Barbara did not need to know the various nuances of working with quadratic expressions, or understand the meanings of various instructions; the prompt for the answer hinted at the required procedure.

Factor.

$$x^2 - 7x - 8$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. $x^2 - 7x - 8 = \square$ (Type your answer in factored form.)

B. The polynomial is prime.

Figure 1: Factor a quadratic (factored answers require parentheses).

Solve by factoring.

$$x^2 - 6x - 27 = 0$$

x =

(Simplify your answer. Use a comma to separate answers as needed. Type each solution only once.)

Figure 2: Solve a quadratic (prompt says use a comma).

On several occasions, Barbara's actions centered around getting through problems and memorizing procedures without focusing on the underlying mathematical concepts. Barbara's tendency to focus on video lectures when examples were modeled and to look for patterns in her answers supports this claim. The narrator of the video lecture presented concepts and procedures to solve examples. Barbara repeatedly said the was narrator boring. Nevertheless she watched the videos, paying attention when examples were modeled, but not when general ideas or connections between concepts were discussed. Even when distracted, Barbara focused when examples were explained.

When I'm doing math homework, I try not to look at my phone but if I'm watching the videos I'm just sitting there. Oh my gosh! And he's just going over stuff I really don't care about and it's not like he's not showing me an example of the problem. When he showed me an example of a problem that's when I put my phone down and like actually pay attention. But when he's just talking, that's when I pull my phone out.

Find any values of the variable that will make the rational expression undefined.

$$\frac{x}{x^2 - 3x - 4}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

A. The rational expression is undefined when x = .

(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

B. There are no values for x that will make the rational expression undefined.

Figure 3: Solve a quadratic in an application (prompt says use a comma).

Barbara focused when an example was explained and would ignore material she did not “care about” including the supporting lecture, conceptual ideas, and connections made to other units.

3.3. *The Consequences of Memorizing*

Although Barbara diligently memorized procedures to use with various problems and developed associations when she saw certain patterns or instructions, her understanding of mathematics was superficial. When faced with problems she could not easily fit into a given pattern, she had significant difficulty answering the relevant question.

While working to solve the equation

$$\frac{2x}{x-3} = \frac{6}{x-3} + 3$$

Barbara got the result $-x = 3$ and asked her instructor Dan to help finish the problem. Dan showed her that $x = -3$. He then asked a follow up question, “What happens if you would’ve got an answer of x equals 3?”, reviewing a concept covered earlier in the module where answers were eliminated if they made an expression undefined. Barbara, after clarifying the question, responded, “then that would’ve been the answer”. Dan followed up: “if you got $x = 3$ so over here [pointing at the answer to the problem] you got $x = 3$ instead of -3 . If your answer was $x = 3$, would it work?” Barbara started flipping through her notes, looked at another problem, then answered, “Oh that’s a trick problem”. Dan asked her to elaborate and she started to consider the other example “Because look $y - 2$ it’s just equal to $y + 20$. Wait these ones didn’t change though.” As Barbara started to discuss an unrelated problem, Dan tried to keep her focused on his main question of what happens when the calculated solution makes an expression undefined. Using a similar example where the calculated answer is $x = 2$ but must be eliminated since $x - 2$ is in the denominator of the original equation, Dan pressed her:

- DAN Mm mm, what happens if you plug 2 into there?
 BARBARA I’d get a positive number.
 DAN No, you don’t get a positive number. What’s 2 minus 2?
 BARBARA Oh, it’s a zero.

