

“I Got You”: Centering Identities and Humanness in Collaborations Between Mathematics Educators and Mathematicians

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Synopsis

Existing literature widely reports on the value of collaborations between mathematicians and mathematics educators, and also how complex those collaborations can be. In this paper, we report on four collaborations that sought to address what mathematics is and who gets to do it. Drawing on the literature and from the careful and intentional work of the collaborators, we offer a framework to capture the richness of those collaborations — one that acknowledges the importance of acknowledging and welcoming the extensive personal and professional experience of each person involved in the collaboration — and a look at how collaborations built with that intentionality and acknowledgment can be impactful for students and institutions and be personally and professionally rewarding for the collaborators.

1. Introduction

In August 2019, the Center for Inquiry and Equity in Mathematics (CIEM, <https://www.ciemathematics.com>) brought together twenty-one professors of mathematics education, seven professors of mathematics from the Center for Minorities in the Mathematical Sciences (<http://minoritymath.org>), and three additional project staff from Education Development Center to spend a week immersed in questions of mathematics and equity [11]. The faculty then collaborated in a variety of ways over the academic year.

The vision of the CIEM project was to address a persistent problem in mathematics about what mathematics is and who gets to do it. Mathematics continues to be widely seen as a static body of knowledge that gets delivered to students. This misperception limits the potential for students to experience the real nature of the discipline, including developing their own mathematics questions. This situation is exacerbated for students of color, partly because the ways in which mathematicians of color have been minoritized has led to a dearth of visible mathematicians of color, which then reinforces cultural narratives that mathematics is not positioned as a space for BIPOC people. Such a pervasive, culturally embedded, insidious idea has real and inequitable consequences for students. So when the project originally envisioned collaborations between mathematicians and educators, the PIs imagined the faculty co-creating opportunities for preservice teachers to disrupt the “who” and the “what” of mathematics with preservice teacher classroom visits from mathematicians of color. One limitation of the original design is that it did not envision mathematics educators visiting mathematicians’ classrooms — in a redesign, the PIs have acknowledged they would incorporate more bidirectionality.

The external evaluator (Dr. Anne M. Marshall) was uniquely positioned to observe several of the collaborations that emerged. In this paper, we draw on evaluation data from the CIEM project and report on four collaborations that occurred between mathematics educators and mathematicians across the 2019-2020 school year. The evaluator personally observed three of the four collaborations described below, either virtually or in-person. The fourth collaboration is a first-person account by two of the collaborators.

2. Framework

Research highlights the benefits of collaboration among higher education faculty. In addition to such collaborations having the potential to be invigorating for those involved, they may also increase productivity and lead to more successful programs [6, 12]. However, as most faculty can attest to, collaboration with colleagues is not a simple endeavor. Successful collaborative efforts take time, commitment, trust, and work by all involved. When working across different departments and/or disciplines on a campus, faculty must also navigate issues and tensions that arise due to differences in discipline knowledge, beliefs, teaching styles, resources allocation, power and status hierarchies, etc. [6, 12, 17]. For example, mathematics educators

and mathematicians may hold different views of mathematics, teaching and learning, and have different personal histories with mathematics. These differences may make collaborative efforts between them even more complex. For faculty collaborative efforts to succeed, members of the collaborative must be committed to shared, co-created goals, the collaborative process, and each other [6, 12, 17].

With these considerations, collaborations between mathematicians and educators should not be entered into lightly, but rather with care and intentionality. An extensive body of literature about collaborations involving mathematicians and educators in the United States illustrates the ways in which those collaborations have been valuable but also complex (e.g., [10, 2, 14, 3, 16]). One such article [15] offers four necessary conditions (M.A.T.H.) for successful collaborations between mathematicians and mathematics educators (page 53):

- M: Motivation to collaborate
- A: Acknowledgement of the strengths of each collaborator
- T: Trust that the motives of each collaborator involve improving student learning
- H: Helpfulness of both collaborators in reaching mutual goals

Articles in [7] and [16] describe outcomes from similarly collaborative efforts. However, there is a gap in the literature in attending to how race and gender intersect with issues of professional status, culture, and standards of practice within collaborations between mathematicians and mathematics educators. For example, in the M.A.T.H. conditions above, words like *trust* or *helpfulness* can read very differently when viewed from personal and professional culture, gender, or racial lenses.

This article illustrates four examples of how leaning into that complexity — instead of running away from it — can have enormous benefits to the faculty themselves, to their students, to the fields of mathematics and education, and beyond. We posit that the four conditions named above are necessary, but yet insufficient when considering collaborations where diverse identities are present and seen as critical to the richness of the collaboration. While our collaboration stories do offer insight into the necessary conditions

of the M.A.T.H. framework offered by [15], we have found that the following conditions more appropriately capture the richness of collaborations that occurred as a result of CIEM.

- M: Motivation to collaborate is about both improving the discipline *and* student learning
- A: Acknowledgement of the expertise and strengths of each collaborator
- T: Thrive on multiple identities of each collaborator (this includes but is not limited to gender, race, professional status, etc.)
- H: Humanness should remain at the center of the collaboration

In what follows, we present, as already mentioned, four collaboration vignettes. Each vignette begins with a brief description of the collaboration followed by faculty dialogue from reflective conversations that occurred post-collaboration. Then we connect a condition of our updated M.A.T.H. framework that was particularly evidenced in that collaboration. While each of the four collaborations embodied multiple conditions of our proposed M.A.T.H. framework, we choose to highlight one condition in each vignette. For example, we found multiple examples across collaborations where collaborators acknowledged the expertise and strengths of their collaboration partner (A) or demonstrated how humanness was central to the collaboration (H).

