

Ethics and Mathematics – Some Observations Fifty Years Later

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Ethics and Mathematics – Some Observations Fifty Years Later

Cover Page Footnote

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Ethics and Mathematics –
Some Observations Fifty Years Later
(In memoriam Friedrich Kambartel (1935-2022))

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Abstract

Almost exactly fifty years ago, Friedrich Kambartel, in his classic essay “Ethics and Mathematics,” did pioneering work in an intellectual environment that almost self-evidently assumed a strict separation of the two fields. In our first section we summarize and discuss that classical paper. The following two sections are devoted to complement and contrast Kambartel’s picture. In particular, the second section is devoted to ethical aspects of the indirect and direct mathematization of modern societies. The final section gives a short categorization of various philosophical positions with respect to the rationality of ethics and the mutual relation between ethics and mathematics.

Keywords: ethics, mathematics, Friedrich Kambartel, application

Almost exactly fifty years ago, Friedrich Kambartel — German mathematician and philosopher from Münster, first professor of philosophy at the newly founded University of Konstanz, later on professor in Frankfurt — presented his reflections under this very title, “Ethics and Mathematics.” His considerations later appeared in print in a quite influential anthology, *Rehabilitation der Praktischen Philosophie* [5]. As in Kambartel’s times, I suspect, the topic still sounds quite “unfamiliar to today’s ears” [page 489]. In fact, it is still a topic of great importance and meriting greater consideration than it has received. However, the “boundary conditions” and the focus of attention have shifted significantly between 1972 and 2022, and so it may be enlightening to report on and discuss Kambartel’s thoughts.

The main concern of this article is exactly this: to recall an essay that deals with the subject in an exemplary, though not exhaustive, manner and which, regrettably, has received far too little attention (Section 1). Moreover, I will sketch some observations on a field Kambartel hardly mentions at all: ethical aspects of the application of mathematics (Section 2). The final Section 3 is devoted to some very brief comments on another gap in Kambartel’s considerations, namely the (history of the) philosophical dispute about the orienting role of mathematical reasoning for ethics.

1. Friedrich Kambartel’s perspective on Ethics and Mathematics

After a brief note that the history of philosophy certainly knows phases — most prominently represented by Plato — of a close relationship between ethics and mathematics (see our Section 3), Kambartel reconstructs the historical origin of a strict “separation thesis” (*Trennungsthese*) in Max Weber’s (1864-1920) demand for a “freedom from value” (*Wertfreiheit*) in the sciences and in its radicalization by the positivist philosophy of the Vienna Circle. In the fight against attempts at ideological appropriation in the formation of the then-young field of sociology, Weber pleaded for

recognizing that the determination of facts, the determination of mathematical and logical facts [...] on the one hand, and on the other hand the answering of the question about the *value* of culture and its individual contents and how one should *act* within the cultural community — that these are both entirely *heterogeneous* problems.¹

If one detaches his conception from its historical context, ethics can be excluded from the canon of sciences, indeed from rationality in general. This ultimately led to the idea that there was an

¹ Cf. his well-known essay “Wissenschaft als Beruf” [22, pages 543–544]: “Nun kann man niemandem wissenschaftlich vordemonstrieren, was seine Pflicht als akademischer Lehrer sei. Verlangen kann man von ihm nur die intellektuelle Rechtschaffenheit: einzusehen, daß Tatsachenfeststellung, Feststellung mathematischer oder logischer Sachverhalte oder der inneren Struktur von Kulturgütern einerseits, und andererseits die Beantwortung der Frage nach dem *Wert* der Kultur und ihrer einzelnen Inhalte und danach: wie man innerhalb der Kulturgemeinschaft und der politischen Verbände *handeln* solle, — daß dies beides ganz und gar *heterogene* Probleme sind.”

insurmountable boundary between scientific considerations and arguments for the foundation of our actions (...) and that this boundary separates reason from irrationality at the same time.²

There is simply no place for ethics, e.g., in Rudolf Carnap's (1891-1970) system of sciences, since ethics neither belongs to its empirical branch nor to the formal or logical branch (including mathematics). Outside these two, however, in Carnap's view there is no rationality. Kambartel quotes the resulting conviction in the words of Alfred Jules Ayer (1910-1989):

There cannot be such a thing as ethical science, if by ethical science one means the elaboration of a true system of morals. [5, page 491]

But how can we understand statements about ethical demands then? Charles L. Stevenson (1908-1979) reconstructs these from a linguistic perspective and claims that ethical statements are just misleading by their grammatical form. They pretend to be the expression of some objective claim. In fact, however, following Stevenson, ethical claims are just *suggestions* to act in some way desired by the speaker and they are of no cognitive or rational content at all. Following Kambartel, all these efforts had "far reaching influence on Anglo-Saxon ethics" (page 492). Within the picture sketched above, mathematics is — like any other science — a piece of "value-free argumentation" (page 492) and it is strictly separated from any form of ethics, since no moral statements have any cognitive or rational content.