DAN What's 2 divided by zero?
 BARBARA Zero.
 DAN What's 2 divided by zero?
 BARBARA Zero.
 DAN What's 2 divided by zero?
 BARBARA It's 2. Zero.
 DAN No. No. No. More no.
 BARBARA 1.
 DAN No.
 BARBARA Seven.
 DAN No.
 BARBARA What's your question?
 DAN What's 2 divided by zero?
 BARBARA 2 divided by 0 is no solution. Undefined.
 DAN Why is it undefined?
 BARBARA [CHUCKLES] Because the limit does not exist.
 DAN Exactly! That was a very calculus answer. Yay!
 BARBARA I got it from *Mean Girls*. [LAUGHTER]

Barbara then described a scene from the movie *Mean Girls* where the protagonist, Cady, in a climactic moment exclaims “the limit does not exist,” allowing her to win a mathematics competition for her school. Barbara’s use of the line from the movie parallels the climactic moment for Cady; both were saved by recognizing that the limit does not exist. But although Barbara applied the line accurately, it was clear that she did not understand the notion of an undefined value, based on her laughing and the increasing randomness of her answers. To deflect attention from her misunderstanding of the content, she called on her lived experience and referenced mathematics from popular culture. The light-hearted reference ended the interaction in laughter and halted further inquiry into Barbara’s (mis)understanding of eliminating potential solutions.

Barbara passed each module exam right after completing the module. However, she had difficulty succeeding in similar problems on her cumulative exams. Cumulative exams, which covered content from the previous five modules, required a grade of 70% or higher to pass. When asked if she had difficulty passing her cumulative exams, Barbara responded “this last one it took me five times . . . it was horrible”.

Barbara's speedy completion of the modules through pretesting often caused her difficulty during cumulative exams, likely because she superficially and quickly advanced through modules. Barbara's difficulty with these exams further demonstrated that associating procedures to answer prompts rather than understanding relevant concepts challenged her retention.

Here, the ethical burden can be viewed as falling on Barbara who chose the expedient route which did not foster deep mathematical understanding. However, it is the course design which did not provide incentive or opportunity to deeply explore concepts. In fact, Barbara could have been penalized if she explored ideas in depth and entered incorrect answers. Working through a pretest and taking multiple cumulative exams gave Barbara the advantage of advancing quickly since she bypassed many questions and the requisite grade required on homework assignments. Ultimately the course design supported her actions since she needed to work quickly to complete the modules before the end of the semester.

4. Discussion: Barbara and Benny

Danny Martin challenges mathematics educators to consider success stories among African American students [28]. This case can be viewed as a success story in that it investigates how Barbara, an African American woman, used the course environment to successfully navigate the MML environment. But, I ask, at what cost? It was hard to miss how much Barbara's progress resembled that of Benny. Both were considered successful in their courses, but it was clear that each left with significant mathematical misconceptions. Note that in both cases, the students' problematic understanding of mathematics was an appropriate response to the course format. If a given course emphasizes correct answers over process or understanding, then students will meet those set expectations.

Barbara, like many students in procedurally-based classes, may not have seen mathematics as more than following procedures since understanding was measured by correct answers. In particular, if what matters in the class is entering the correct answer in the empty box or choosing the correct option, whatever Barbara thought, understood, or realized did not matter if she did not enter the correct answer. Indeed, when presented with a situation

where she needed to apply knowledge, such as eliminating an answer which would leave a rational function undefined, Barbara was confused and obfuscated the situation with funny references and jokes. Knowledge in MML was strictly demonstrated by entering correct answers and not by making connections, seeing relationships, or understanding larger mathematical patterns. Although pattern recognition and number sense falls under mathematical thinking, the value of these skills did not lie in Barbara's ability to make connections between concepts or to motivate procedures, but in her ability to quickly determine the correct answer so that she could move quickly to the next module.

Overall, Barbara was adept at moving through modules using whatever means she could to complete the MATH075 course before the end of the semester. This included violating the prescribed practices and expectations by focusing on recognizing patterns in answers and memorizing directions and prompts rather than working to develop a full understanding of the mathematical concepts covered in the course. However, I would argue that this is related to the design and nature of the course and not related to Barbara nefariously undermining the course. Barbara responded to the structure of the course appropriately. When advancement is based only on correct answers, the answers become the focus. By understanding how to think about the mechanics of the software and how the software defined success, Barbara used the course structure to her full benefit to complete the developmental sequence and qualify for college-level mathematics courses.