Two important things to note: First, these collaborations took place after a range of interactions — in-person at the CIEM workshop, phone calls, coffee dates, emails, texts, Zoom sessions, etc. They did not occur without relationship building to varying degrees. What is presented here is a snapshot of each collaboration and does not necessarily reflect the time invested for faculty to build trust and establish shared goals for their collaboration. Second, it was a collective decision by the authors to use professional titles in the introduction of faculty collaborators and then to use first names in the narrative and dialogues that follow. We acknowledge the depth of historical racism that many faculty of color have faced, and the fight for professional titles and status as a result. Therefore, we introduce readers to the faculty with their professional titles. However, as shown throughout the interactions, the humanness and personal relationships that emerged over the life of the collaborations engendered the use of first names between collaborators.

3. Collaborations

3.1. Dr. Pamela Harris & Dr. Steven Greenstein

This collaboration occurred between Dr. Pamela Harris, then Associate Professor of Mathematics at Williams College, and Dr. Steven Greenstein, Associate Professor of Mathematics Education at Montclair State University. This collaboration occurred at Montclair State University. Drs. Harris and Greenstein collaborated to provide an experience for students in Dr. Greenstein’s *Critical Mathematical Inquiry* graduate course. Students in the class were exploring the teaching and learning of mathematics through a critical perspective.

The Collaboration The set of interactions between Pamela and Steven took place in the Fall semester of 2019, before the COVID-19 pandemic. Steven and Pamela built structures that would allow them to develop relationships online. It was decided that Pamela would visit the class via Zoom. Prior to the Zoom session, Steven had his students watch Pamela’s video-taped MAA talk, “A Mathematical Journey of Culture, Community, and Collaboration” (available online at <https://www.pamelaeharris.com/post/maa-invited-address-video>) about her journey of becoming a mathematician and her experiences as a Latinx mathematician. She talked about her journey as a student and the challenges she overcame as a woman of color with a passion for STEM. After Steven’s students watched the videos, they sent short thank yous to Pamela, about what they saw in her video, or how her video affected them. Pamela reflected on that opportunity for students to learn about her and interact with her prior to her Zoom session: “I think it’s really great — they knew about me — and it would be awkward for me to just show up and do the math . . . so we built a relationship.”

This set of interactions created a space in which Pamela *could* build that relationship quickly enough that her feedback was far more meaningful than it might have been without those prior interactions. Pamela created multiple avenues online for students to get to know her without even bringing her to the classroom.

During the Zoom session, students had brought lesson plans and ideas for critical mathematics lesson plans. The lesson plans ranged in their focus including climate change, distribution of wealth, health care, immigration, etc. Steven provided a space in the corner of the room for students to have

one-on-one meetings with Pamela over Zoom. Students were able to share their lessons and ask questions about both math content and a range of critical mathematical/social justice contexts. Pamela was positioned as a consultant with mathematical expertise and she provided feedback to students on their social-justice lesson plans. The conversations varied in length and several students were vocal in how much their thinking was impacted by the conversation, thus impacting how they were going to revise their lesson plans.

Some students did not have a completed lesson plan to discuss, but wanted to engage in a conversation with Pamela to get some ideas and help focus their thinking. Pamela offered both mathematical and contextual suggestions for the lesson plans but prompted students to think about the impact that conversations about social inequities might have on the students in their classes. For example, while a lesson focused on the mathematics behind the root causes of homelessness could be powerful, it could also be traumatic for students in the room who might currently be experiencing homelessness or who have experienced homelessness.

More generally, Pamela encouraged students to ask themselves the question, “Who benefits during a lesson like this?” It was evident that she made a positive impact on how the students thought about creating and implementing lesson plans that integrated mathematics and social justice. Several students expressed a desire for Pamela to talk to their K-12 students. After the class, Steven shared with Anne that Pamela had not only really pushed students’ thinking about inquiry and equity but also that she had an impact on his own thinking and teaching. He specifically shared how the collaboration had impacted the design and direction of his course.

Her feedback was typically along two dimensions: first, looking at the mathematical content, and second, pushing students to think about how their social justice lessons — while well intentioned — might affect students who are in their classrooms. Pamela had a lot of feedback for students, her overall report was that she was “very pleased to learn how many discussions pre-service teachers are having about social justice and mathematics. It gives me hope for the next generation of mathematicians and how they will be better prepared to tackle the hard problems of addressing inequity in the mathematical sciences.”

Faculty Reflections on the Collaboration

Steven:

I feel like all the benefit went from Pamela to me. I don't know what I contributed to the collaboration, but I guess I shouldn't speak for Pamela. This collaboration with Pamela came at the right time when I was teaching a social justice course—she visited and had resources. I don't know how that gets framed as like a math-math educator kind of collaboration, but it's just two people who care about the work collaborating together.

