Walking Alone: My Career in Mathematics

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Synopsis

In this article, dictated by Maohua Le and arranged by Yongzhong Hu, Professor Le briefly recounts his legendary experience of self-study mathematics, which reflects the life experiences of his generation of Chinese people.

Keywords: self-study mathematics; memory

1

I was born to an ordinary civilian family in Shanghai on April 10, 1952. My father used to be a clerk in a company, and my mother kept house at home. At that time, we had our own house, and the family was of tolerable means. My parents had five children, four boys and a girl. Since I was the youngest in my generation — my siblings were all twenty years older than me — I had a happy childhood with the love and care of my family.

In 1956, my father died unexpectedly. This was undoubtedly a heavy blow to my family. Fortunately, at that time my elder brothers and sister had already grown up, and my father was killed in the line of duty, so my life with my mother was still secure.

Since 1958, I had been attending a primary school near my home. Because I was less interested in the stereotypical classroom teaching since I was young, my school record has always been lackluster. In addition, perhaps because of congenital reasons, I have a poor physique since childhood; I also suffer from deep myopia. At school I was required to participate in a variety of vigorous sports. Therefore, the six years of primary school life did not leave me many happy memories. On the contrary, physical and capacity defects made me always worry about how I would make a living when I grew up.

I remember there was a watch shop near my house called Hendry. Every time I passed by, I would stop and look through the vitrine to watch the master who would be repairing a watch in the shop hall. I would see him sitting motionless in station, holding the magnifying glass with his eye socket, his hands deftly fiddling with tools and parts on the table. Gradually, I began to think that this might be the suitable job for me in the future. Although my parents only attended private schools in their hometown, their attention to their childhood's education was productive. All three of my older brothers graduated from top universities in China and were engaged in technical work. By contrast, my ambition to become an apprentice in a watch shop when I grow up was a far cry from the experience of my brothers. Would my wish come true?

2

In 1964, after graduating from primary school, I was admitted to Shanghai Number 51 Middle School. The school, known before and after as Shanghai Weiyu Middle School, is also a local key middle school of some renown.

During this time, two things happened that affected my life. One was that the teaching reform in the school made me very interested in mathematics and gradually led me to embark on a road of self-study. I remember that year when I just entered junior high school. The school implemented a series of teaching reform measures, one of which was to encourage first-grade students to prepare for mathematics courses. The goal was to ask students to master the material that has not been covered in class, and to be able to complete related exercises correctly. Although this reform measure did not achieve a wide success in the end, it greatly stimulated my enthusiasm about learning as I was not interested in classroom teaching. At that time, it took me only a year to preview all the mathematics courses from Grade One to Grade Three. I independently completed all of the exercises, and was praised by the teacher.

This experience not only made me interested in mathematics, but also made me confident in my ability to learn mathematics. What is more, I gradually developed the habit of self-study, which I have continued to pursue since then.

The other thing that happened then was China's remarkable Great Proletarian Cultural Revolution (for short "Cultural Revolution"), which began in the early summer of 1966 and lasted for a decade. As is known to all, this political movement ultimately had a great negative impact on China's development, so this period of history is sometimes called the decade-long catastrophe.

I was in Grade Two when the Cultural Revolution began. As a result, schools were closed indefinitely. I dropped out of school and never had a formal education again. In spite of this, the interest in mathematics I developed in school did not wane. In the three years after dropping out, I not only continued to self-study the mathematics courses in high school and college, but also applied the knowledge I learned to solve problems outside of textbooks. I remember that I used the method of finding the extremum in advanced mathematics to solve a problem on the optimal distribution of fuel for a kind of multistage rocket. I was not yet sixteen when I wrote my first article on this topic. I continued to learn without a teacher and enjoy myself. My progress in the process of self-study helped me gain a lot of sense of achievement. From then on, I made up my mind: I'd make something of myself in the field of mathematics, no matter where I was going.

3

Since dropping out of school, my social identity had changed from a student to an educated youth. In the spring of 1969, I left Shanghai with a huge contingent of educated youth and was sent to the countryside two thousand kilometers away to work as a farmer. For those of us who grew up in a big city, life in rural China was very hard. I lived in the country for nearly three years. During this period, I never gave up on self-study, but due to the unfavorable conditions, I made little progress.

At the end of 1971, I got a chance to be transferred from the countryside to a factory in Changchun as a worker. I worked in that factory for thirteen years. Compared with the countryside, the conditions in the city were much better. Here, in addition to the security of my daily life, there were first-class universities and large public libraries to provide me with favorable learning

conditions. I was no longer satisfied with learning the knowledge of books and working out the exercises in textbooks, but wanted to try my skills through discussing the same problems with experts, so I began to pay attention to learn about and determine the research subjects that would be suitable for myself by exporing various mathematics publications.