4.1. Similarities with IPI

Novelty in form does not equate to novelty in structure. MML shares many similarities with Benny's program, IPI. Just like MML, IPI was mastery based; students could not advance until they achieved a requisite score on previous assessments. Also just like MML, IPI's content was sequenced; students worked through ordered modules broken into units. Unlike MML, which is accessed online and whose content is introduced via video lecture, IPI was taught through a series of skill booklets and worksheets. This difference was likely due to technological limitations at the time of IPI's creation, and seems to be only a surface-level difference. Overall, a student's progress through the course in IPI was similar to that of MML.

In IPI a placement test identified a student's starting module. From there, each student was given an individualized packet of booklets (units), which would provide everything the student should need to pass each test with minimal support from the teacher. These booklets were individually chosen for each student based on their skills and needs. A booklet was included in a student's lesson if they could not demonstrate mastery of the associated learning objective through multiple assessments, a pre-test and a post-test given at the start and end of a module, and a curriculum embedded test (CET). The CET was a short test which was to be completed as a student finished working through each unit. The CET tested students on the specific learning objectives of the current unit and the next one in the sequence, giving teachers the ability to further personalize students' progress by eliminating upcoming units already familiar to the student. This allowed the CET to act as a pretest for each unit. IPI differed from MML on pretests in that IPI students were automatically assessed on whether they knew the upcoming topic, while MML students choose to take the pretest.

Like the MML class, IPI teachers administered more than taught. They focused on grading assignments and creating personalized booklets based on students' test scores. IPI booklets were self-sufficient so students did not rely on the teacher for extensive mathematical support. Teachers intervened if a student needed significant support, however both programs were designed so students learned independent of instructor support. The technological limitations of IPI required those teachers to do the work automated by the MML software: compiling the books, checking students' work, and tracking their progress.

4.2. Answer-Driven Course Format

Computer-centered programs can help students develop a richer understanding of mathematics. In a college-level calculus class in Australia, students who used the DERIVE software to explore multiple representations of functions or test conjectures found DERIVE added a layer to the classroom environment which expanded students' knowledge [32]. Although students were confident their calculations using DERIVE were correct, they felt they learned better when required to work with a pencil and paper. Such software which helps students see relationships or test conjectures supplements rather

than usurps mathematics teaching. Furthermore, answer-based computer programs incorporated into a mathematics course can provide immediate feedback to students, while relieving instructors of the burden of grading low-stakes assignments. Thus, students and instructors can benefit depending on how didactic software is used in a course [22]. In the case study presented in this article, the structure of the course allowed Barbara to succeed by being answer-driven and by using the predictable computerized format, rather than focusing on pondering the underlying foundations and connections within mathematics. This work therefore demonstrates how the introduction of technology can mask an accounting of the quality of education [39].

I3C classes may be more appropriate for students who understand basic arithmetic and algebra and need to review the content to meet requirements than for students who lack basic mathematics skills, see [2]. This may explain why short computer interventions are successful [26]. The touted success of I3C classes for developmental students could be explained by inaccurate placement rather than by the course structure [7]. Belfield and Crosta's analysis of reading and mathematics computerized placement scores found that up to one-third of students placed in developmental courses were able to succeed in college-level courses. Successful I3C mathematics students may already possess the necessary mathematics skills and agency to pass these courses without support from these programs.

For many students, the primary goal in a mathematics class is to earn a passing grade. In MATH075, a student does so by entering correct answers, valuing answers over understanding. In a survey, 20% of students indicated they could correctly answer MyMathLabs questions by guessing or mimicking the support features [1]. Students will focus their actions on what in the course will bring them success. The assumption that students demonstrate their mathematical understanding by entering correct answers is faulty since students can learn how the software is programmed and enter correct answers, gaming the system, rather than demonstrating their mathematical understanding. Here we see an ethical challenge behind the design of answer-driven learning software.

