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Caleb Holloway
West Virginia University Institute of Technology

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Cover Page Footnote

The work that was the basis for this study was funded by a grant from West Virginia University Libraries and Teaching and Learning Commons. My thanks go out to my students for their help in identifying bugs in my exercises throughout the spring 2020 semester. I would also like to thank the reviewers for their time and assistance in preparing this article for publication.

More than Free Textbooks: Labor and Pedagogy in Implementing Open Resources in a Trigonometry Course

Caleb D. Holloway

West Virginia University Institute of Technology, Beckley, West Virginia, USA
caleb.holloway1@mail.wvu.edu

Synopsis

This paper reports the implementation of open educational resources (OER) in a university trigonometry class, with an emphasis on the pedagogical considerations and academic labor involved. To date these two matters have been underreported in the literature on OER. I provide an account of the work involved both in choosing an open textbook and in creating hundreds of accompanying homework exercises for an online learning platform. I also present the pedagogical lens that informed this implementation, discuss how it informed my adoption of an open textbook, and provide specific examples of how it guided the creation of these exercises. Based on my experiences I make some observations for those who might be considering OER in their own classes, and I present the results of a survey I gave my students on the use of OER in their class.

1. Introduction

1.1. OER Overview

Open educational resources (OER) are "teaching, learning or research materials that are in the public domain or released with intellectual property licenses that facilitate the free use, adaptation and distribution of resources" [30]. This right to "retain, reuse, revise, remix, and redistribute" [31] is what makes OER so appealing. Students have easy access to free course materials (freedom of use), saving money on expensive textbooks; instructors can improve these materials and adapt them to their needs (freedom to adapt); and modified materials can then be made available for others to use (freedom to redistribute).

Past studies on OER have focused primarily on their efficacy in the classroom compared to that of traditional materials [15]. Such studies have frequently had methodological shortcomings including, but not limited to, failure to control for confounding variables [12]. Additionally, these studies often do not report the name of the commercial texts against which OER is being compared. Regardless, multiple meta-analyses have supported the claim that OER are at least as effective as commercial materials in meeting the needs of the classroom [13, 6].

The focus on compared efficacy is hardly surprising, especially considering the common skepticism about the quality of OER [3, 23]. However, it leaves unanswered many other questions related to OER, their use, and their implementation. McDermott [15] identified three major areas wanting for investigation: the potential for OER to promote inclusive access to education, the pedagogical considerations of selecting and implementing OER in the classroom, and the academic labor required to implement OER.

1.2. Goals of this Report

My goal in this paper is to shed light on the last two of these questions. For those considering OER, what is the work involved in making the transition (especially if new materials must be created rather than simply adopted)? And what sort of pedagogical reasons (if any) may there be for choosing OER over commercial materials, or one OER over another?

I do this by reporting on an introductory math course that I taught using OER at a small public university in the spring of 2020, a course that up to that time had been taught using commercial materials. I first provide background information on the university and the course (§§2.1). I next describe how I received the opportunity to transition this course to OER through a grant from my university's parent institution (§§2.2), and give a brief account of the adoption, creation, and implementation of OER in that course (§§2.3). Then, I describe in greater detail the work involved in this process (§3) and discuss the pedagogical considerations involved (along with an explanation of the pedagogical lens through which the whole process was viewed) (§4), before adding some remarks about the transition to online learning (§5). I conclude with a brief discussion of my overall experience (§6) and a call to other instructors to join in the OER movement (§7).

2. Account of Adoption of OER

2.1. Background

The university under consideration, West Virginia University Institute of Technology, is a small, public university in the eastern United States. A good number of our students are engineering or computer science majors, so for these students a strong foundation in mathematics is essential. Additionally, roughly a third of our students are first-generation (that is, students whose parents did not earn a four-year degree), and about two-thirds can be classified as either first-generation or low-income [7].

