

Book Review: Reckonings: Numerals, Cognition, and History by Stephen Chrisomalis

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Book Review:
Reckonings: Numerals, Cognition, and History
by Stephen Chrisomalis

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Synopsis

This review of *Reckonings* shares our thoughts on the diverse insights presented by Stephen Chrisomalis's version of the history of numerical notation. Chrisomalis suggests that members of distinct cultural groups write numbers as an active choice in accordance with their own sociocultural contexts, which reflect the influences of historical, cognitive, social, economic, political, environmental, and cultural factors. This book integrates comparative, cognitive, and evolutionary understandings on numerical cognition with historical and linguistic evidence on the use and transformation of numeral systems through the historical advancement of numeracy. Chrisomalis offers an interesting historical perspective on numbers that builds upon three main themes: numerals, cognition, and history. *Reckonings* can inspire us to view the concept and constructs of numeracy through a historical and sociocultural lens.

Reckonings: Numerals, Cognition, and History. By Stephen Chrisomalis. The MIT Press, Cambridge, Massachusetts, 2020. (ISBN: 978-0-262-04463-9. 245 pages.)

1. Initial Considerations

Stephen Chrisomalis, a professor of anthropology at Wayne State University (Michigan, USA), has written a compelling and thoroughly entertaining account of how numbers came to be historically used and represented. In this context, the author situates numeracy within a social and cognitive frame, by viewing the peculiarity of shaping numerals by diverse groups of people as a way of communication. His book, *Reckonings*, situates numbers and mathematical knowledge as sociocultural, historical, and cognitive phenomena. In this perspective, numeracy is a sociocultural phenomenon, as well as a product of interactions between members of different cultures, whose mutual influences and interactions make constant changes in the complexity of the universe. In this regard, numbers can be viewed as mathematical objects, and numerals can be viewed as cultural artifacts of observation as well as technological tools that involve diverse ways of representing, understanding, and manipulating aspects of the daily life of these members [15].

Chrisomalis presents an engaging and accessible view of the cognitive and cultural history of numbers and numerals as well as the diffusion and spread of this knowledge from generation to generation. This approach is relevant for those who seek to understand how members of distinct cultures count and represent numbers in the present, as such understanding of contemporary representations and numeral systems needs to be rooted in an understanding of the relevant developments in the past.¹

Reckonings offers a cross-cultural cognitive analysis of numbers and written numerals over the past 5000 years; Chrisomalis investigates the relationships between individuals' cognition and the broader social, political, cultural, environmental, and economic processes within this expansive time frame. He also discusses the ways that these systems differ from each other, explaining, for example, how members of distinct cultural groups built on the ancient tally systems they used and how they developed their own writing systems.

¹ D'Ambrosio [5] affirms that instruments, objects, and artifacts such as buildings, number systems, numerical representations, musical instruments, and computers provide information about their makers and users since they were developed to meet their basic necessities, as well as provide recreation, entertainment, and ways of communication. Similarly, D'Ambrosio and D'Ambrosio [8] comment that members of distinct cultural groups develop a diversity of ways to communicate, using spoken and written language, signs, gestures, numbers, numerals, and visualization.

Reckonings acknowledges that humanity has developed more than a hundred different systems for representing numbers over the last fifty centuries.² It also offers a fresh account of the hows and whys of the ever-evolving number systems over this period of time and into the future, and in which diverse methods of numerical notation and distinct ways of writing numbers have been developed, represented, and used by the members of specific sociocultural groups. Reading this book, one gets a glimpse into how prior cultures and groups calculated and measured, often without using any written supportive machinery that could allow them to do so.

Chrisomalis considers how members of distinct cultures both developed and used numerals by reinterpreting historical and archaeological representations of numerical notations, and by exploring their implications and role in diverse societies. Through his work, he demonstrates how both numerical words and symbols interact in specific sociocultural contexts as well as how they have been used and structured through history, thus helping us to rethink our own assumptions about the evolution of numbers and its numeric representations.

As ethnomathematics scholars, it was fascinating for us to read how over millennia, people calculated physically by using devices such as manual calculators (abacus, yupanas, and quipus), and then used written notation to share the obtained results. What we read resonates with [14], where Rosa states that numeral systems, number words, numeral symbols, and numerical notations all seem to follow logical and mathematical principles whose development is influenced by the sociocultural context in which they were rooted, developed, accumulated, and diffused among and between members in their own cultural groups.

Chrisomalis in his book also shows us how historical numeral systems can be replaced and/or modified. For example, a look at the past of these systems shows that their evolution is predictable and progressive. Similarly, a look at their potential in the future shows that the development of numbering ideas, procedures, and practices is still in place. Once again, this resonates with our understanding of the essential sociocultural character of numeral systems.