When students reflect on the structure of an I3C course and how to be successful, they may not see the value of understanding the concepts over mem-

orizing since conceptual understanding undermines the ability to advance quickly. This course encouraged students to advance through the content by learning to work the system, not understand the content. Barbara illustrates that students direct their energy appropriately to meet the goal, namely entering the correct answer. Without internal or external motivators to help students understand the content, the course structure leaves little encouragement for students to focus on “inefficient” learning strategies. There was no reason to consider anything but the path of least resistance, particularly if other factors were irrelevant, and might even be a distraction from the actual goal.

This path of least resistance is a result of the assumptions inherent in MyMathLabs and other procedurally-based mathematics curricula. There is an assumption in procedurally focused courses that mathematical manipulation will lead to conceptual understanding. MyMathLabs seems to support such a hypothesis. Underlying this assumption could be the belief that a more conceptually focused program is unnecessary for developmental students who need to master basic mathematics. Perhaps it is not important for certain students to have the opportunity to explore the richness of mathematics. Not only are these students not in a course where “why” is never a central question, but the content is deconstructed and prescribed to such an extreme extent that asking such a question would be inappropriate. At no point is the question of who is or is not given the opportunity to explore mathematics in depth is explored.

4.3. Barbara and Benny

Barbara is similar to Benny in that Barbara progressed through her mathematics curriculum with great success (compare with [18]). Both students were observed when they were experienced with their respective program and therefore provide an informative picture of the consequences of what happens when procedurally-based curricula are worked through independently by students with minimal context and conceptual accountability.

Benny was regarded as one of the most successful students in the course, thus his teacher assumed Benny understood his work. However, under scrutiny Erlwanger learned that Benny created his own mathematical world rooted in misconceptions. His significant misconceptions were developed to align with

the behaviorist program, and he answered the necessary questions. Over forty years later, Barbara's case, while not as extreme, is similar and reinforces the limitations of the behavioristic features in MyMathLabs.

IPI and MML sequenced the mathematics curriculum linearly, required 80% mastery on exams to advance, and allowed students to advance at their own pace. Like Barbara, Benny had a limited conceptual understanding of mathematics. When both students were pressed to explain their thinking, each stated their own mathematical rules. However, Barbara's rules were centered on applying procedural steps based on the instructions, whereas Benny created his own mathematical world. Barbara's case, like that of Benny, warns of the consequences of focusing on answers rather than explanations. Both were successful in the course because they learned how to answer the given question correctly, not because they fundamentally understood the subject. Both demonstrated a desire to put the seemingly unrelated ideas into perspective. Both worked to advance despite the seemingly arbitrary rules and procedures in the content.

In both cases the course structure dictated that correct answers were important. Neither program required the student to explain their thinking or understanding even when questioned. Barbara's reference to *Mean Girls* was likely meant to obfuscate her lack of understanding and she avoided any pressure to explain her reasoning. Without mechanisms to explain or clarify thinking, Benny and Barbara focused on what they believed would help them pass the course: correct answers irrespective of the concepts. For Barbara, this included associating procedures with answer prompts. For Benny, this meant going on a "wild goose chase" [18, page 221] so his calculated answer would not differ from the final authority in his class, the answer key. Both students directed their efforts to entering correct answers, to the point where they developed their own math rules, such as Barbara's rules for simplifying terms.

Barbara had a strong number sense and could easily discern patterns in answers. She worked to develop connections in the problems, seeing how to apply one set of answers to another. Barbara's skill in arithmetic and pattern recognition helped her be successful in MATH075. Without some minimal mathematics skills, Barbara would likely have had a difficult time working through the modules as quickly as she did [2].