Pamela:

The first thought I had, which is interesting because Steven said, “Well, maybe the only benefit was mine and maybe the students’.” And it's funny how when we collaborate, you never know what ends up happening afterwards. And so for me, I kept thinking about how powerful that [the experience] was to have somebody who's external to a class to come in and just share some space with students. It gives students a completely different perspective than what we can provide, because we're in the room with them all the time. In fact, I've now implemented this very thing in my own teaching. My class that I'm teaching right now is a research course in graph theory. I do some graph theory, but I'm an amateur, so to speak, in that realm of mathematics. I found some small amount of funding and I've paid people to come in and be external mentors for the groups. I picked five external mentors and they each created a video in advance of the class starting. The students watched all the videos and then picked who they wanted to work with and what kinds of math problems they wanted to do based on that asynchronous video that the mentors provided. The mentors check in with the students once a month, just a one hour meeting once a month to do the kind of thing that we did, which was, “. . . here's the thing we're thinking about . . .” or “We're kind of stuck here”.

. . . And the thing that's been really powerful about that experience is that it taught me that I don't always have to be the expert in the room, which of course I know that in theory, but in practice is very different because I still work at a predominantly

white institution. Society is what it is. I'm never thought to be the expert. And so it's hard for me to step back and be able to build a classroom in which I can be comfortable being uncomfortable and not knowing things. This was a new experience for me and the students are thriving. They're proving conjectures that these mathematicians provided, and then we're excited. And I feel really engaged because it's content that I don't know. I don't understand it. I don't get to be the one that holds the knowledge. So we're co-creating as we go, but very much this was inspired by our collaboration and I just revved it up. Because I was like, this was great. How can I leverage this in my own teaching?

Steven:

Yeah. I've been thinking as I got to know Pamela through this workshop. I didn't know her before and then followed her—she's published a book. I've been paying attention to what's been going on, but I think I do a lot more reflection and too little action. Maybe practice can be collaborative. We don't have to do all of it, because I have a collaborator here who can supplement my action steps for me when I don't know how to do them, or I'm not doing them or I don't have to take on that entire trajectory.

Connecting to a Condition of M.A.T.H.: Motivation to Collaborate is About Both Improving the Discipline *and* Student Learning

Again, it's worth noting the ways in which this collaboration as well as the other collaborations could illustrate any of the four elements of the revised M.A.T.H. framework, but we will focus this analysis on the "M" — motivation to collaborate is about improving the discipline AND student learning. In this interaction, each collaborator reported feeling like they were the ones reaping the benefits of the collaboration — they both expressed surprise that the benefits were experienced by the other person (and in Steven's case, their students). But when Steven described the collaboration, he said, "I don't know how that gets framed as like a math, math educator kind of collaboration, but it's just two people who care about the work collaborating together."

The primary motivation for collaboration, and their primary roles as collaborators were "*two people who care about the work.*" They came to the collaboration as people who care about the work, and as experts, but they

also came as learners—this stance reflects that motivation of caring about improving the discipline and student learning. Pamela expressed learning from Steven’s modeling of vulnerability in his expertise—he brought in an outside expert mathematician because he thought his students would benefit, both from her mathematical expertise, from her personal experience, and from her ways of thinking about how students would experience their curricular choices, particularly around choosing social-justice oriented problem settings. She then thought about how she could draw on outside expertise in her own classroom. Steven also acknowledges the enormity of the task of preparing teachers, and describes Pamela as a person “with resources.” What they can offer students is greater than what they can offer individually, and this provides motivation to collaborate.

It’s worth noting that the students experienced this collaboration as beneficial, too. The students in Steven’s classroom reported benefitting from this collaboration, and from having two experts with different perspectives support their thinking about the intersection of equity and mathematics. Several students reached out to Pamela outside of her visit because they were eager to learn more with her.

3.2. Dr. Theodore Chao & Dr. Terrance Pendleton

Dr. Theodore (Teddy) Chao, Chinese American mathematics education faculty at Ohio State University, collaborated with Dr. Terrance Pendleton, Black mathematics faculty at Drake University, to co-facilitate a class session in Dr. Chao’s undergraduate Elementary Mathematics Methods course. While the course was offered through Ohio State University, the course was held onsite at a local elementary school in Columbus, Ohio. Dr. Chao and Dr. Pendleton then repeated the morning activities with teachers during a professional development session in the afternoon.

The Collaboration Teddy and Terrance co-planned during several virtual sessions leading up to the onsite visit. Teddy shared the goals and objectives of the course, described his students and the context in which they lived, and also how he hoped Terrance might share and interact with students. Because opioid addiction is rampant in Ohio, Terrance relied on his applied mathematics background to create authentic modeling activities for prospective teachers (PSTs) and in-service teachers about opioid use in Ohio. He also talked about his own pathway to mathematics and his own research.

The two shared their research foci and interests. In addition, they had more personal discussions about their lived experiences. Together, they planned for an activity that highlighted Terrance's applied mathematics expertise and was also relevant to the students' lives.

Terrance began the activity by sharing his background and personal story of becoming a mathematician. He mentioned having failed Algebra in middle school and what inspired him to persevere and where he found passion in mathematics. Then, Terrance and Teddy co-facilitated an activity on data related to opioid addiction, a topic that was of great concern to the PSTs, many of whom were from parts of Ohio that had been affected by the opioid crisis. PSTs were invited to make predictions about a data set and later engaged in doing some data calculations and drawing graphs of the data. Teddy and Terrance encouraged them to think about what they might do when the data no longer fits a hypothesis/model. The focus of the following conversations was on how to "mathematize" new data as it was shared. During the whole group discussion, Terrance told the PSTs, "You guys approached it like undergraduate mathematicians...you made some predictions based on how the numbers are growing and used averages to look for patterns." Teddy added that they were engaged in reading and writing the world with mathematics (see [9]), an idea that the class had been talking about throughout the semester. Teddy and Terrance exemplified a powerful co-teaching relationship between a mathematician and mathematics education.