² According to Steen [20], numerical concepts were developed in relation to the often unique and distinctive theoretical, social, and historical foundations across cultures, and these distinct circumstances have come to influence the decision-making processes as developed by the members of diverse cultural groups in using equally diverse numeral systems.

In [15], we comment on the importance of understanding and explaining both current and historical varieties of mathematical ideas, procedures, and practices related to numbers and numerals, as these change across time, culture of origin, race, ethnicity, gender, with interactions across cultures, and other sociocultural characteristics.

2. Reading *Reckonings* Chapter by Chapter

In his book, Chrisomalis presents research on cross-cultural anthropological methods and theories related to numeracy,³ literacy, matheracy, technoracy, and the history of anthropology. He shows that culture plays an important role in the development of numerals through history; he also makes explicit connections to theories of cognition.

Chrisomalis explores how number systems have configured, as well as been shaped by, diverse societies, and how members of distinct cultures have invented number concepts and numerical representations and notations. Through his investigations, he conveys both the joy and the importance of the development of numeral systems; this multifaceted exposition might interest a broad range of readers, educators, and researchers.

The basic organizational structure of *Reckonings* is as follows: there is an introduction and seven interlinked chapters that are both macro-historical and cross-cultural. These all are developed with a specific and particular focus on the origins, decline, and fall of the Roman numerals. These chapters all together also offer a vast bibliography related to numbers and numerals, cognition, and history, all providing good fodder for readers who wish to think deeply about what it means to be human, what it means for humanity to sustain and pass on its heritage, and so on.

³ We argue in [15] that numeracy is the skill set and habits of mind needed for the interpretation of graphs and tables, numbers, and numerals, and other ways of informing members of distinct cultures. In our opinion, even reading includes understanding the condensed language of codes. The competencies included in the construct of numeracy have much more to do with screens and buttons than with pencil and paper. There is no way to reverse this trend, just as there has been no successful censorship to prevent people from having access to books over the past 500 years. You can censor them, remove them, even burn them, but over the long term, they have tendency towards returning to the shelf! Getting information through new media and technological forms supersedes the use of pencil and paper and numeracy is achieved with calculators [3].

It is important to state here that, according to Chrisomalis, numerals, cognition, and history are the three central themes of the seven chapters that compose *Reckonings*. Chrisomalis writes, “each element of this triad is essential for understanding human numbering practices and the social context of written numbers” (page *xv*).

In the introduction, entitled *Three Reckonings*, Chrisomalis reflects on the evolution of knowledge regarding the three reckonings, which are: to reckon to calculate, to reckon to think, and to reckon to evaluate. These neatly parallel the three themes mentioned above: numerals, cognition, and history. Chrisomalis displays this parallel structure by presenting a scenario in which the scale of biological evolution and the process of cultural development must acknowledge the role of the human element in evoking the processes of selection and adaptation, as the latter innovates daily.

Chapter 1, entitled *The Limits of Numerical Cognition*, raises issues related to constraints against universals and particulars; it deals with constraining infinity and spoken and written numbers, providing an example regarding the 99% problem. In the conclusion of this chapter, Chrisomalis provides us with a valuable and interesting discussion regarding the constraints, as well as history and cognition, in which he develops a series of arguments regarding the historical evolution of numbers, numerals, and, and what he calls *Western numerals*. In particular, Chrisomalis writes that history and cognition can be integrated into a holistic view of the development of numeracy. Thus, numeracy cannot be seen as a static set of strategies related to *basic facts* that might contribute to members of diverse cultures reaching their creative potential and stimulating and facilitating common action necessary for the evolution of social coexistence, ethics, and citizenship.

Historically, scholars interested in the development of *numeracy* primarily focused on the advancement of basic mathematics skills, such as measuring and calculating [15]. Currently, however, the advancement of these skills is no longer confined to the ability to count, calculate, and master common numbers, numerals, and/or algorithms. It is important to state here that:

This limited form of mathematics is insufficient to help individuals perform their basic civic roles in society. Thus, it has become necessary to propose a new role for numeracy; one that emerges as a response to increasingly relevant sociocultural demands of a contemporary society (page 587).

Even though the specific concrete tools involved are constantly evolving, as we noted in Footnote 3, mathematization remains an important intellectual tool for the development of numeracy because it is a transformative process that translates real world phenomena into mathematical representations. It handles this translation by developing and communicating notations, representations, numbers, numerals, vocabularies, and algorithms in order to enable members of distinct cultures to generalize and make conjectures and as result to promote the development of numeracy [15].

Chapter 2, entitled *Conspicuous Computation*, builds on the framework of cognitive constraints and patterns previously outlined, which configures a rational, comprehensive, and understandable environment for the decision-making process that members of distinct cultural groups develop in response to the sociocultural contexts and the many day-to-day challenges they face. This framework helps to delineate an outline for understanding how numerical texts create meanings; Chrisomalis applies these insights to the case of Roman numerals, and thus is able to pose and address questions and problems rooted in social, cultural, and linguistic contexts.