In the 1960s and 1970s, Professor Luogeng Hua, a famous Chinese mathematician, responded to the call that science and technology must serve industrial and agricultural production, and led his disciples to go into grassroots factories and mines to popularise mathematical methods such as the overall planning method and the optimization method. His action was once praised by the top leader of the time (see [19]). In this series of works, there is a handing wheel problem about machine tool processing. In 1975, three domestic experts proposed a solution to this problem in their paper [1]. Shortly afterwards, I used the properties of number theory about the best fractions of real numbers to come up with a new and more efficient solution, and I corrected some of the errors in the handouts that promoted the optimization method. I wrote up these results and sent them to the relevant journals, but I never got a reply.

In order to verify whether my results were correct, I visited the Department of Mathematics of Jilin University with apprehension. In the political environment at that time, the university people were very polite to me, a young worker from the factory. They told me to leave my manuscript with them and promised to ask the teachers in the department to review it. Soon after, on the recommendation of Professor Guochen Feng of the department, my paper was published in the *Journal of Jilin University*, which had just been revived after the end of the Cultural Revolution (see [9]). This was my first published academic paper. In the following years, I published several papers in important domestic academic journals such as the *Chinese Bulletin of Science* and the *Acta Mathematica Sinica*. These achievements further strengthened my confidence and courage in mathematical research.

The year 1976 was a turning point in modern Chinese history. The Cultural Revolution officially ended, with the national economy on the verge of collapse, and the changes ushered in a hope of revival. Also in 1976, I fell in love with a woman who was also working in Changchun, and we got married. My wife understood my obsession with mathematics. She took on all the housework after work, so that I could have more time to study mathematics.

Three years later, we had a daughter and we were happy as a family.

In 1977, China resumed university enrollment. Since I was little interested in learning in the classroom from childhood, I still choose to study mathematics by myself.

4

Since the earliest known numbers are integers, number theory, which discusses a variety of properties of integers, is one of the oldest branches of mathematics. Known as the prince of mathematics, the German mathematical master C. F. Gauss once literally compared mathematics to the queen of science, and number theory to her crown. Since I had successfully solved the hanging wheel problem with the method of number theory in 1970s, I focused my self-study on number theory and made the solution of equations discussed in number theory as my research goal.

As is known to all, mathematics is a science that studies the quantitative relations and spatial forms of objective things. In mathematics, the identities that describe equality are the most common quantitative relationships; an equality that contains unknowns is called an equation. Therefore, the specific content of many mathematical research projects is to solve various equations.

As early as the third century of the Common Era], the ancient Greek mathematician Diophantus had extensively discussed the equations that limit unknowns to positive integers. Since then, such equations have been referred to as Diophantine equations. In addition, since there are usually more unknowns in Diophantine equations than there are equations to solve them completely, such equations are also called indeterminate equations. Diophantine equations make up an important branch of number theory; many famous problems in number theory can be summed up as solving problems of this kind of equations. For example, the seventeenth century French mathematician P. Fermat proposed that, for any positive integer n with $n > 2$, the sum of nth powers of two positive integers cannot be the nth power of another positive integer. In the language of Diophantine equations, the above proposition can be expressed as,

if n is a positive integer with $n > 2$, then the equation

$$a^n + b^n = c^n \qquad (1)$$

has no positive integer solutions (a, b, c).

For more than three hundred years after it was put forward, the proposition has attracted the attention of countless mathematicians and mathematics enthusiasts with its simple form and profound connotation. In those days I first became interested in Diophantine equations by Fermat's proposition.

Due to the long history of Diophantine equations, predecessors have carried out a large number of systematic studies in this field and achieved many results. So if you want to build on that, you have to innovate. My road to innovation was tortuous and long. Around 1980, I completely gave the formulas for general solutions of a class of exponential Diophantine equations by using the properties of the representation of integers by binary quadratic primitive forms. Not only does this result provide a unified method for solving many Diophantine equations concerned by people, but also it allows us to overcome some shortcomings of classical methods. At first, my above result could not be published in important academic journals in China, so I had to publish them in the journals in my universities (see [10] and [11]). Ten years later, the English version of this paper was published in the *Journal of Number Theory*, an important international journal in the field of number theory (see [12] and [3]).

Later I used these results of mine to solve an equation problem about Pythagorean triples. This problem was an important part of a famous conjecture put forward by Polish mathematician L. Jeśmanowicz in 1956. A team led by Professor Zhao Ke of Sichuan University did a lot of research on this problem, but only solved some very special cases (see [8]). In 1981, I used my method to completely solve this problem. However, in the process of submitting to domestic journals, the attitude of reviewers was disappointing. Far from understanding the key ideas of my method, the reviewer simply asserted rudely that it was wrong. Although, through the editor of the journal, I kept repeating my academic views to the reviewer and refuting his errors, my efforts did not change anything in the end. The paper was not published until forty years later, and then only abroad (see [2]). This experience made me start losing faith in some of the so-called academic authorities in China.