This report focuses on a first-year course in trigonometry. This course deals with trigonometric functions, equations, and identities, along with related topics such as vectors and complex numbers. It is a prerequisite for calculus and as such is taken by many of our engineering and computer science majors. It also serves a variety of other majors including the natural sciences and premed.

The format for this course as I taught it was the standard lecture delivered three days a week. Homework assignments were assigned frequently and completed on an online learning platform; in-class quizzes were assigned about once a week. The majority (80%) of a student's grade came from tests, of which there were four: three regular exams plus a final. The remaining 20% of the grade was split evenly between homework and quizzes.

2.2. Motivation and Grant Opportunity

A study by Florida Virtual Campus [11] found that "the high cost of textbooks is negatively impacting student access, success, and completion." Students in this study reported that high textbook costs had caused them to drop or withdraw from a course (26.1% and 20.7%, respectively), earn a poor grade in a course (37.6%), or fail a course (19.8%).

The cost of education is a special concern at my institution due to the high percentage of low-income and first-generation students. Our mathematics department has a history of taking measures to reduce student expenses in our courses. Price is a key factor in every textbook adoption process, and we frequently adopt older editions and books that can be got secondhand. When in early 2019 a grant initiative to implement OER on our campus became available, it seemed an obvious fit.

2.3. Adoption, Creation, and Implementation

There were two parts to this initiative. First, there was a workshop where attendees could learn about OER and select an open text to review (receiving a $200 stipend for their efforts). Second, there was a call for grant proposals to adopt and/or create OER to be implemented in a university course, the amount of the grant depending on the size of the undertaking.

In late February I submitted a proposal to adopt an open textbook in conjunction with the MyOpenMath online learning platform for our trigonometry course, and to implement these resources in one section of Plane Trigonometry in spring 2020. In March I attended the OER workshop, where I selected *Trigonometry* by Ted Sundstrom and Steven Schlicker [25] to review. After reviewing it, I decided this would be an appropriate text to use in my own course. So, in May, when I received word that I had been awarded the grant, I had already chosen both my text and my homework platform.

At this point in time my trigonometry class had been using *Algebra and Trigonometry, 4th Ed.*, by Stewart, Redlin, and Watson [24], published by Cengage; along with WebAssign, an online learning platform also supported by Cengage. Therefore, the contents of the textbook and online platform were closely aligned. This was not the case for MyOpenMath and our new open textbook, however. For this reason, I decided to code the exercises contained in Sundstrom and Schlicker [25] into MyOpenMath, along with supplemental exercises of my own creation, to serve as the basis for the homework assignments in my course. This would be the bulk of my work in the transition to OER.

I began work on this project in early July 2019 and continued until just before fall classes began in mid-August, at which point I continued to create new exercises as I could. By January 2020 I had coded most of the textbook's exercises, along with many supplemental problems of my own – enough to fill most of the course assignments.

I taught trigonometry that semester using these open materials. I created a course in MyOpenMath containing a PDF copy of the textbook, homework assignments, and various resources, and gave my students instructions for accessing this course. As the semester progressed, I continued coding exercises to ensure that new assignments were ready when students needed them. I also fixed any bugs that cropped up in my exercises as my students reported them.

In March the university bowed to the inevitable as it was announced that, due to the COVID-19 pandemic, all instruction would move online following our spring break. The transition was relatively easy for this class as much of it was already based online. At the end of the semester, I gave my students a survey on their satisfaction with the open resources.

3. Academic Labor

3.1. Selection

The selection process was straightforward, so I will keep this section accordingly brief. During the OER workshop I surveyed five or six open trigonometry texts on the Open Textbook Library available at https://open.umn.edu/opentextbooks. Of these I was most impressed by Sundstrom and Schlicker's book [25]. I found its tone inviting to students, and I appreciated its frequent attempts to make students active participants through what it calls Progress Checks (page 6). There were also several pedagogical considerations that went into my selection, on which I elaborate later.

I would have chosen MyOpenMath as a replacement for WebAssign for no reason other than it is the only free and open math-focused online learning platform that I know. Thankfully, I had the additional motivation of two years' experience using the IMathAS assessment tool on which MyOpenMath is based at my previous job, including much experience coding questions of my own on that platform.