Chrisomalis in this chapter also deals with conspicuous computation, which he defines as “the intentional use of large numbers for their visual or psychological effect on the reader” (page 42). This concept, as we can imagine, has significant effects in hierarchical and inegalitarian societies, in which numeracy is valued, yet unequally distributed among its members, because it serves as a textual tool of social control.⁴

Chapter 3, entitled *The Decline and Fall of the Roman Numerals I: Of Screws and Hammers*, evaluates the merits and demerits of numerical notations related to Roman numerals by presenting curious and peculiar facts, texts, and materials related to their use in the past. Some of this involves interesting historical details about the Roman empire. Thus, Chrisomalis shares with his readers “a number of Roman and Post-Roman medieval techniques [such as] finger arithmetic, the abacus, tallying. To this, we could add mental and verbal arithmetic, in whatever form they took” (page 72).

⁴ Here we are reminded of [4], where D’Ambrosio states that numeracy is not limited to the study of traditional mathematical concepts because it is related to critical applications of mathematical knowledge and procedures through the use of numerical, statistical, probabilistic, and measurement tools that people use to resolve their problems and situations, and by linking these competencies to ethics and citizenship in modern societies.

Today, most consider Roman numerals to be inefficient and cumbersome, and even archaic, obsolete, and awkward. It is often assumed that computation and standard calculations would be impossible if we used them. However, Chrisomalis presents examples of the use of these numerals in classical, medieval, and early modern historical contexts, showing us that in their context, they were quite feasible as notational tools for computation; people just needed different algorithms to use. And yet their applications declined and became unpopular; this too is carefully detailed by Chrisomalis.

This chapter (and perhaps the next), for those curious to learn more about Roman numerals, might be worth reading on its own. For us as mathematics educators interested in the history of mathematics and mathematical notation, it was interesting to see how for millennia, scientists and vendors alike used concrete means for calculations, again abacus, yupanas, or quipus, and used the Roman numerals to record their findings in diverse knowledge areas.

In Chapter 4, entitled *The Decline and Fall of the Roman Numerals II: Safety in Numbers*, Chrisomalis analyzes a single characteristic related to frequency and use of Roman numerals and a single outcome related to the adoption of its innovative and/or unusual usage. We also read about how these interact with three kinds of frequency dependence: frequency-dependent bias, prestige bias, and frequency-dependent and prestige biases. Here we learn that the frequency dependence of Roman numerals is related to how socio-cultural surroundings influence the decision-making process of members of distinct cultural groups to adopt a particular numeral system through *cultural dynamism*. In this context, cultural dynamism refers to the exchange of systems of knowledge that enables members of distinct cultures to exploit or adapt the world around them [14]. Thus, cultural dynamics enable the incorporation of human invention, which relates to the idea of changing the world in order to create new abilities and institutionalizing these changes. In turn, this serves as basis for the development of more skills, abilities, and competencies.

This chapter also deals with networks and frequency in communication systems by showing how spoken and written numbers are affected by frequency dependence, as well as the development of Roman and Western numerals as a case study of frequency. In particular, we know that at some point Roman numerals were supreme; “Western numerals had to come from a position of radical *infrequency*, and the process of replacing Roman numerals took centuries to complete” (page 108).

In Chapter 5, entitled *Number Crunching*, Chrisomalis examines a case study related to “the broader context of the development of numerical notation since the Western numerals achieved their present level of ubiquity (page 121). Thus, we learn about “Cherokee numerals, which is part of a numerical notation system that never came into common use, and which has actually been so sparsely attested that it is unmentioned in most of the systematic history of writing and number systems” (page 123).⁵ Through this case study, Chrisomalis tries to understand what kinds of numeral systems members of distinct cultural groups might develop and be using in the future.

Chapter 6, entitled *How to Choose a Number*, discusses the need to use the concept of modality when choosing numerical systems. This includes: a) the medium when a number is expressed both visually and auditorily and b) modes of expression that show how numbers convey meanings as well as the degree to which they claim to be representations of some underlying reality. For example, number 4 (four) can be expressed and communicated in different modalities, such as in speech, in writing as in Roman numeral IV, in Spanish (*cuatro*), in Portuguese (*quatro*), and in French (*quatre*), and in a hand with four fingers extended. In this chapter, Chrisomalis states that multimodal representations provide writers with opportunities to emphasize and facilitate specific readings by making numerals more evident. In this approach texts are used to enable a particular communicative relationship with some audience. He also discusses how different number modalities can be mixed together to give us blended modalities, in which numerical notations contain a cue or index to a lexical reading; hybrid modalities, where single numbers are written by using two or more distinct notations, such as lexical and graphic, and parallel modalities, in which the same number is written two or more times in different ways.