5

1984 was a turning point in my life. At that time, a government official described my self-study situation to his leader. After assessing my learning and my results via the relevant departments, the state transferred me from the

factory to Changchun Teachers College (it changed its name to Changchun Normal University in 2013) to teach (see [20]). Since then, I have worked as a mathematical professional.

When I first arrived at the college, people wondered whether I, whose formal education did not go beyond that of a junior high school student, could be qualified to be a university teacher. Before long, however, the facts proved that those fears were misplaced. Although I had not taken classes since I dropped out of junior high school, and I have not had the opportunity to teach before, either, my understanding of mathematics enabled me to play freely in the class. Especially after I unexpectedly found a counterexample of an exercise in the textbook (see [14]), my colleagues recognized my professional foundation more.

At the end of the Cultural Revolution, when higher education in China was limited in both quantity and quality, there was a big push to encourage people to develop their talents through self-study to make up for shortages in various areas. In 1985, the state awarded the first self-taught staff award, in recognition of one hundred self-taught outstanding individuals and I was lucky enough to be among them. At the award ceremony, I got to know Professor Weixuan Li, who also became an expert in operations research through self-study of mathematics. Similar aspirations and experiences mahlde us feel like old friends at the first meeting.

In my first few years in the university, although the work was very smooth, life was not satisfactory. The treatment of Chinese university teachers is largely linked to their professional title, so title acquisition and promotion naturally become major events in their career. In Chinese universities, the professional titles of teachers has been divided into four grades. From the highest to the lowest are (full) professor, associate professor, lecturer and teaching assistant. And each grade is divided further into several levels. In the three years I worked at Changchun Teachers College, I didn't even have a teaching assistant title, so you can imagine how bad my situation was. At that time, I wrote confidently in my career training plan that I would self-study to become a professor in ten years. Faced with surprised and skeptical looks from colleagues, I secretly determined that this goal must be achieved, and that this goal would certainly be achieved.

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Since I had neither the educational background nor any of the other formal qualifications, the only way to get the promotion of the professional title I desired would have to rely on outstanding scientific research achievements. As a researcher of basic mathematics, my scientific research achievements include academic papers, research projects and scientific awards. Obviously, of these three aspects, the former is the basis of the latter two. Therefore, under the current system, publishing as many papers as possible in high quality journals would be the key to success.

So, how could I determine, in the vast sea of journals such as the sea of smoke, which are high quality journals? The Chinese thought of using the American *Science Citation Index* (shortened for SCI) to screen out undesirable journals. As a result, the journals included by SCI have become the first choice for Chinese authors to post their papers. Although the authorities have criticized this political direction after three decades, its objectivity is still recognized today.

Since 1989, I have published many papers in *Acta Arithmetica*, *Journal of Number Theory*, *Proceedings of the American Mathematical Society*, *Transactions of the American Mathematical Society*, *Journal für Reine und Angewandte Mathematik* and other important international mathematical journals. Although summarizing this experience is easy, it took more than a little courage to write papers and submit them to foreign journals with just only a few sentences of pidgin English I had learned in junior high school. In 1992, my research project received its first grant from the National Natural Science Foundation of China (shortened for NSFC).

In 1994, I was transferred to Zhanjiang Normal University (it changed its name to Lingnan Normal University in 2014) and appointed as a professor. So far, I had completed the growth process from a worker to a professor.

7

I have worked in Zhanjiang Normal University for eighteen years. It is been a stable period in my career. During this period, I was either lecturing or doing research every day, fulfilling my professorial duties step by step, and living a relaxed and happy life.

As Zhanjiang Normal University is a local university, the teaching task of the teachers is rather heavy. I usually would teach twelve classes a week, not

including many correspondence courses. Since the University had not yet obtained the qualification to recruit graduate students, my teaching subjects were all undergraduates, so although I spent a lot of time in class, I felt very relaxed.

Due to my own interest, and also because universities all over the country were vigorously carrying out the quantitative management of scientific research and implementing a corresponding reward system at that time, I spent most of my time outside of teaching on research. During those eighteen years, I presided over and completed seven projects funded by NSFC; six hlwere general projects and one was a special project for monograph publishing. As the research results of these projects, I have published more than six hundred papers and one monograph; more than one hundred of my papers have been included in SCI.[1] Part of my independent research results won the second prize of Science and Technology Award of Guangdong Province.

Because of these reasons, I have won the special government allowance issued by the State Council, and been awarded the National Advanced Worker, the National May Day Labor Medal, the National Outstanding Teacher and other honorary titles.