However, Kambartel recommends here a more detailed analysis. Indeed, a closer look at the history of mathematics and at its current form reveals deeper reasons for the strict separation outlined above and may also point to ways to overcome it. He first mentions some historical episodes illustrating the development from the Euclidean ideal of mathematics as an axiomatic-deductive science — based on true principles and aiming to prove true theorems — towards the modern conception of mathematics as a purely formal science. In particular, he mentions the debates about the parallel postulate

² [5, page 491]: "(...) es bestehe eine an der Basis unüberwindliche Grenze zwischen wissenschaftlichen Überlegungen und Argumenten zur Fundierung unsres Handelns (...) und diese Grenze scheidet zugleich Vernunft von Irrationalität."

in the 19th century and the controversy between David Hilbert (1862-1943) and Gottlob Frege (1848-1925) about the status of mathematical axioms. In Kambartel's view, the result of this development is a radically formalist conception: mathematical axioms only have the status of purely formal statements (*Aussageformen*) without any claim to be true, and

the mathematical reasoning consists now only in making understandable the steps, which lead from first formal expressions to further expressions of that formal character. Whoever makes such deductions, the mathematician according to a currently widespread view, no longer considers himself responsible for the choice of the axioms, only for the proper execution of the “formal” steps of deduction.³

An even more radical approach could also choose the rules of logical derivation or the rules of formal ‘proof’ almost completely arbitrarily and without further justification. Kambartel claims that this view of mathematics is following the mainstream of his own time. We might call it a strictly binary caricature: It is based on a completely free or arbitrary decision about axioms (and rules of derivation) on the one hand, and on the absolute necessity or validity of the derivations within that system on the other — provided that these are correctly formed. Whether that picture holds true for the real mathematics in its historical form of any time is of secondary importance here. For Kambartel's purpose it is only important to observe its implications for the discourse about any possible connections between ethics and mathematics. Indeed, in this picture, the mathematician is only responsible for the correctness of the derivations constructed within the arbitrarily chosen formal system, while the *choice* of the formal system itself is made without any reference to external norms whatsoever. If, however, this shape of mathematics were to become the pattern of scientific work in general, any claim to

³ [5, page 496]: “Hier ist es nun so, daß das mathematische Begründen jetzt nur noch im Einsichtigmachen der Schritte besteht, die von ersten formalen Ausdrücken zu weiteren solchen Ausdrücken führen. Wer solche Deduktionen tätigt, ex officio nach einer heute weit verbreiteten Ansicht der Mathematiker, hält sich für die Wahl der Axiome nicht mehr für verantwortlich, nur noch für die ordnungsmäßige Ausführung der "formalen" Deduktionsschritte.”

reason would automatically be excluded from the discussion of moral questions since “moral questions are of a substantive nature and therefore cannot be reduced to statements about sign operations” (page 497). Summing up, the preliminary punch line of his reflections is the following:

The orientation towards the formal-deductive understanding of mathematical argumentation prevailing today has the consequence that for the field of ethics it is precisely not the Socratic-Platonic, but the Sophistic judgment that appears valid.⁴

However, Kambartel underlines that it appears far too convenient if a scientific system, which is after all extremely privileged and is subsidized by society, refers to purely internal, “esoteric” (page 497) standards for evaluating and justifying its own actions. Although mathematics mostly does so this is hardly convincing. At this point he mentions two different points for an *external* justification sometimes made by mathematicians. First, mathematics is claimed to have an aesthetic value, to be “beautiful” and, second, research in pure mathematics is “pragmatically,” albeit in a naive way (page 498), justified by the possibility of useful external applications. The first claim, however, is qualified by Kambartel to be hardly comprehensible outside mathematics and the second to be almost empty of real content. While it is not logically impossible for a piece of mathematics to be applicable at some point, this assertion is almost trivial. Summing up: given the embarrassment over good reasons to justify one’s scientific practice, it is therefore hardly surprising, but nonetheless unacceptable, that one should fall to radically excluding the question of such reasons from the realm of the scientific.