Teddy's math methods students, the PSTs, worked with 5th grade "Buckeye Buddies" at the elementary school. The school, located near the University, serves a historically low-income community with an extremely diverse student population (36% Black, 27% Latinx, 22% White; 90% eligible for free or reduced lunch). After the data activity, the methods students collaborated with their Buckeye Buddies in small groups. Teddy and Terrance circulated to watch the math methods students work with fifth graders. Terrance was able to talk to several Buckeye Buddy groups and with their fifth grade teacher about the math they were doing together in groups and about the mathematics that Terrance engages in as a mathematician.

In the afternoon, Teddy and Terrance facilitated a professional development session for the in-service teachers in the building. They repeated the mathematics activities conducted in the morning and also included some time for reflections and conversations about the lesson. At the end of the activity, a teacher asked Terrance, "What jobs do your grad students get after they

leave you? Because my fifth grade students always want to know, ‘Why do I have to learn this?’” Terrance shared a wide range of the jobs that his former students have taken on — and that they make lots of money in jobs that mathematize the world. Several weeks later, Teddy shared, “I don’t know how I would have done this (referring to the collaborative session) on my own. It was great. Some of the in-service teachers have been telling their own kiddos about Terrance and his story.”

Faculty Reflections on the Collaboration

Terrance:

But for me, it did have quite a tremendous personal impact. It allowed me to interact with groups of people that I had never considered interacting with including young kids. I think it’s been more than a decade since I stepped foot in an elementary school or middle school. And so I never get to really see how kids of that age get to process math. And then in talking to future math educators who are going to go in that classroom and then having to kind of stand up there and try to convince them that maybe some of the things that I do, might affect how you teach mathematics to students of that age.

And Teddy was very instrumental in helping me, I had an idea in mind since I was in Ohio and we were thinking about the opioid crisis. And I was trying to think of ways of how to mathematize that and turn this into a way that you can really see how math is being applied to these real-world situations. And Teddy said something that I essentially I’ve stolen from him now and I use it now all the time about *reading and writing the world with mathematics* [Teddy referencing *Reading and Writing the World with Mathematics* by Rico Gutstein [9]]. And he used that phrase, I was like, “This is amazing. I’m taking this now and I’m going to use it.”

And so anytime I talk about applied math in my class now, that’s what I go back to. And the way that together we were able to redesign that activity in a way that things spoke better to what pre-service educators were used to doing in regards to math and the way that they were used to thinking about it. And in the way that they could perhaps translate it to an actual, real third,

fourth or fifth grade math class. That really kind of empowered me in making this kind of bold claim that anyone can be a mathematician. . . . I don't know, I think it helped me reaffirm the question, "Who gets the opportunity to do math?"

Teddy:

Interacting with you helped me understand, when you do mathematics and you're in a mathematics department, the focus is more on the doing and the presenting the very particular way of thinking of mathematics, as opposed to, the work that I do, which is often, I call it evangelizing mathematics, right? We're basically trying to convince people that they are also mathematical people too. And maybe interacting with you helped me realize just in particular what the differences are in the ways that we're positioned, and at two different universities and in two different sorts of departments and how that sort of works, to me, it seems like two sides of the same coin.

But I keep thinking, there still isn't a huge canon of just great go-to activities that I think teachers can feel they have access to, a lot of these, the teachers find them from connecting to someone like us, or connecting to someone in a professional development session. So I wonder...the stuff that Terrance and I are thinking about, it's not the activity itself that I think is the magic, it's the fact that we're connecting and talking. And then from that connection, comes this magical activity and then the focus on what is the core nifty part about the mathematics. And it is the core nifty part that we want our students to focus in on that develops that ability to say, "I am experiencing a lot of power and fun here from engaging in this mathematical activity."

Terrance:

It was so much fun. They were talking about things I wasn't even thinking about or hadn't considered. And I was like, "Oh, that's really a good idea. I need to go back to the drawing board and think about that some more." But yeah, those pre-service teachers were just, I mean, they were love. That is best way I can describe it. They were just an incredible group and made me feel very comfortable. And I felt like I was able to resonate with them a bit. I don't know, it was a great, great experience.

Teddy:

I want to hear about that, because Terrance you keep bringing up the fact that it's been a long time since you've been in the classroom. And I wonder about that magic, right? Things happen when we're around kids, or things happened when we were in schools. Things happened when we just physically were in those same spaces together, and yet, how hard is it to sometimes make just that aspect happen? I think about it. When's the last time I was... in my mind, I keep thinking, "I'll go hang out in the math department."

Terrance:

And I'll admit, before this experience, I don't want to say, I didn't think I belonged in that space, I just didn't think I would do well in that space because I was so afraid. I don't know how to talk to a kid about... the closest kid that I know is my niece who's thousands of miles away from me and I still barely know how to talk to her. And so, I don't know, there was some trepidation there, it feels like. I'm so used to being in this one kind of environment, and I've been working on trying to, I guess, have a genuine math conversations no matter what experience or where you come from mathematically.