3.2. Creation

Trigonometry consists of twenty-seven sections across five chapters, each having between two and eleven homework exercises. Very often a single exercise consists of several separate questions (for example, exercise 3 on page 58 asks students to calculate and draw the reference arc for six separate provided arcs), so the actual number of questions is somewhere in the hundreds, and possibly over one thousand.

There is, of course, more to coding a homework question for an online learning platform than simply transcribing the problem. To begin with, for each question I had to create a prompt for student responses, then write the solution into the code for the program to check these responses against. Since I did not have a solution manual, this meant that I also had to solve each problem myself. I also had to account for the difference in media.

For example, a textbook exercise might ask students to sketch the graph of a sine curve, but such a question will not work on MyOpenMath; at least, there is no way for the system to check the student's hand sketch. I often had to devise creative solutions to deal with such problems.

The coding process was greatly complicated by my choice to create algorithmically generated versions of most homework exercises. What I mean is that, instead of translating a question into MyOpenMath verbatim, I would, as often as possible, create in the code of the question algorithms to randomly generate the numbers to be used in the problem according to set parameters. This obviously complicated the code, but it also added an extra layer of work in other aspects of the creation process. When initially solving a problem, I had to pay special attention its pedagogical aim to ensure that, in each random iteration of the problem, this aim would be maintained. (To give a very simple example, working with positive versus negative angles require different understandings, so a question that has students calculate the measure of one of several possible random angles would need to be coded so as not to generate a negative angle by mistake.) I then had to set parameters for the numbers to be generated, solve the exercise in terms of these parameters, and then code both the algorithms to generate the needed numbers as well as the solution in terms of the parameters. Finally, I had to test and debug the question — a necessary process regardless, but one now made quite a bit more difficult.

Despite the already large number of exercises on my plate, I felt there was still room for more. Some sections of the textbook had rather scanty homework sets, and there were important aspects of students' understanding that I feared would not be sufficiently developed with the present exercises alone. (I provide specifics on this latter point in the section on pedagogy.) Thus, in addition to the exercises from the text, I decided to create supplementary questions of my own as needed.

I spent six weeks, from early July through early August 2019, coding homework exercises at a rate of about twenty hours per week. I spent considerably less time on this endeavor once classes began in the fall, though I was able to finish the work in time to add the last few assignments to the end of my spring 2020 course. (Debugging throughout the spring semester consumed a lot of my time, as well.) When all was said and done, I had written about 350 homework exercises total.

There were other affairs that took far less time but that I still wish to mention. Of course, the adoption of a new textbook and homework platform necessitated changes to course materials, primarily the syllabus. My class notes needed modification, as well, as did certain class worksheets from previous semesters. I also created several animated and interactive graphs on Desmos to be used both in class and alongside homework exercises. Finally, I had to set up a class section in MyOpenMath, organize my library of homework questions into assignments, and prepare the section for my students' use.

4. Pedagogical Practice

4.1. Pedagogical Lens

Over the past few decades, research in math education has led to a greater understanding of how students form mathematical conceptions and which conceptions are most fruitful in furthering their mathematical development. Approaches that focus on procedural understanding at the expense of quantitative reasoning tend to produce students who cannot apply learned techniques to solve novel problems [20, 22]. Common assumptions on how students view mathematics have turned out to be wrong [10], and popular approaches to mathematical ideas were shown to be ineffective at producing the desired understanding [21]. Such results illustrate the need for the constant evaluation and reevaluation of how mathematical concepts are taught.

There are three closely related areas of study that bear strongly on the teaching and learning of trigonometry: the notion of "quantity" and quantitative reasoning; the "process view" of functions; and covariational reasoning. Here I will give a brief description of each and their relationships to each other. In the next section I will describe how my understanding of these topics influenced my adoption and creation of OER.