After this critique, Kambartel presents a sketch of his own alternative conception “to make intelligible again the kind of reason inherent in ethics and mathematics alike” (page 498). His goal is therefore twofold: first, he must demonstrate a cognitive or rational content in ethical discourse. Second, he must show at least some aspects of normativity within mathematics.⁵

⁴ [5, page 497]: “[S]o hat denn die Orientierung an dem heute vorherrschenden formal-deduktiven Verständnis der mathematischen Argumentationen zur Folge, daß für den Bereich der Ethik gerade nicht das Sokratisch-Platonische, sondern das sophistische Urteil gültig erscheint.”

⁵ Shortly after Kambartel, Hilary Putnam argues on a quite similar line (see his later

Concerning the first goal, he presents a “proposal for a trans-subjective understanding of ethical arguments” (page 498). The task is thus to show that ethical statements and arguments are not merely subjective suggestions to act in some ‘desirable way,’ but that they have an objective and rationally justifiable content. To fulfill that task he sketches essentially a variant of *discourse ethics*.⁶ Roughly said, in this variant of basic ethical theories one formulates only abstract rules for the common discourse and defends these to be necessary. This ‘ideal discourse’ on the given ethical questions is then supposed to ensure that the ‘ethically required’ is found. Kambartel follows the traces of Jürgen Habermas (b. 1929) to reflect on the common ground for any rational discourse aiming at some kind of consent. This leads to principles, which he summarizes in the following way:

and encompassing work [20]. Their common starting point is the critique of the positivist strict division between ‘objective’ statements about facts and ‘merely subjective’ statements about (ethical) values. Beyond Kambartel’s analysis, Putnam intends to show that this strict dichotomy has strong parallels with the sometimes equally over-interpreted distinction between analytic and synthetic judgments (he leaves it at very short references to Immanuel Kant with the remark that his philosophy is too complex to sketch it in his book). Accordingly, he wants to perform an operation similar to that of Willard Van Orman Quine, who showed that the strict separation of analytic and synthetic judgments is meaningless. Again like Kambartel, Putnam outlines the historical genesis of positions in favor of a strict fact/value dichotomy, but this time starting with David Hume. Later on he gives a careful and critical examination of Carnap’s position — mostly based on an analysis of the (natural) language. Putnam then takes great pains to highlight the differences between his own position and Habermas’ (a major point of Putnam’s seems to be whether values exist in an objective sense). Again, in contrast to Kambartel, who bases much of his argumentation on the historical development of mathematics, in Putnam’s picture mathematics comes explicitly into play only as Kant’s predominant example of synthetic, however, *a priori* knowledge. The field of application for Putnam’s theoretical considerations is economics (reference is made especially to Amartya Sen), which he elaborates in great detail. There is not enough space here for a more detailed appreciation, but there seem to be strong points of contact with my own intention (see point (C) in Section 2). Unfortunately, there is no reference to Kambartel in Putnam’s book. The author thanks the anonymous referee for pointing out the similarity of Kambartel’s and Putnam’s efforts.

⁶ Kambartel explicitly refers to the work of Paul Lorenzen (1915-1994), who presented with Oswald Schwemmer (b. 1941) a joint justification of ethics and logic. They reconstruct logical conclusion and ethical argumentation by standardized dialogues: provable or justifiable are exactly those propositions for which there is a safe ‘winning strategy’ in such ‘dialogues’; cf. [7].

According to the terminological determinations made, it can be said when a dialogue should be called rational, namely exactly when it is unbiased, unconstrained, and non-persuasive. In the terms of Habermas one could speak in this case also of an *undistorted* or *ideal communication situation*. The consent of all participants obtained with the help of a rational dialogue is called *rationally obtained*.⁷

In the above formulation it already becomes clear that the subject of such ideal discourses may well be of an ethical nature, but that this need not be the case. Thus, mathematics is dependent on corresponding starting points of a similar character, e. g., one inevitable prerequisite is the common ground of our natural language and of the communicative rules for rational argumentation. Before tackling any mathematical subject, we must explicitly enter the mathematical discourse, and therefore we inevitably start from natural language.⁸ Thus the first step into mathematics is agreeing (explicitly or implicitly) on the fundamental rules of argumentation to be followed. Although mathematical texts seem to be “monologist and purely theoretical chains of insight,” they must nevertheless appeal to “the reader’s willingness to make the author’s proposals the basis for further joint action” (page 502). Already the entry into the mathematical discourse and the determination of its first elements cannot take place without a reasonable justification, which again would have to be of exactly the kind as it was proposed above for the justification of ethical norms. By that observation Kambartel finally achieved two things: On the one hand, for ethics,

a first overcoming (...) of Max Weber’s thesis that moral judgments are ultimately incapable of justification (...). Whoever ad-