In addition to doing research and teaching, I inevitably took on some administrative work. I served as the dean of the College of Mathematics and Computational Science for eight years. Although this type of work took a significant amount of my energy, fortunately it also provided a rich social experience, which made it a lot more manageable than expected.

Since the end of the last century, the various reform and opening policies in China have brought about a rapid growth of social wealth. Domestic universities are getting richer and better resourced, and inter-university travel has become more common and frequent. Personally, I believe that multiple unnecessary visits and meetings are both a waste of funds and time. Because of my few social engagements, I have become an eccentric maverick in the eyes of many domestic colleagues.

In 2012, at the age of sixty, I retired honorably as a second level professor, but my career in mathematics did not end there.

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[1] A complete list of my publications is included in the article supplement, available at https://scholarship.claremont.edu/jhm/vol13/iss1/20/.

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My life after retirement is pleasant. Freed from the constraints of various assessments and the boring disputes over personnel, I finally have complete freedom to control my own time and settle down to do what I want to do.

In the last decade, I have written a handful of survey articles summarizing my own work and that of others (see [15], [16] and [17]). Years of practice made me realize that it is very important for me to fully understand the history and current situation of domestic and foreign research in related fields while working on my own basic mathematics research. Although there are now international mathematical literature retrieval tools such as *MathSciNet* (initially known as *Mathematical Reviews*) and *Zbl Math* (initially known as *Zentralblatt fur Mathematikand*), due to the rapid increase in the number of published papers, it is difficult to gain a solid and complete overview of a particular topic from a large number of scattered information. Therefore, I think it is a worthy endeavor to spend a lot of time writing such surveys, both for other scholars and for myself.

While summarizing previous research, I sequentially went on to publish some new results together with my collaborators. A good example of this is the progress we have made in our work on the ternary purely exponential Diophantine equations. Let a, b, c be fixed coprime positive integers with $\min\{a, b, c\} > 1$. Since the equation

$$a^x + b^y = c^z \tag{2}$$

is related to the generalized form of Equation (1) in Fermat's proposition, this is a basic and important type of purely exponential Diophantine equations. Further let $N(a, b, c)$ denote the number of positive integer solutions (x, y, z) of Equation (2). As early as 1933, German mathematician K. Mahler used his p-adic analogue of Diophantine approximation method to prove that for any triple (a, b, c), $N(a, b, c)$ is always finite [18]. However, his result is ineffective; that is, this result does not give us an effective computable upper bound for $N(a, b, c)$. In 1940, Russian mathematician A. O. Gel'fond used his original method of transcendental number theory to give the first effective result [4]. His method was later greatly extended by English mathematician A. Baker and became known as Gel'fond-Baker method, and it has a wide rang of applications in Diophantine equations (see [13]). Following studies have made the upper bound for $N(a, b, c)$ gradually smaller (see [16]).

Around 2015, combining the Gel'fond-Baker method with some elementary techniques of number theory, I and my collaborator analyzed the upper bound for the solutions and the lower bound for the distance between different solutions of Equation (2), and proved that if $\max\{a, b, c\} > 10^{62}$, then

$$N(a, b, c) \leq 2 \tag{3}$$

(see [5], [6] and [7]). Note that, if $(a, b, c) = (2, 2^r - 1, 2^r + 1)$, where r is a positive integer with $r > 1$, then Equation (2) has at least two solutions $(x, y, z) = (1, 1, 1)$ and $(r + 2, 2, 2)$. So we have $N(2, 2^r - 1, 2^r + 1) \geq 2$. It implies that there exist infinitely many triples (a, b, c) with

$$N(a, b, c) \geq 2. \tag{4}$$

Since we know from Equation (4) that it is impossible to improve the upper bound Equation (3), our result gives the best upper bound for $N(a, b, c)$ in general.

Through the years, I have recorded my learning experience in the process of studying mathematics. Most of the notes that could stand on their own as papers have already been published. I have not yet had the time to sort out the rest of the content, which includes tentative ideas and conjectures about some unsolved problems. I intend to put these documents together in my lifetime and share them with my friends and others who might be interested in them.

Time flies and years have gone by. I have unknowingly entered the twilight years. Although I am getting weaker in health, I will continue to work hard to move forward and live up to God's gift to me for as long as I can.

9

To recap, I was born in Shanghai of China, and I am approaching seventy years of age. For historical reasons, I dropped out of middle school after only two years. After I entered the society, I have worked as a farmer in the countryside and then a factory worker. During this time, mathematics has always been my hobby. Later, I became a professional worker of mathematics through self-study, and stayed there until I retired. Looking back on the past, I feel a lot of emotions. If it is a great blessing for a person to be able to do what he is interested in and be able to live this life, I should be one of the lucky people.

Acknowledgement

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