This chapter also explores the case in which number words and numerical notations are used in distinct contexts. This approach is classified as a sociolinguistic model, because it provides an example of code choice in which expressions are selected based on the same goals, interests, or principles. Yet, even when not used for the same purposes or in the same contexts, numerical modalities can influence each other.

⁵ We would like to emphasize that this numerical notation system is due to Cherokee polymath Sequoyah (1770–1843), who developed it alongside his celebrated syllabary.

This chapter reminded us of how, according to D’Ambrosio and Rosa [9], culture is transmitted both in space and time, through encounters, communication, and information. To develop human values we need to understand the dynamics of this transmission. Culture, as life, is not static; it is in constant evolution, through inter and intra-cultural encounters. Hence, culture is transformed and, as a consequence, values change as they are influenced mutually.

In Chapter 7, entitled *To Infinity and Beyond?*, Chrisomalis returns to reckon with the central questions posed in the book regarding cognition, history, and linguistics changes. As he writes, “holistic, comparative, and historical anthropology is witness to diachronic and social processes at a multimillennial scale” (page 208). He argues that there is:

(...) a need for a much closer coordination of archaeological and ethnographic data, not despite the fact that they are different, but because they are different, and because we need to explain that difference if we are to have any success in understanding the range of constraints on social formation. I am far more sympathetic of the call for cross-cultural comparisons of archaeological as well as ethnographic data (page 190).

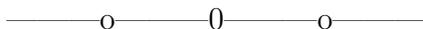
3. Some Connections and Recollections

Reckonings shows that every time we work with numbers, we are participating in a multi-millennial history of writing and communicating, as we are using numerals that evolved historically and are applied in diverse sociocultural contexts. Thus, as we wrote in [16], cross-cultural comparison can be viewed as a global approach that seeks to comprehend or explain alternative ways of doing mathematics, as well as to study the development of numbers and numerals throughout history.

Helfrich [11] notes that in order to use cross-cultural comparison effectively to compare multiple cultures, “both the objects and the standards of comparison must be equivalent across cultures” (page 132). Yet, Chrisomalis states that:

(...) dehumanizing comparativism is worse than analysis that look only at the present, because data are ripped out of any context that might be relevant to understanding them and retrofitted

to a rigid hierarchy of social standing. The problems of unilinear cultural evolutionary theories are so well in modern anthropology (...) (page 193).



This book offers us a fresh perspective on numerals and numeracy, encouraging us to view them together as one of humanity's most powerful cognitive tools, tools that involve using symbols to perform basic daily activities such as read clocks, check calendars, dial phone numbers, organize the family budget or do arithmetic.

Members of distinct cultures perceive how most of the numerals used in their own environment are applied in calculations and in representational systems taken from their own realities. Understanding how these members make these choices in relation to number systems helps us understand motivation and decision-making about advancement of numeracy in diverse sociocultural contexts.

In this context, we are reminded of D'Ambrosio, who in [3], addresses the topic of literacy, matheracy, and technoracy as a curricular proposal. This approach is imbued with holistic conceptions that offer a consistent transdisciplinarity and are compatible with various contemporary trends for pedagogical action, with communicative, analytical, material, and technological instruments that enable the development of numeracy. Let us think a bit more about this tri-partite proposal and its ingredients.

Literacy is the ability to process information by applying techniques of reading, writing, representing, and calculating in a variety of diverse media contexts [15]. It can be understood as the conglomeration of competencies related to the processing and creation of information, which include abilities such as checking prices, times, schedules, using units of measurement, performing mathematical operations, as well as numeracy competencies such as representing numbers and interpreting graphs and tables [8]. This communicative instrument is best understood as the integration of school and cultural contexts through the process of cultural dynamism. It is the ability to use and combine instruments according to needs and situations, which, from the perspective of transdisciplinarity, constitutes a way of overcoming the barriers of disciplinary relationships.

Matheracy is understood as the capability to interpret and manage signs and codes as well as the ability to both propose and use models in everyday life. It helps students develop statistical competencies that include the abilities to collect, read, understand, propose hypothesis, infer, produce and interpret data in order to assess their validity and draw conclusions [3]. It provides students with symbolic and analytical instruments that help them to develop their creativity and enable them to critically understand and solve problems and situations [8]. To clarify, as we write in [15], matheracy does not refer only to the meaning of mathematical abilities, but also to the competencies needed to interpret and act in social, cultural, political, and economic situations structured by numbers and mathematics. As such, it is an important analytical tool for the development of mathematical knowledge structures that need adequate pedagogical methodologies so that members of diverse cultures may achieve the comprehension of their own realities.