⁷ [5, page 500]: “Nach den getroffenen terminologischen Bestimmungen lässt sich sagen, wann ein Dialog rational heißen soll, nämlich genau dann, wenn er unvoreingenommen, zwanglos und nicht persuasiv ist. Mit Termini von Habermas könnte man in diesem Falle auch von einer *unverzerrten* oder *idealen Kommunikationssituation* sprechen. Die mit Hilfe eines rationalen Dialoges gewonnene Zustimmung aller Beteiligten heiße *rational eingeholt*.”

⁸Accordingly, Socrates’ first question in the famous *Meno* dialogue about doubling the square refers to the slave boy’s ability to understand and speak Greek.

vocates this thesis cannot even understand how mathematics as a science is possible.⁹

On the other hand, for mathematics we get a propaedeutic role for ethics explicitly in the sense of Plato:

Thus (...) mathematics is able to be a school in which the possibility of non-sophistic normative argumentation can be tested, however, only if mathematics fulfills an already Platonic and today in the discussion of fundamentals again necessary request, the request namely (...) not to banish the justification of the first steps from mathematics.¹⁰

Before entering into the discussion, I would like to emphasize that I consider Kambartel's essay to be an important pioneering work that has emphatically placed an apparently long-forgotten and at the same time eminently important topic on the philosophical agenda. Nevertheless, in the following I will argue in some contrast to Kambartel in many places.

2. Ethics in (applied) mathematics

First of all, it is extremely striking what Kambartel almost forgets to address, but which today does not only shape the external image, but in many cases also the self-image of mathematics. It is felicitously characterized by the catchphrase of mathematics being a 'key technology.' Kambartel, in contrast, draws the picture of a self-centered, pure and formalist mathematics.¹¹

⁹ [5, page 503]: "eine erste Überwindung (...) der These Max Webers, daß moralische Urteile letztlich einer Begründung nicht fähig (...) seien. Wer diese These vertritt, kann nämlich nicht einmal begreifen, wie Mathematik als Wissenschaft möglich ist."

¹⁰ [5, page 503]: "So vermag (...) Mathematik eine Schule zu sein, in der die Möglichkeit nichtsophistischer normativer Argumentation erprobt werden kann; allerdings nur dann, wenn die Mathematik einer bereits Platonischen und heute in der Grundlagenauseinandersetzung wieder nötigen Aufforderung genügt, der Aufforderung nämlich zum nirgends verweigerten *logon didonai*, der Mahnung heißt das, die Rechtfertigung der ersten Schritte nicht aus der Mathematik zu verbannen."

¹¹ A very different 'point of contact' between ethics and mathematics, also missing from Kambartel's reflections, was highlighted by the anonymous referee. The choice of

Of course, this fits well with the “Bourbakian Zeitgeist” of the 1970s and there may indeed have existed a large number of examples for this attitude among professional mathematicians. Even when applications were noticed or even acknowledged, a strict separation was emphasized between these applications and the proper task of mathematicians. One particularly clear example may be added here. Jean Dieudonné (1906–1992) formulates the typical position, published under the pseudonym N. Bourbaki used by a group of prominent, mainly French mathematicians [3]:

the basic axioms might be seen as a *question of value* similar to questions of value in ethics. And these axioms might also come under the name of a “conjecture” as it is, for example, the case with Cantor’s *continuum hypothesis*. In fact, there are sometimes discussions within mathematics about the “natural,” or “good,” or “well justified” axioms. The most prominent episode of that type might still be the so called foundational crisis at the beginning of the 20th century and, in the sequel, the quarrels between classical and intuitionist or constructivist mathematics. One could also mention disputes between standard and non-standard analysis or even about assuming the existence of certain large cardinals in variants of set theory. Kambartel did not mention these disputes since in his — slightly woodcut-like — picture of mathematics the choice of the primary set of rules is completely free and also free from any obligation to justify it. In fact, following Kambartel’s story, the only strict criterion any choice of axioms must fulfill is logical consistency. This view about mathematics might be in accordance with the view of some of his contemporary mathematicians and it is certainly in accordance with the view of some (of his contemporary) philosophers. And it is precisely this view that Kambartel criticizes to be responsible for the radical divide between ethics and mathematics on the one hand and to be hardly convincing on the other. Within the professional culture of mathematics, however, there *are* of course “good reasons” to choose a *certain set* of axioms and there is sometimes also dispute about these reasons. These kinds of argumentation are, however, beyond Kambartel’s interest. Moreover, for mathematics there exists a very easily accessible way to avoid long lasting quarrels about these kinds of decisions about (epistemic) values: the branching into subdisciplines. If you don’t like scalar products, you just decide to work in Banach spaces; if you don’t like to work with limits, you switch to non-standard analysis. Initially perhaps ideological questions can thus be defused into harmless questions of taste or habits. In fact, this type of liberality has been proven to be very fruitful for mathematics because it can thus evolve in — often similarly promising — alternatives without too much efforts spent on foundational disputes. This phenomenon is, of course, in sharp contrast to any philosophical ethics that aims at the universality of its principles.

Still somewhat different is the dispute between classical frequentist and Bayesian statistics. Here we have common acceptance on the mathematical character of both approaches. There is no dispute about whether the math is wrong. The center of dispute lies on the question whether the respective (empirical) situation is “correctly modeled.”

Why have some of the most intricate theories in mathematics become an indispensable tool to the modern physicist, to the engineer, and to the manufacturer of atom-bombs? Fortunately for us, the mathematician does not feel called upon to answer such questions, nor should he be held responsible for such use or misuse of his work.¹²

In fact, Kambartel could have quoted Dieudonné, who fits well in his picture of contemporary mathematics. But he could also have critically questioned the complete rejection of any responsibility for the applications of one's own science. Moreover, even in Dieudonné's and Kambartel's time the image of mathematics being a purely formal science was rather incomplete. Applied mathematics is indeed a conspicuous white spot on Kambartel's mathematical map. This is all the more astonishing since, for example, the military applications of mathematics — think of Alan Turing and the decryption of ENIGMA or the involvement of John von Neumann and other mathematicians in the Manhattan Project — were quite spectacular. Robert Musil's (1880–1942) monumental novel [11], published as early as 1932, could already have served as a seismograph in this respect:

[I]t is in any case quite obvious to most people nowadays that mathematics has entered like a daemon into all aspects of our life [...] and that mathematics forms the source of an evil mind that, while making man the lord of the earth, also makes him the slave of the machine [...] this only went to convince them, later on, that mathematics, the mother of the exact sciences, the grandmother of engineering, was also the arch-mother of that spirit from which, in the end, poison-gases and fighter aircraft have been risen.¹³

¹² A very instructive study on the self-image of the scientific discipline of mathematics based on case studies of German history during and after World War II can be found in [9].

¹³ [11, pages 38ff]: “[E]s ist den meisten Menschen heute ohnehin klar, daß die Mathematik wie ein Dämon in alle Anwendungen unseres Lebens gefahren ist (...) und daß die Mathematik die Quelle eines bösen Verstandes bilde, der den Menschen zwar zum Herrn der Erde, aber zum Sklaven der Maschine macht. (...) Damit war später für sie

Today, applied mathematics is given an extremely high priority in a wide variety of fields, although “application” is understood to mean very heterogeneous things — just compare the so-called applications of mathematics in the context of elementary or secondary school with the content that mathematical researchers call “application,” which in turn is very different from the “applications” of mathematics in the eyes of an engineer. Today’s widely ramified ethics within the sciences, however, still concentrates largely on the natural sciences (of course, also on medicine) and on engineering and, at most, computer science comes into view.¹⁴ However, insofar as mathematics shapes modern society in a fundamental way, this shaping must be accompanied by reflection. Otherwise, we would simply follow the various processes of mathematization without reason. As with the hard sciences, the strict separation of “pure knowledge” and “application” of this knowledge, which is even more effective in the self-image of mathematicians, must be critically questioned for this purpose. Of course, the heading “Ethics and Mathematics” does not *only* require a critical discussion of *applied* mathematics (see in particular Kambartel’s reflections, which, by the way, are probably more influenced by the Bourbakian picture of mathematics than Kambartel himself would like). A complete account must not forget to address the question of applications. I will not present an elaborated *ethics of applied mathematics* here, but I would like to structure the broad field a little and point to initial contributions.