Barbara's case demonstrates the advantage students with some mathematical capabilities have. Having arithmetic skills and algebraic understanding helps students focus on manipulation and processes necessary in an environment where much of the content must be worked through independently. Unlike Benny, a child who was still developing his quantitative reasoning skills, Barbara, the adult, was able to use her quantitative reasoning to advance.

5. Social Justice and Ethical Implications of IPI and MML

In the dissertation where his study originates, Erlwanger listed Benny as a middle class student but did not report his race [19]. Reporting class and not race implies that Benny's race was somehow unremarkable. One can therefore assume Benny was White due to the normativity of Whiteness in research practices in 1970s America, if not currently.

The IPI curriculum, a progressive program based on behaviorism, was developed at the University of Pittsburgh and was designed as an individualized, adaptive instructional practice. It is easy to see that Benny and his classmates were privileged: they learned from a progressive curriculum developed by university researchers using the most current psychological theories of the time. University researchers were on-hand in the classroom to observe and support the students. Benny himself received "remedial" instruction from Erlwanger, the graduate researcher, getting personalized attention from a highly qualified individual. Nothing but the very best was provided to Benny and his classmates.

Modern, progressive learning theories are constructivist in nature with curriculum centered around inquiry-based and student-centered instruction. Students are encouraged to gain a conceptual understanding of mathematics so they can use mathematics to solve problems they encounter on an exam or in life. Current learning theories are supported by cognitive science and are aimed at helping students develop rich neuropathways between ideas. By building on students' personal experiences and with the aid of a master facilitator or educator, instructors can help students learn multiple approaches to achieve their math learning goals. The emporium classroom is not learning based on the latest in educational research; Barbara is not experiencing a learning environment derived from current learning theories.

Although the programs are similar, time has further differentiated the meaning of these experiences. Today, the students who need the most mathematical support are more likely to take a class using MML. In other words, the marginalized, developmental students who need the most support are learning using outdated means. The comparison between Barbara and Benny cannot ignore the fact that Benny was learning using state-of-the-art practices, while Barbara experienced an outdated instructional model. Indeed, it was Benny who illustrated the limitations of behaviorist-style mathematics content and propelled mathematics education researchers to explore better ways to educate students. Yet, marginalized students are still subject to learning on their own with no context which will ultimately undermine their long-term success. If Benny experienced a learning gap from a well-intentioned curriculum which fell short, Barbara experienced an opportunity gap resulting from a well-known phenomenon. I use the term “opportunity gap”, not “achievement gap” to reinforce that Barbara’s achievements were due to the limited societal opportunities available to her based on the inequitable educational system imposed on her. Barbara, like Benny, simply did the best she could under the given circumstances.

6. Conclusion

Although this study was set in a class which used MML, I do not intend to singularly focus on MML. Readers should consider the pedagogical framework from which any EdTech product they intend to implement is derived, and how the product is designed to be used in the classroom. We must consider the ethical question of why students who need the most support are subjected to outdated learning models, while being told they are experiencing the latest that technology and educational research have to offer.

When I first read *Experience and Education* [16], I was taken by how modern Dewey’s words on how experience helps the child learn sounded. I imagined a classroom where students would explore the world and come to conclusions based on critical thinking and their own experiences. I pictured how such a school could have shaped me as a student, as a learner, and as an adult. However, the largest impact on me from reading Dewey was the publication date. I was born well after Dewey wrote this piece, yet I had never experienced a classroom described by him.

Barbara experienced the reverse. Her learning environment mimicked a well-studied and well-understood setup. However, this classroom format was not the ideal to which we should all strive as educators; it was the warning. Erlwanger's work demonstrated how an answer-driven, behaviorist learning environment could do such a disservice to the students. Benny slipped through the cracks. But this slip could have been for the greater good. Erlwanger's study should have been the beginning of the end for such pedagogical practices. It wasn't. Now Barbara slipped through the cracks. How many more students will slip through the cracks?

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