Technoracy refers to people's ability to critically use and combine different material and technological tools, from the simplest to the most complex, and their ability to evaluate possibilities and limitations in diverse everyday situations in order to make suitable decisions for themselves and for *others*. It also makes use of numeracy competencies, such as the understanding of the condensed language of codes and numbers, through the use of technology, such as calculators and computers [6] and other technological resources. Thus, as we point out in [15], technocracy can be viewed as the mastery of material and technological instruments that help the development of peoples' critical and reflexive familiarity with technology, which in turn allows them to use technological instruments in order to evaluate diverse ways to present and represent numbers, and mathematical ideas and practices, and to assess the reasonableness of the results in distinct contexts.

These three make up a solid foundation for a desirable curriculum, according to D'Ambrosio [3]. Together they support all in their individual and collective aspirations. In this context, what does Chrisomalis's book tell us? What can we learn from it?

In our minds, perhaps the most important point made in the book is regarding the diversity in the development of numbers and numerals. This in turn reveals a creative existence that resists a "false acceptance of a cultural and cognitive homogeneity" [4, page 106] that can inhibit members of distinct cultural groups from overcoming and transcending challenges they face daily.

Chrisomalis affirms that “an anthropology that aims to be relevant to contemporary life must, therefore, build on its foundations as a science of constraints, and to seek to become an anthropology of change” (page 208). In our opinion, this approach seeks to find meaning in the reality of the members of distinct culture by transcending the criteria of superiority between cultural manifestations related to the development of numeracy, which enables the establishment of a coherent relationship between numbers and numerals.

4. General Remarks

In his book *Reckonings*, Chrisomalis establishes three understandings related to reckoning: calculation, thinking, and evaluation. He wants us to recognize that numbers provide a basis for the development of social practices that energize the world, from trade and calendaring, to recipe-making and record-keeping.

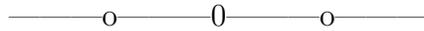
Studying the history of the development and use of numbers and numerals shows us how they were applied, as well as certain constraints on human thinking. This can help us understand, explain, share, and communicate ideas related to the development of cognition and the evolution of science and mathematics. This approach highlights how our worldview can both reflect and influence how members of diverse cultural groups make meaning of the world (as one of us pointed out in [14]).

In this regard, Chrisomalis compares arguments for the cognitive efficiency of calculations and claims for the goodness of fit between the representation and the tasks members of distinct cultural groups complete, as well as to explore the relative advantages of material technologies underlying numeral systems. His exposition reveals the cultural and political aims of advancing or transforming societies through understanding of numeracy.

Chrisomalis states that numerical cognition is related to techniques and strategies that members of distinct cultural groups are able to learn from experiments, as well as how numerals are used in everyday practices throughout history. Crucially, numerals are primarily tools for *representing* numbers and for doing arithmetic.

Chrisomalis further shows that numeration is a social practice by arguing that written numerals are conceptual tools that are transformed to fit the perceived needs of their users, and that the sorts of cognitive processes that

affect decision-making around numerical activity are complex and involve social factors. Thus, we can reckon with related scholarship in literacy, math-eracy (numeracy), and technoracy (in the sense of [3]) developed by members of distinct cultural groups who are infrequently represented in the development of mathematical knowledge.



Drawing on the triple meaning of “reckon” (which are: a) think, b) calculate, and c) evaluate) as a framing device, Chrisomalis argues that the history of numeral systems is best considered as a cognitive history of language, writing, mathematics, and technology. We agree with Andrew Simoson, who writes, in [19],

[Chrisomalis’s] threefold approach in *Reckonings* is to examine number with respect to structure, purpose, and transience, and in doing so, he cautiously avoids overgeneralizing, ever mindful that the Latin maxim *exceptio probat regulam* ought to be translated as “the exception probes the rule.” Thus, Chrisomalis habitually inserts curious richness into his narrative.

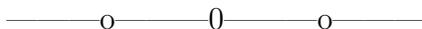
Chrisomalis roots his arguments in the practical goals of exploring the ways in which cognitive constraints interact with distinct environments such as economic, cultural, technological, historical, and the social context of science, and the cultural anthropology of cognition. In addition, he points towards relevant connections to epistemology, anthropology, history, cognitive science, and creativity.

Thus, in our point of view, this work serves both as a program for conceptualizing cognition in specific cultural, social, historical, and political contexts, and as a guide to the development of a research program and an accompanying list of pedagogical actions needed to teach numbers and numerals, as well as mathematics and history, for more comprehension and for better cross-cultural understanding.

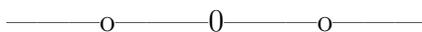
Chrisomalis has both sociologically and culturally located the coalescing interest in reckonings. He is able to explicitly point out the key role that researchers played in questioning the central dogma of universal development of numbers and their representations and values in the history of humanity.