Let us begin with the chain so aptly characterized by Musil: mathematics leads to (the theory of) natural sciences and, finally, to technology and to the military (for more details concerning this point, see [12]). This *indirect* influence of mathematics does not only accompany the history of the 20th century. For example, in the 16th and 17th centuries, the early mathematical theory of dynamic systems is closely linked to questions of ballistics. A precise knowledge of the trajectory of a cannonball is both mathematically interesting and militarily appealing. Yet even Plato in the *Politeia* frequently mentions the

bewiesen, daß die Mathematik, Mutter der exakten Naturwissenschaft, Großmutter der Technik, auch Erzmutter jenes Geistes ist, aus dem schließlich auch Giftgase und Kampfflieger aufgestiegen sind.”

¹⁴ Compared to mathematics, computer science in fact plays a special role inasmuch as a politically alert scene traditionally forms the social background of the discipline.

importance of mathematics in the same breath with warfare, and one of the mathematical founding fathers, Archimedes of Syracuse, can be described as an archetype in this respect. He was already famous among his contemporaries for his military inventions. During the conquest of his hometown in 212 BCE, he had — busy with a mathematical construction — gruffly snapped at the invading Roman soldier: “*Noli turbare circulos meos!*” This request by Archimedes not to disturb his circles was answered by the soldier with a deadly sword stroke — in accordance with his own profession. Sadly that was much to the annoyance of the victorious commander who would have been all too happy to take the prominent opponent and his skills alive into his power. Archimedes thus also stands for ignoring problematic applications which, after all, had often already determined the theoretical direction of the question. Meanwhile, a number of recent contributions, especially from historians of mathematics, address the contributions of mathematics to enhancing the techniques of warfare; see for example [2, 8, 4]. From a systematic point of view, however, it would be worth examining more closely why and to what extent “pure” mathematical theory repeatedly harbors the potential for (sometimes quite unexpected) military applications.¹⁵ Of course, there is much more to say about the influence of civil technology on societies, which is enabled by sciences based on mathematical theory; for more details, see [13].

The far-reaching shaping of the life-world through the use of computers could be called a *semi-direct* form of mathematization. I would only like to mention this phenomenon, which is discussed intensively and from various perspectives under the catchword “digitalization,” and I would like to point out that the underlying mathematics practically never comes into focus.

At least as profound as the indirect influences, but much less considered, is the *direct* shaping of society through mathematically codified social norms.¹⁶ Such a mathematical basis was already essential for early civilizations — for example, in the design of a tax system. For modern societies, however, this is true in a dramatically intensified form. I would like to mention just a few examples here. First, the increasing use of mathematical methods in

¹⁵ Some hints may be found in [10].

¹⁶ For more details, see [14]. From a historical point of view the mutual shaping of mathematics and society is discussed in [6].

economics and especially in finance raises various ethical questions. One way to avoid such a debate is to claim that mathematics is merely a neutral language for describing a given situation, completely independent of that language. However, this position ignores the fact that mathematical concepts in finance are essentially used to *create* new types of actions or trading opportunities in the first place (see [16]). Moreover, if one follows the political discussion about democratic election procedures — gerrymandering is only the ugly side of this question — about health, pension and tax systems, it becomes clear that mathematics is hidden beneath the surface of many political disputes — a kind of mathematics normally mastered by only a few. But again, mathematics works here not just as an uninvolved descriptive language of given conditions, but as a stock of possible rules for society. Finally, the self-observation of society, which is often perceived as overly complex (among other things due to implemented mathematics!), is increasingly shaped by using simple numbers as evaluation and orientation criteria. The most prominent examples, again, are economic indicators such as GDP.¹⁷ Quite recently in the COVID-19 pandemic we observe, for example, incidence values and hospitalization rates (see [18]). Key figures also dominate (political) decisions in the education and science system, for example in various rankings or by using bibliometrics. The drastic reduction to a few, easily comparable, numerical values trivializes difficult and possibly controversial evaluation problems and at the same time may hide a discussion about the quality of the evaluation standards. The sometimes dramatic effect on the shaping of society, which is exacerbated by the use of digital procedures and which not infrequently runs counter to the original intentions of the responsible political actors, is described by Cathy O’Neil essentially through a precise discussion of individual case studies under the telling title *Weapons of Math Destruction* [19].

After having briefly mentioned some examples and fields of a massive influence of mathematics on the shape of modern societies, I would like to point out three aspects of the mathematical that are important for the aforementioned effects:

¹⁷ Oliver Schlautdt lucidly examines the effects of quantification made possible by mathematics in the field of economics, cf. [21].