Teddy:

Yeah, I feel it. I mean, I think a lot of things came together really well with your visit and those things need to be unpacked. Because I've been on so many projects where we put great people together and it still doesn't work right. So I think like one, I would say Terrance, the fact that you are a mathematician, you carry yourself with a lot of humility. So that when I'm talking to you, I don't feel as if I have to prove my math chops to you. Whereas other mathematicians I've talked to sometimes, there is this, I don't want to say arrogance, but there is this sort of like, they see me and they see my math level and we're not on equal footing when we're having this conversation. I think that's an important part of it.

Connecting to a Condition of M.A.T.H.: Acknowledgement of the Expertise and Strengths of Each Collaborator Throughout their dialogue, we see the ways in which both collaborators were motivated to work together, thrived in drawing on each other's identities as mathematicians, mathematics educators, and people of color, and kept humanness at the center of the collaboration. They found a lot of joy working with each other. Teddy and Terrance acknowledged each other's strengths, illustrating the "A" of the MATH framework. Teddy shared, "I don't know how I would have done this [the collaborative session] on my own. It was great. Some of the in-service teachers have been telling their own kiddos about Terrance and his story." Terrance has also echoed that sentiment in reflecting back on the collaboration: it was work that was made much better because of the professional strengths that each brought to the collaboration.

Teddy notes that Terrance did not try to "assess" his mathematical knowledge, or enter into the collaboration as the all-knowing mathematical expert. He says, "you carry yourself with a lot of humility," as opposed to other experiences he has had, in which mathematicians see an educator, "and they see my math level and we're not on equal footing when we're having this conversation. I think that's an important part of it."

Terrance points to three aspects of Teddy's expertise: first, using language that resonated with Terrance about an experience: "Teddy said something that I essentially I've stolen from him now and I use it now all the time about *reading and writing the world with mathematics*. And he used that phrase, I was like, 'This is amazing. I'm taking this now and I'm going to use it.'" And he does use it. Terrance also points to the collaborative nature of building the curriculum for work with pre-service teachers, in-service teachers, and children: that they had an idea for what they wanted to investigate (the opioid crisis in Ohio), and they worked together to build the materials for the session. Terrance doesn't diminish his own expertise as an applied mathematician, but he also looks to the expertise that Teddy has as an educator, especially when he notes that "Teddy was instrumental in helping me."

Finally, Teddy has access to populations that Terrance would not normally be able to reach easily — elementary school students and teachers — and ways to think about making good use of that access to bring new ways of thinking about what mathematics is and who does it into schools. Each of Teddy and Terrance brought significant expertise, experience, perspectives,

and knowledge to the collaboration, and each worked in a spirit of acknowledging each other’s strengths.

3.3. Dr. Michael Young & Dr. Mollie Appelgate

This collaboration occurred between Dr. Michael Young, a Black mathematician in the Mathematics Department and Dr. Mollie Appelgate, a white mathematics educator in the School of Education. Both faculty were at Iowa State University at the time of the collaboration. This collaboration involved Dr. Young visiting one of Dr. Appelgate’s elementary mathematics methods courses.

The collaboration During the semester, Mollie had been talking about the kinds of mathematics that mathematicians do, and thinking in particular about teaching children to develop mathematicians’ perspectives on mathematics.

One of the big ideas she addressed was that mathematicians get to think creatively [5]. So when Mollie was preparing the students for Michael’s visit, she connected his visit to their recent work on what it means to do mathematics like a mathematician. She told them, “Today we are going to have a guest professor from the mathematics department come in and talk about math education with us.”

She then asked the students to think about three questions, all related to Michael’s identity as a mathematician:

- If you could ask a university mathematician and professor a question, what would you like to ask?
- Thinking as a future mathematics teacher, what would you like to ask?
- What would you like to know so that you can share it with future kids?

After the students thought about those questions, she asked them another:

- He’s also Black — does that change any questions you might have?

In other words, she didn’t try to pretend that his identity as a Black man was not relevant. She heard one of the students remark what might have been in the minds of many students: “It kind of does; does that make me racist?”

Then Michael visited the classroom, and students asked him questions about his life as a mathematician. He shared stories about working with K-12 students as a professional mathematician, and as a volunteer in his own daughter's classroom. He also talked about being an elementary school student, and what he enjoyed, and how teachers positioned him. The students were really interested in that — they could relate those stories to their careers, because they see themselves as future elementary school teachers.

This visit mattered because in the education department, white and female students are very overrepresented — fewer than 10% of the students are people of color. So to have a Black man share his perspective was useful.

In preparing the students for Michael's visit, Mollie made different aspects of Michael's identity as a Black male mathematician who sometimes works in schools visible. This created space for the students to ask about different aspects of his experience that would be relevant to their careers as elementary school teachers.

The school of education at Iowa State made a collective decision to have students call all instructors "Dr. . . ." for equity reasons. But when Michael came in, he said, "Call me Michael." He let the students know what to call him, even though that reflected norms from a different department. It's worth noting that those norms *were* different and needed to be acknowledged—one more way in which surfacing things that might be hidden (even what we call each other!) was worth the time it took to do so.

Faculty Reflections on the Collaboration

Mollie:

My fear is that I don't have as much to offer as Michael does.

Michael:

Don't have as much what?

Mollie:

I don't have as much to offer you (as you offered me) in our collaboration.

Michael:

That's false. You want to know what I need right now?

Mollie:

Sure.