By offering concrete reckonings, Chrisomalis makes an eloquent appeal for an entirely new approach to conceptualizing mathematical knowledge, and more generally, knowledge related to numbers and numerals, an approach that embraces the full thrust of human and cultural diversity. As such, this type of approach can address the urgent need to provide members of distinct cultural groups the necessary tools to become ethical, creative, reflective, and critical individuals prepared to participate in an emergent *glocalized society*, the context in which members of each distinct cultural group must be “empowered to act globally in its local environment” [5, page 76].

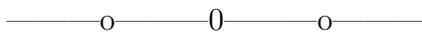
Glocalization has emerged as the new standard when it comes to reinforcing positive aspects of worldwide interaction in textual translations, localized marketing communication, sociopolitical considerations, and in the development of scientific and mathematical knowledges [17, page 197]. We appreciate that Chrisomalis recognizes that this is the case; he explicitly acknowledges the necessity to work with different cultural environments/contexts and, as acting as ethnographers, to describe the many diverse mathematical ideas, procedures, and practices in order to give meaning to these findings.



This important book encourages an awareness of historical constructions of numbers and numerals, and, through this awareness, aims to create a new understanding of numeracy, by focusing on the process of generation, intellectual and social organization, and dissemination of this kind of knowledge by establishing multiple relationships with other theories and references to reality.

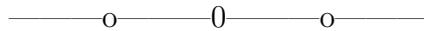


Reckonings is not a mathematics book per se. Yet, it deals with interesting technical, disciplinary, and interdisciplinary issues, themes that could be of interest to all people of mathematics, because it reflects on everyday practices of working with and manipulating numbers through their numerical representations that were developed in diverse sociocultural contexts by members of distinct cultural groups. As such, it explicitly situates mathematical activity in its social, cultural, and historical contexts, and squarely puts mathematics in the human sphere, as a clearly human activity.



One of the key arguments in this book is that numerical notations are mainly used for representation and communication. We agree that these notations can be applied in the context of writing and reading practices, as well as in mathematics, which was a motivation of the development of their concepts thought history.

Chrisomalis understands the meaning of *reckon* from calculation to general thinking. Consequently, in our point of view, he leads us to *reckon*, in a manner that involves the understanding of general cognitive processes that lead to the comprehension of reckoning as judgements. This context involves numerical notations that face their own reckoning as creators and users decide to adopt, transmit, diffuse, modify, and abandon it in order to create new numeral systems.



In *Reckonings*, Chrisomalis discusses the eventual obsolescence of numeral systems throughout history, using, specifically, the Roman numerals as a case study. Hence, he echoes an important decision-making process, a process of judgement, by which users decide to abandon a numeral system in favor of another.

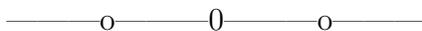
In this context, we have written elsewhere [15] that diverse systems of numeration are writing systems used for expressing numbers, which includes mathematical notations for representing numbers of a given set using digits or other symbols in a consistent manner as developed by members of distinct cultural groups. We all can easily visualize how a series of accidents probably led us to the Hindu-Arabic numerals founded on base-10 arithmetic, and how we might be thinking differently, and what direction science and commerce might have taken, if something else had arisen during the development of humanity.

Some readers may think about written numbers and more generally the numeral systems we use as a cognitive technology that each of us applies every day. Thus they may question the assumptions we are making here about the historical development of numerals as technological artifacts.

We, however, agree with D'Ambrosio [3], who views technological artifacts as devices conceived and created by humanity to solve their survival needs and/or to enable, facilitate, and transcend the development of daily activities and tasks. They are objects developed to help the fulfillment of functions

that usually seek to extend the limits of the human mind and body. As such, written numbers are clearly a technology that evolved historically.

Harper [10] states that numerals can be viewed as technological artifacts that are constituted by both form and function because they have the necessary internal structure such as symbols for numbers that must be distinct from one another and located in a sequential progression. In addition, numerals have an external structure since they are developed and situated in sociocultural contexts that confer them qualitative (used as identifiers) and quantitative (used as the identification of set sizes) functions. Historically, numbers and numerals are the product of diverse, constant, and continuous evolutionary processes, which include social and cognitive selection that help their learnability by human users in their daily routines in distinct sociocultural contexts.

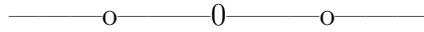


Yet, it is necessary to emphasize that the operative aspects of technology, in most cases, are inaccessible to the members of minority cultural groups. More generally, the basic ideas behind technological devices, their possibilities and dangers, the morality supporting the use of technology, are essential issues to be raised and discussed [15].