- (A) By the term *complexity paradox* I would like to denote the strange phenomenon that mathematics on the one hand allows an extreme reduction of complexity, but on the other hand can unfold the most complicated structures from the simplest principles. For example, on the one hand, counting a crowd of people is a radical renunciation of the most diverse qualities, so that in the end only the discrete quantity, i.e., number, is taken into account. On the other hand, however, the mathematical number theory with its immense abundance of results and structures emerges in a remarkable way from extremely few and easily overlooked principles — expressed, for example, by the axioms of Peano. It seems to me that the first aspect is more often in the focus of public debate: The widespread view here is that mathematics is a tool for simplifying or solving problems and that the tool is essentially constant in structure and content. Indeed, the use of mathematics makes it possible to reduce our relationship to a “world” experienced as complex to the “essentials,” to structure content and thus to simplify it sometimes drastically. The creative potential of mathematics is, however, underestimated or even ignored in this picture. Mathematical research is currently producing completely new results, but also methods and questions, to an ever-increasing extent. In doing so, however, it also provides society with entirely new structures and rules, and at the same time causes additional internal complexity in society. Afterwards, in the best case, mathematics can be used to reduce that complexity, thus to solve problems that we would not have without mathematics.
- (B) The *transparency paradox* refers to a related, at least equally remarkable, epistemological aspect of mathematics. If we assume in a simple sense that a mathematical fact is “understood” by those who can operate correctly with the corresponding terms and methods, and who can also actively connect further mathematics, then the following obviously applies: The very largest part of mathematics is completely incomprehensible to the very largest part of mankind.¹⁸ This is not only true for mathematical laymen, but also for those working in mathematical science. To an extent that is hardly known to outsiders, the mathemat-

¹⁸ This phenomenon is further analyzed in [15].

ical subdisciplines have become incomprehensible to each other. This phenomenon stands in strange contrast to the fact that mathematics, more than any other science, insists on explicating all its methods and terms used, on clearly distinguishing definition, assumption and theorem from each other, and on proving the latter to be universally valid by, again, clearly separated and transparent proofs. However, the effect of this effort for absolute clarity is — as stated above — largely the opposite: mathematics becomes quite esoteric (maybe precisely because of its effort for transparency). This is particularly irritating because it is not being said that mathematics is about secrets that can only be understood by some, however selected, circle of initiates. On the contrary, it is emphasized that everything that is necessary for understanding is stated completely, precisely and clearly. Only slightly exaggerated, the strange paradox could be stated: Mathematics obscures itself through transparency.

- (C) I have already briefly referred to the *descriptive-normative double face* of mathematics. In my view, the willingness to accept and consistently adhere to certain principles or rules that are necessary for mathematical discourse, such as the axioms of set theory or logical principles, already has a normative flavor — Kambartel has argued similarly.¹⁹ Beyond the inner realm of mathematics, however, in the context of shaping society through mathematics, I would like to distinguish two levels of normativity: First, it is by no means normatively neutral to put a phenomenon into mathematical terms. For example, the quantitative description of a social situation may sometimes be helpful, sometimes annoying but harmless, but sometimes counterproductive — and in any case its appropriateness is by no means self-evident. Second, even if one decides — hopefully with good reasons! — to apply a mathematical methodology, the chosen “mathematical modeling” is by no means determined by

¹⁹ See [3] and [9] as mentioned in Footnote 12 above. In fact, we may distinguish the following two normative aspects. First, the decision to enter the mathematical discourse and to obey the respective rules of a consistent argumentation seems to be at least an analogue to the decision to discuss ethical issues in a “decent way.” Second, within the mathematical discourse, we can choose between different principles (axioms), guided by aesthetic or epistemic values, or simply by habit.

the “nature of the matter,” but is subject to the decisions about what is considered “important,” “interesting,” “worthy of attention” and therefore taken into account. The distinction and demarcation of these two normative aspects from the inner-mathematical precision and objectivity seems to be by no means easy, especially since the experts involved do not usually direct the main focus of their expertise precisely to the normative side. So the convenient thesis that mathematics is a purely descriptive tool for a precise analysis of objectively given features of a situation is still very popular among professional mathematicians. Here is a particularly clear example from the concluding remarks in Robert Aumann’s article on game theory (1987), reprinted in [1]:

While game theory does have intellectual ties to ethics, it is important to realize that in itself, it has no moral content, makes no moral recommendations, is ethically neutral. (...) Game theory is a tool for telling us where incentives will lead. History and experience teach us that if we want to achieve certain goals, including moral and ethical ones, we had better see to the incentive effects of what we are doing (...) Blaming game theory (...) for selfishness is like blaming bacteriology for disease. Game theory studies selfishness, it does not recommend it. [page 98]

3. Some comments on positions from the intellectual history.

In this last section, I would like to follow up briefly on Kambartel’s reference to phases in intellectual history that show a close connection between mathematics and ethics. First, I would like to emphasize that ethics and mathematics are similarly universal, but maybe at the same time quite different in their characteristic features. They are culturally or historically conditioned, but point to objectivity beyond what is culturally or individually conditioned in each case. Thus, it is not surprising that philosophical positions take a look at both — even comparatively. And in fact, there are prominent authors relating ethics and mathematics, although their positions are clearly more diverse than the binary alternative proposed by Kambartel

between positions that claim a related rationality for ethics and mathematics and those that regard ethics as fundamentally irrational and thus at the same time distinct from mathematics. I would like to follow Kambartel’s proposal to distinguish philosophical positions with respect to two different characteristics. First, a position can prefer a “parallel” treatment of ethics and mathematics, it can regard both as essentially similar endeavors — or, on the other hand, it can plea for a different treatment, it can thus stress the mutual differences with regard to the subject under consideration or — more important — with regard to the respective method. Second, we may distinguish philosophical positions with respect to the question whether there is any rationality attributed to the ethical field. Both characteristics should be treated separately, since there are prominent examples for all four possible combinations. The following categorization of philosophical positions is very woodcut-like, however, I hope to make thereby the point a little clearer that in Kambartel’s picture some very important stances were missing — namely positions on the secondary diagonal in our Table 1 below.

	Mathematics and Ethics are Similar	Mathematics and Ethics are Different
Ethics is rational	Plato, Descartes, Leibniz, Lorenzen, Kambartel, ...	Aristotle, Pascal, Kant, ...
Ethics is irrational	Skeptics: Sextus Empiricus, Hume, ...	Positivism: Carnap, ...

Table 1: Table of Philosophical Positions regarding Ethics and Mathematics

First, no positions were mentioned that call for or state a rational approach for both mathematics and ethics, yet strictly separate the two disciplines methodologically. And second, skeptical positions that concede neither ethics nor mathematics any rational dignity beyond their pragmatic usefulness should not be forgotten. In my opinion, however, the dominant dispute in the course of history is between the two different types of “ethical rationalists” who, although united in their opposition to skepticism, answer the question of what role a methodological orientation to mathematics should play in this in very different ways.

I will now just briefly mention a few authors from the history of philosophy who hold contradictory positions here.²⁰ First, Plato and Aristotle open the long sequence in this dispute, and at the same time they represent the opposing positions in a paradigmatic way. Thus Plato, in a manner yet to be clarified, grants mathematics at least a propaedeutic function for the discourse about the question of the good, and possibly even goes beyond this in his so-called unwritten doctrine. In contrast to Plato, Aristotle, in explicit criticism of his teacher, separates the principles and discourses of ethics and mathematics relatively strictly. Second, in contrast to René Descartes' search for an absolute, universal unified science modeled on mathematics, which was primarily intended to resolve ethical questions, Blaise Pascal insists on a fundamental difference between the *esprit de finesse*, which deals with questions of morality and decent behavior in individual cases, and the *esprit geometrique*. Third, Immanuel Kant's critical theory on a fundamental difference between the method and achievable certainty in mathematics and philosophy are directed against Gottfried Wilhelm Leibniz with his efforts to create a formal universal language that is to replace all political and ideological disputes with simple arithmetic. Deepening the discussion of these particular constellations goes far beyond the intention of the present paper. So I come to some concluding remarks.

Since mathematics massively shapes modern society, directly and indirectly, it requires accompanying (professional) ethical reflection. A closer look at the sometimes equally massive consequences of mathematics for ethical theorizing, which I could only hint at here, and the synopsis of both in the field of tension between freedom and determination,²¹ which must be reserved for a later opportunity, could lead to a deeper understanding of both ethics and mathematics. Anyway, I hope that Kambartel's efforts were not in vain and that in due course the topic of "ethics and mathematics" no longer sounds quite so unfamiliar to more and more ears.

²⁰ Of course, there are also contemporary authors fitting well in that categorization.

²¹ Some considerations about the role of freedom — not just arbitrariness — for mathematical proof can be found in [17].

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