Michael:

I need... who's the DOGE over there in the school of education right now?

Mollie:

We have no DOGE.

Michael:

Okay, so *I need you* to be DOGE. [DOGE stands for Director of Graduate Education. A DOGE coordinates and oversees administration of graduate degree programs.] So I have lots of students that are interested in co-majoring with education and some of them have tried to start taking classes. But someone told our students to stop registering for the classes and that it wasn't likely that they'd be able to do co-majors. But I've got a lot of students-

Mollie

Wait, why?

Michael:

These are mostly minority students who are interested in doing a co-major there in Education. Like our students are required to pass four qualifying exams, and many of our minority students are unsuccessful, however, if they pass two qualifying exams and set up a co-major, they'll get the math PhD, right? And so we've had many students pass two who wanted to do an education co-major. But we need a DOGE there to help make that happen.

Mollie:

So wait, they're PhD students?

Michael:

Yeah.

Mollie:

Wait, wait, wait, wait. I just want to understand this better. So they're PhD students, and then they would co-major, like get a math education PhD with a math PhD?

Okay, yeah. The reason this is falling through the cracks is because we have no DOGE.

Michael:

And then we would get lots of people who would be doing math PhDs and already doing stuff in the education space.

Mollie:

We need to do this ASAP.

Michael:

But wait, first you have to say, "I do have something that I could give to Dr. Young."

Mollie:

I do have something . . .

Connecting to a condition of M.A.T.H.: Thrive on Multiple Identities of Each Collaborator (Including Gender, Race, Professional Status) Michael and Mollie chose to thrive on their multiple identities throughout the collaboration. We see how Mollie positioned Michael's identity in her classroom as first a mathematician and then also a Black man. His visit surfaced important conversations and questions about how identity matters (e.g. when the student questioned . . . "*It kind of does, does that make me racist?*"). Mollie's students were affected by Michael's visit and were inspired by the mathematics that he shared. Mollie used the classroom visit to expand her students' understandings of what mathematics is, what it means to do mathematics, and who does mathematics. In return, Michael reported learning about the work of mathematics educators as a result of the collaboration, which in turn affected the advocacy work he engaged in (both local and state level committees that affect math education policy that impacts students, school, and teachers).

Months after the collaboration, Michael and Mollie were able to imagine potential avenues for impactful collaborative work on campus and in their shared state of residence. Michael leaned into Mollie's professional identity in the School of Education as an asset for the needs of doctoral students of color in the mathematics department. This collaboration started with one class of elementary math methods classrooms in mind. However, that joint work grew into more complex conversations that could lead to greater and more effective collaboration between departments. The role of multiple identities between the collaborations was key in making it powerful and effective.

3.4. Dr. Jennifer A. Wolfe & Dr. Kamuela Yong

This collaboration occurred between Dr. Jennifer A. Wolfe, a biracial Thai American and Associate Professor of Mathematics Education at The University of Arizona and Dr. Kamuela E. Yong, a Native Hawaiian and an applied mathematician at University of Hawai'i–West O'ahu. This collaboration occurred entirely virtually. Dr. Yong collaborated with Dr. Wolfe to design a talk for Dr. Wolfe's Noyce seminar students. While the talk was offered through the University of Arizona, the talk was open to the broader community.

The Collaboration Those who attend the University of Arizona Noyce seminars are typically undergraduates who are future middle school and high school teachers. Undergraduate STEM majors were invited as a way to potentially recruit them into teaching, so the mathematics of the talk was geared toward early undergraduates. Prior to the talk, Kamuela and Jen met several times over Zoom to talk about possible content and the needs of the group, all while fully attending to the current realities of the COVID-19 pandemic. The evaluator was invited and attended two of the planning meetings. Jen and Kamuela worked together to create a meaningful opportunity for students to learn about the mathematics behind the COVID-19 pandemic by drawing on Kamuela's experience and expertise in mathematical epidemiology.

The Zoom talk invitation was extended to CIEM leadership, CIEM fellows, and students of CIEM fellows. In the beginning of the talk, Kamuela shared his own experiences in becoming a mathematician and the first Native Hawaiian to earn a PhD in applied mathematics. This was followed by a lecture on the mathematics of COVID, including but not limited to how to understand and calculate the Case Fatality Rate and Flattening the Covid Curve. There were opportunities for students and attendees to interact and participate throughout the talk. It was evident that participants were engaged by the level of interaction, responses, and discussion. The timing of the talk was incredibly responsive to the concerns of all participants given the COVID-19 pandemic during late Spring. The talk was recorded and shared with CIEM leadership and fellows. The collaboration exemplified how to use mathematics to help students better understand relevant real-world mathematics. It was also a powerful example of using a virtual space for teaching and learning.

Faculty Reflection on the Collaboration

Jen:

Kamuela's expertise in infectious diseases was really timely, because he was going to give his talk in the spring and we were all transitioning into teaching remotely because of everything that happened with COVID. There was a lot of uncertainty, I think, in the general public but with my undergraduates, lots of us didn't really understand some of the terminology and things that were coming out in the media. What does this mean? Why should we wear a mask, not wear a mask? How does the disease spread? Why is it that this is a little bit different than your typical flu? He was able to break it down for them and take something that seems very complicated and telling the story where you can understand what's happening, and then from there take action. That's something that I felt he was able to do — to present this work to the audience and then the audience was also left with, "Okay, I have more knowledge, but I also have an idea of an action or what I could do, to do my part to not spread the disease."