Thompson [26, 27] defined a "quantity" as a person's conceptualization of an attribute of an object in such a way that it can be measured. A quantity does not exist in the "real" world; it is a mental construct of the person imagining it [27]. It is schematic, being "composed of an object, a quality of the object, an appropriate unit or dimension, and a process by which to assign a numerical value to the quality" [26]. The driver of a car might perceive, for example, the distance (quality) driven by the car (object) on a trip, measured in miles (unit) as displayed on the odometer (means).

Understanding the nature of quantity is important to help students develop quantitative reasoning. Quantitative reasoning is important for elementary mathematics (e.g., learning to count, basic arithmetic) and for application problems. But it has also been shown to be a powerful tool in understanding higher mathematical concepts. Oehrtman, Carlson, and Thompson [20] showed students who used quantitative understanding were more adept at solving algebra problems than those who employed purely procedural methods, while Moore [16] demonstrated how a quantitative approach can help students better understand the underpinnings of trigonometry.

Rarely is the relationship shared between two quantities static, so it is important to understand how the values of related quantities vary together. The coordination of the values of two (or more) quantities and the corresponding changes in those values is called "covariational reasoning" [28]. Many studies support the idea that covariational reasoning is important to students' mathematical development [8, 19, 28, 29]. The coordination involved distinguishes covariational reasoning from variational reasoning, that is, reasoning about changes in a single quantity's value. While necessary, the ability to reason variationally does not imply covariational reasoning [18].

Closely related to the idea of covariational reasoning is that of a function, a concept that forms a through-line from algebra into calculus and beyond. A typical definition of function is "a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B" [24]. While useful for advanced mathematics, this set-correspondence definition lacks any notion of variation (historically seen as an important aspect of functional thinking [14]) and has been criticized as inappropriate for students of introductory mathematics [28].

Instead, Thompson and Carlson [28] propose a covariational definition of function as

> a conception of two quantities varying simultaneously such that there is an invariant relationship between their values that has the property that, in the person's conception, every value of one quantity determines exactly one value of the other.

Observe the emphasis on the person's conception of a relationship. Such a definition not only implies that one person may see a function relationship where another does not, but also that the way two people perceive the same function may differ.

This brings us to our last point: what Breidenbach and colleagues call the "process conception" of function. This conception "involves a dynamic transformation of objects according to some repeatable means that, given the same original object, will always produce the same transformed object" [4]. A person who sees a function in this way can imagine it existing independently of themselves. By contrast, one who holds only an "action view" of function must themselves carry out the action at hand (e.g., evaluate the function at a given input) for the function to have any meaning. One who holds a process view, therefore, can better comprehend a function that cannot be evaluated using algebraic procedures, as is the case with the trigonometric functions. Additionally, the process view better enables one to understand functions as having properties such as "one-to-one," which is essential for understanding the inverse trigonometric functions.

4.2. How this Lens Informed Adoption

Moore [16] has described trigonometry as a topic that "lacks coherence in mathematics education." Trigonometry can be used in different contexts and for different goals, yet students of trigonometry often see no connection between these contexts or goals [5, 16]. It is common for students to develop a conception of trigonometric functions dominated by right triangles, often to the point that any problem that involves these functions in another context must first be converted entirely to the right-triangle context — if these students find the problem comprehensible at all. Such an understanding severely impairs students' ability to work meaningfully with trigonometric functions [27].

My goal when selecting an open textbook was to find one that would present trigonometry coherently. To this end, I sought a book that emphasized the more versatile unit-circle approach to trigonometric functions over the restrictive right-triangle approach. Such is the approach taken by *Trigonometry* by Sundstrom and Schlicker [25]; the authors first introduce the unit circle and use it as the basis for the sine and cosine functions. Then, related topics such as arc length, reference arcs, and angular velocity are discussed, and the remaining four trigonometric functions are introduced at the end of chapter 1. Chapter 2 deals with graphs of trigonometric functions, inverse trigonometric functions, modelling applications, and trigonometric equations. Then, in chapter 3, right-triangle trigonometry is introduced as a special case of the more versatile approach and alongside appropriate applications.

