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Figure-Ground Perception: A Poem Proof

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Synopsis

This is a proof, in poetic form, of a bit of real analysis, more specifically involving the topology of accumulation points, that exploits the human optical phenomenon of figure-ground perception. Sometimes it is not a change in content, but a snap shift in point of view that yields a proof.

Definitions

Sets here are sets of real numbers, also called points.

Let a and b be real numbers.

The open interval (a, b) is the set of all real numbers both greater than a and less than b.

A real number x is an *accumulation point of a set* if an arbitrary open interval containing x contains at least one point of that set different from x.

The notation E' represents the set of all accumulation points of the set E.

So, (E')' represents the set of all accumulation points of the set E'.

Proposition

The points of (E')' are contained in E'.

Proof

Assume x is a point contained in (E')'.
So, x is an accumulation point of E'.
Let (a, b) be an arbitrary open interval containing x.
Then, (a, b) contains at least one point y of E' different from x.

Suppose $x < y$.
Since y lies in E', y is an accumulation point of E.
So, any open interval containing y contains at least one point of E different from y.
Note that y lies in the open interval (x, b).
Since y is an accumulation point of E and y lies in (x, b), there is at least one point z of E lying in (x, b) different from y.
So, z lies in E, and z lying in (x, b) means both z lies in (a, b) and z is different from x.
But (a, b) was chosen to be an arbitrary open interval containing x.
So, x is an accumulation point of E.

Suppose instead $y < x$.
Since y lies in E', y is an accumulation point of E.
So, any open interval containing y contains at least one point of E different from y.
Note that y lies in the open interval (a, x).
Since y is an accumulation point of E and y lies in (a, x), there is at least one point z of E lying in (a, x) different from y.
So, z lies in E, and z lying in (a, x) means both z lies in (a, b) and z is different from x.
But (a, b) was chosen to be an arbitrary open interval containing x.
So, x is an accumulation point of E.

Therefore, x is contained in E', as desired. ■

Translation to Poem Proof

The "sea" is (a, b).
"I" am a point of $(E')'$.
"You" are a point of E'.
A "point of green" is a point of E.

Poem Proof

Within this sea
in which I live,
I only know
that I see you
and you in turn
tell me you see
as near to you
a point of green.

I do not know,
I do not see,
if there might be
a point of green
so near to me.

But wait.

The point of green,
the one you see
(your point of view)
within the sea
in which you live,
the sea we share,
since I see you,
that point of green
I see it too;
it's always near
both me and you.

I see it now,
we share it there,
we share, we share,
I am no long-
er unaware.
We are the same
within the sea,
we are alike
both you and me.