When we collaborated, he drafted his whole entire talk and then we met and then we went through the talk and talked about, "Do we really need to talk about this part? What if we had more visuals here? Do we want this part to be interactive?" Through those conversations and then being a participant in the seminar, I felt like I got a lot out of that.

I'd also say that . . . It's kind of my piece of trying to be engaging in activism by telling my story, because I don't look like what you think about when you think about a math educator or someone who can do mathematics. I think Kamuela has had those experiences as well. I think that having seen two people that do this work and they're both identified as people of color, doing this is huge just because they're seeing people that look like them. Right? Representation, I mean, it's critical. Then sharing those stories, I think is another piece of that, so ways that we're able to share our stories out into the community is important. I mean, I feel like that's part of the collaboration, I think that that made for the collaboration a little bit more at ease.

Kamuela:

One thing I want to add too, is that this is at the start of our remote learning process and kind of joining in, seeing how Jen was working Google Docs or the slides and having all that collaboratively, posing questions and having people work on that. It was a nice learning opportunity for me to just kind of be in her classroom and see how she ran things with her students to be there at that moment too. I was learning from that.

Jen:

One thing I'm finding through this collaboration, just in talking now, is that often I don't realize the impact that I have. I think that just hearing Kamuela say the ways that the collaboration kind of helped with his own teaching, but that it spurred even more action. I had no idea that after this and putting everything together and tightening up the presentation, he was like, "Well, this is good to go, let me get it out there. Let me start the series." That's another level of not only did the collaboration help just us, but there was action that was taken that got disseminated and used in other ways that I wasn't aware of, so that's awesome . . . the collaboration not only helped us in different ways, but it also ended up having an impact on a broader community.

Because I think it definitely helps, in working with Kamuela, that I feel like he takes it from a very humanizing perspective, he thinks about mathematics in a humanizing way, we don't see that a lot in, at least, my current environment. I don't see that piece of it, right. That you're not taking into account people's identities and their experiences, their lived experiences, into the space. I feel like, at least, that that is there and present.

Kamuela:

I did like that. I was kind of making mental notes back when some of them were in person, "Oh, I can do this with my students and whatnot," and then when we were with the remote learning on using Google as a tool to have students work collaboratively and just even having students work collaboratively, right, I mean, in many math classes that's a foreign thing, to have students work collaboratively on problems. It was just something that I really appreciated having that opportunity to learn from.

Jen:

I think that that's really important, I think when we think about the collaborative process because I had that in mind too. I know Kamuela was like, "Oh my gosh, she wants to meet a week before" but we were making the transition to online, right, it wasn't going to be in person. We knew he was probably going to do an online thing, anyway, but it felt, I guess, more urgent because we were all transitioning to online learning. I don't know if I came at it just from just being human, I wanted to make sure he knew I had him. When he was going to go into the space, he didn't have to worry. I can manage the chat, I could send him to breakout groups. We could have this transition, so he could really focus on the interaction and presenting the content in collaboration with that group.

That was actually a part of why I wanted to meet the week ahead of time, to look at what he was thinking about presenting but also let him know he wasn't alone in that. It's hard enough to make those transitions and maybe he was much more fluid than maybe I knew, but I just wanted to make sure that if there were ways that I could support during the talk that he felt like he had someone who was managing certain aspects of the space so that it did feel like it was collaborative in that sense there.

Kamuela:

That's good because at that time we're all just starting to learn Zoom, and if you're sharing a screen can you see the chat? She explained how she used Google Docs in real time with her students. She offered if you want to do a poll, I can manage this whole poll. If you want to pose a question and send them into a little breakout, I can create the breakouts for you. I can moderate all this. It gave me a lot to think about on how I want to do that. I didn't quite feel comfortable or necessarily had the materials to run a breakout so we didn't do that, but Jen kind of was planting all these seeds saying like, "I can do this. I can make this available. We can do these other things." Planting seeds on things that I could do, but as well as for me professionally, like, "Oh, you can do breakouts and you can do this and then they can make it more . . . add their comments."

Again, mind you, this is the second week of April, so we're moving to remote teaching. Those are possibilities I didn't even know you could do online. I did learn a lot and in hindsight I don't have problems but where you just like, "I got to have a talk ready a week in advance." I distinctly remember when I was a graduate student and I was giving a talk and people are like, "Yeah, I just finished my talk last night." And they're like, "Yeah, me too." And this one was like, "Yeah, last time I got my talk together this early was when I was in grad school." And I go, "Well, I'm a grad student, so I had my talk last week." But it was just kind of funny, I chuckled to myself thinking like, "I mean, that was a while ago."

Jen:

Kamuella knowing a little bit about my story and me knowing a little bit about his story made us much more comfortable about sharing certain aspects of our journey. I think that that's part of it too. Because I am in a math department, but even though I'm a math educator in a math department, there aren't, one, a lot of scholars of color in the department and certainly the mathematicians, well, they have a lot of work they need to do on their own identities. I think you could see a lot of that just coming out in the things with AMS and MAA in the last year. It was definitely very healing, I think, in some ways to be in community with the Center, right. Because you got to see other people that it's like, "Okay, I'm not alone in this journey." I think that definitely contributed to the collaboration.