History shows that values are intimately related to technological progress. Thus, it is possible to offer the possibility of harmonious relations in human behavior and between humanity and nature. Both interactions are intrinsic to the advancement and enhancement of an *ethics of diversity* [2], which deals with:

- respect for the others (the different).
- solidarity with the others.
- cooperation with the others.

Consequently, it is not possible to relinquish our duty to cooperate, with respect and solidarity, with all human beings who have the same rights for the preservation of their own good. The essence of an ethics of diversity involves a deeper mindfulness and respect for, indeed solidarity with, and cooperation with the others. This leads to quality of life and dignity for all if knowledge acquired in distinct sociocultural contexts are equally valued and respected.



This book also considers the possibility of including new pedagogical strategies embedded in technological innovations that reflect the complexity of life and that are imbued with transdisciplinary perspectives, within an ethics of diversity. We highlight the importance of the non-linearity of the ideas presented in this book as an invitation to reflect on various themes within an exploratory investigation that is complemented in each chapter.

This context synthesizes the cycle of knowledge, as reality [natural and cultural environment] informs [stimulates, impresses] individuals and peoples who, as a result, generate knowledge to explain, understand, live with reality [4]. The problem lies in the expropriation, institutionalization, and filtering, through the power structure, of the knowledge generated and, intellectually and socially, organized, which is returned to the people, often in limited and limiting ways, even hindering their access, for their survival and servitude to power.

The ideas, as shared in *Reckonings*, offer the ability to critically and reflectively engage members from diverse cultural contexts in the development of new and innovative situations. For example, D'Ambrosio [2] signals that the difficulties inherent in creative and speculative know-how must be avoided in order to oppose the facilitation of standardizing judgment. Therefore, this approach presents myths that these members must be freed from, in order to decide in favor of progressive learning along with the development of concrete examples to facilitate their daily routine.

As mathematics educators developing investigations regarding the connection between culture and mathematics, we understand that mathematical and scientific knowledge are culturally constructed and subject to context variations. Thus, these knowledge fields are human products of particular sociocultural environments, which are related to the advancement of numbers and their numerical notations and representations.

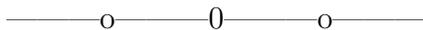
5. Final Considerations

Reckonings is an import addition to the literature in culture, history, anthropology, philosophy, and the sciences. Because this book transcribes the development of human beings who nourish essential dreams of peace, ethics, and happiness for humanity, reading it is a joy. However, this aspect does

not deny its academic character or its value to serve as a guide for formal educational praxis.

Themes covered in the chapters of this book are transversal and demand a holistic and transdisciplinary approach. Ethics, in our opinion, is a content that is coherently suggested by discourse and evidenced by action. The author's clear and consistent stance throughout the book in relation to ethical issues gives this manuscript an implicit behavioral character, indeed an authenticity.

We recall that the ethics of diversity proposed by D'Ambrosio [4] "rests on respect, solidarity and cooperation" (page 142) and turns to the conduct of peace, which must be present in the encounter with the different, as condition to the existence of life itself. As one of us wrote a while back in [14], ethics may be viewed as a regulator of human behavior, because its holistic view seeks to historically understand the human being as an individual whose knowledge is acquired for survival and transcendence.



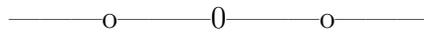
Reckonings shows that numerals must be understood in a broader context because they offer a compelling case study in how cognitive technologies are shaped by the vagaries of history, the dynamics of culture, and the constraints of the human mind. It draws on the disciplines of anthropology and linguistics, but extends its conceptions to cognitive science, archaeology, classical philology, the history, and philosophy of mathematics, evolutionary theory, ethnohistory, and semiotics.

Chrisomalis introduces general readers to topics of numerals, cognition, and history. He sets several goals that must be accomplished within a holistic comprehension of the universal (global) principles of rationality as well as the local and particular ways of understanding cultural reasonableness through a dialogical approach. He writes:

Here we are playing around two central theories in social scientific and specifically anthropological approaches. On the one hand, *universalism* is the idea that humans are basically the same, that human behaviors differs only to a small and relatively insignificant degree, and thus that findings from one part of the world can be easily extrapolated and generalized. On the other hand, *particularism* argues that the local, historical, and particular conditions

of human existence are sufficiently variable that large, generalizing explanations are grossly oversimplifying, and that we would be better to focus ourselves on the panoply of local conditions that cause particular people to behave the way they do (page 5).

This is not a small challenge! This book crosses disciplinary lines to examine language as much as mathematics, philosophy, psychology, and as well as history, but the ease with which it connects and moves through these subjects takes the readers joyfully along with interest and motivation.