I think it shows a lot of the way that the collaboration was approached in the mutual kind of respect that you had for what was happening. I mean, to some degree it feels like a successful collaboration, from my perspective. If I felt like, "Kamuella got something from me and I got something from him." Right? There was a mutual kind of beneficial reciprocity between the two of us getting things and growing from the experience. I think if it's one directional, that doesn't feel collaborative and it doesn't feel like we can then go the next step and feel like we've gotten growth out of that, I guess. I'm just kind of talking.

Connecting to a Condition of M.A.T.H.: Humanness Should Remain at the Center of the Collaboration In her reflection on the collaboration, Jen stated, “I don’t know if I came at it just from just being human, I wanted to make sure he knew *I had him*. When he was going to go into the space, he didn’t have to worry.” This moment shines a light on the humanness of their collaboration. The evaluator had observed differences in approaches to preparation and implementation for the talk, which could be attributed to personal preferences, styles or discipline culture. However, both Jen and Kamuela were committed to centering each other and compromised on their preparation styles and worked together to create powerful learning experiences for students.

One thing that stands out about this collaboration, as well as in many of the others, is the extent to which the collaboration positively affected both people in the collaboration. Both Jen and Kamuela reported on the mutuality of impact. Kamuela also shared, “after that experience, I was able to go and do this workshop and I wasn’t able to do that before (reflecting specifically on integrating technology and interactive opportunities for students), and it’s because Jen helped me put together the presentation”. Their collaboration inspired Kamuela to develop a COVID-19 speaker series at his university (see <https://westoahu.hawaii.edu/ekamakanihou/?p=12235>). He attributed much of his feeling successful because of Jen’s support and knowledge.

We see humanness throughout this vignette. Specifically, the humility of both Jen and Kamuela as they admitted learning from each other, expressed appreciation for the collaborative work and each other, shared stories of each other’s identity, showed both vulnerability and mutual respect, and encouraged each other.

4. Concluding Thoughts

The four CIEM collaborations highlighted here offer a broad definition of possibility; One mathematician supporting PSTs in writing social justice mathematics lesson plans for K–12 students, a mathematics educator and mathematician co-teaching a math lesson taught to offer preservice teachers and in-service teachers an opportunity to do mathematics in authentic ways, in a context that was relevant to people in their state, a mathematician visiting a class of prospective teachers to share some of his story, and a math educator supporting a mathematician to give a widely shared mathematics talk on the mathematics of Covid-19.

Some believe that mathematicians and math educators should work together because of the value to students and to the discipline, and this is widely recommended in the literature cited in the introduction. We agree and can certainly imagine all the ways that these collaborations are good for students. But witnessing and reflecting on the CIEM collaborations, we believe these interactions, in addition to being good for students, are also good for both of these disciplines, school and campus communities, and individual faculty members. These collaborations have affected each faculty member differently. Evidence emerged that the collaborations had an impact on professional learning, personal learning, and on the identities of the collaborators.

Historically, lines have been heavily drawn between the two communities — mathematics and mathematics education. The reflective conversations illuminated how our professional communities still sometimes operate so differently. But the conversations also showed evidence that the collaborations had a significant impact on the individuals and communities where the collaborations belonged. The symbiosis that emerged from these collaborations are a starting point for what is possible and offer a model of critical professional development. As one of the faculty collaborator reflected, “I keep thinking about how special these collaborations are. They’re much more than just synergy. They involve loving people who are focused heavily on making the world and the people around them better, and who had an opportunity to create a shared bond. I think this relationship and outlook is crucial, otherwise these partnerships just descend into the standard professional collaboration.”

The collaborations and their broad effects are a reminder to see what is possible beyond the original vision of this project. Entering into the CIEM project, the shape of faculty collaborations was not clearly defined — and most likely was imagined to look most similar to that of Steven and Pamela’s. That is, a mathematician and mathematics educator working together to plan a ‘visit to a classroom’ — although we could not have foreseen the layered richness and value of that particular collaboration. What we learned ourselves, and what we hoped has come through in this article, is some of the magic that happens when collaborators from education and mathematics come together through shared motivation and purposefulness, acknowledgement of different expertise and experience, welcoming of identities, and centering each other’s human spirit.

The process of reflection and interaction with the collaborations has allowed us to see beyond the boundaries of the initial collaborations. That is, we have seen several ripple effects. Because of the initial collaboration, Terrance gave a colloquium that was open to Teddy's department and consulted with the Dean of the College of Education and Human Ecology. Kamuela developed additional talks and made them available to a broader audience, including his entire home state via the local library system. Pamela regularly invites outside experts to her classroom to broaden the expertise that her students see as part of their typical classroom learning. And mathematics graduate students at Iowa State have broader avenues open to them for academic investigation.

We wonder and are optimistic about the potential ripple effects if more mathematicians and educators had opportunities to work side by side, with care and attention to creating effective collaborations. The possibilities for creating richer and more equitable learning environments for our students are breathtaking.

In the introduction [8] to the 2018 NCTM Annual Perspectives in Mathematics Education volume, Rochelle Gutiérrez wrote, "Beyond being seen as a legitimate participant (a "doer" of mathematics), a student should be able to feel whole as a person — to draw upon all of their cultural and linguistic resources — while participating in school mathematics." We see a parallel in these collaborations: each collaborator was seen not just as a "participant" in the collaboration, or "doer of mathematics," but as a whole person, drawing on extensive personal and professional resources to benefit students who are learning — and learning to teach — mathematics.

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