In our own research with a diversity of students from diverse continents, in ethnomathematics, mathematical modelling, and ethnomodelling; we currently identify three approaches: *global*, *local*, and *glocal*. These three approaches help us to investigate, study, and understand mathematical ideas, procedures, and practices developed by members of distinct cultures. These approaches are similar to the concepts of *universalism* and *particularism* as conceptualized, defined, and studied in this book.

The *global* approach (corresponding to Chrisomalis's *universalism*) is related to the outsiders' view on beliefs, customs, and scientific and mathematical knowledge developed by the members of distinct cultural groups. In our opinion, globalization has reinforced the Euro-western-academic-utilitarian approach of school mathematics, and the Western bias in prevailing mathematical development, as well as in helping to globalize pervasive scientific ideologies.

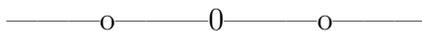
Thus, mathematics is criticized as a cultural homogenizing force, a critical filter for status, a perpetuator of mistaken illusions of certainty, and an instrument of power. In this approach, comparativist researchers attempt to describe differences among cultures. Sue and Sue [21] say that these individuals are *culturally universal*.

The *local* approach (corresponding to Chrisomalis's *particularism*) is related to the insiders' view on their own culture, customs, beliefs, and scientific and mathematical knowledge. Local knowledge has typically been tested and validated within internal contexts, often over extremely long periods of time [1]. As we wrote elsewhere [15], local knowledge creates a framework from which members of distinct cultural groups can understand and interpret the world around them.

For example, we have stated that, currently, there is a recognition of the importance of local contributions to the development of scientific and mathematical knowledge [18]. In this approach, the members of distinct cultural groups describe their culture in its own terms. Sue and Sue [21] say that these individuals are *culturally specific*.

Glocalization (cultural dynamism) represents a continuous interaction between globalization and localization related to the same phenomena [13]. It involves blending, mixing, and adapting two processes in which one component must address the local culture, and systems of values and practices [12]. In this context, it is “necessary to work with different cultural environments and, acting as ethnographers, to describe mathematical ideas and practices of other peoples. It is fundamental to give meaning to these findings” [5, page 79].

Through focusing on local knowledge first and then integrating global influences, we support members of distinct cultures and collective groups, rooted in their own local cultural traditions, but are also equipped with global knowledge, in creating a sort of localized globalization [1] of the mathematical practices developed daily in diverse contexts.

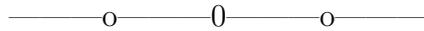


Reckonings is an ambitious book. Its philosophical content seems almost a consequence of its urgency. Nothing less is at stake than the remarking of numerals, cognition, and history.

In general, we recognize philosophy as the mental strategies (abstract) to critically understand and explain how humanity developed ways to cope with the natural and social environment (to survive) and to understand and explain reality (cosmic, natural, and emotional), mythology and ontology, aesthetic, and ethics (to transcend survival). Mathematics are strategies (concrete and abstract) developed by the human species with the same objectives. According to D’Ambrosio [7], philosophy can never be reduced to mathematics. Philosophers try to recognize the mental strategies (abstract) to critically understand and explain how individual mathematicians advanced in their strategies (concrete and abstract) to reflect upon facts and ideas about the natural, social, cosmic and emotional environment, motivated by mythology and ontology, aesthetic and ethics, and even the imaginary that transcend survival.

Thus, it is important to explore the nature of mathematical knowledge by paying attention to philosophy and cognition, in particular philosophical domains beyond those traditionally associated with the history of mathematics.

Reckonings does just that. It is a cultural and cognitive history of how humanity develops, uses, and thinks about numbers by representing them with numerals. We live in a world written by numbers, surrounded by their intuitive or practical presence. This is a relatively new phenomenon; numbers were once rare in our linguistic landscape. But nowadays they are so commonplace that we often do not feel the need to think about them in order to solve problems we face daily in our own sociocultural contexts. This is exactly what this book does! It reminds us that we *should* think about numbers.



In short, Chrisomalis has written a compelling and thoroughly entertaining account of how historically numbers came to be used and represented. The book serves as an enlightening example of how the fundamentals of knowledge cannot be understood by a single discipline; they require the complementarity of several knowledge fields such as philosophy, sociology, and education. We strongly and enthusiastically recommend this book to all readers interested in the broad understanding of human history, as well as the history of human mathematics, from these informational lenses.

The book leads its reader to the important realization that the current number system we use, just like many other aspects of our civilization, is likely not going to persist forever, and as the planet continues to evolve, and civilization moves on (forward or backward is the question these days), new adaptations to counting, numbers, and cognition will arise as tools and crises move us on.

We thank Chrisomalis for this important work. He has left us with much wonder about numbers and numerals, and the possibility that perhaps someday another numeral system will evolve. During these dark and difficult times, this book is a breath of fresh air and hope. *Muito obrigado!*

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