I found another merit to the organization of the book in the way it withholds four of the six trigonometric functions until the end of chapter 1. Since these functions can be defined entirely in terms of the sine and cosine, there is little need to introduce them right away. Instead, students can focus their attention on the two fundamental trig functions, getting to know them better through the exploration of related topics, and turn to the four remaining functions once a firm foundation has been laid. Similarly, the graphs of sine and cosine are studied in depth before the graphs of the other four functions.

I also found several opportunities to focus on quantitative and covariational reasoning in this text. The text opens with a quantitative presentation of the unit circle by literally wrapping the number line around the unit circle (page 3). Using this illustration, students can associate the quantity of distance (measured on the number line) with the corresponding terminal points on the unit circle; an accompanying animated web applet helps strengthen the point. The section on angular velocity (pages 35–44) emphasizes the variational nature of angles that is often underemphasized in trigonometry texts. A student who learns to think of angle measure as a changing quantity can better see the covariational relationship between it and related quantities.

I have already mentioned one web applet associated with the unit circle. Links to other web applets are provided throughout the text. For example, one animated applet illustrates the relationship between the unit circle and the graphs of trigonometric functions and highlights the covarying quantities at play (see Figure 1 below). Studies have shown that interactive computer software can aid students' learning of trigonometric functions [2, 9].

4.3. How this Lens Informed Creation

In addition to the exercises from the textbook, I also coded many supplementary problems designed to develop student thinking in one or more of the three areas discussed previously (quantitative and covariational reasoning, and function-as-a-process). I will now provide examples of these problems.

As illustrated by Moore [17], students of pre-calculus math often lack a quantitative understanding of angle measure. In his study, Moore demonstrated how teaching activities designed to draw attention to the quantities involved in the measuring process can lead students to conceptualizing an invariant multiplicative relationship between the measure of an angle and the length of a subtended circular arc.

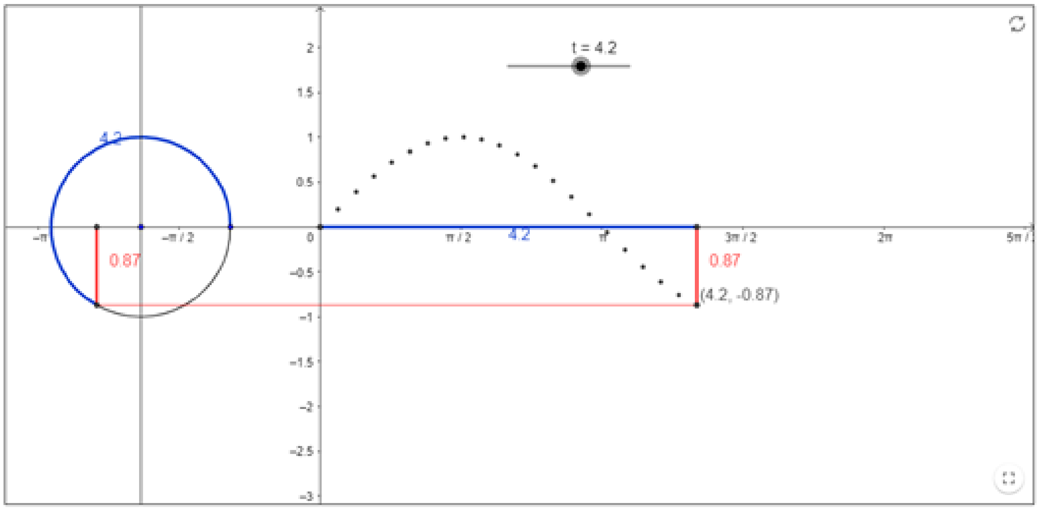


Figure 1: Animated graph of the sine function. Note the correspondences between quantities on the graph, and quantities on the unit circle.

Following Moore's example, I designed several questions aimed at reinforcing the link between angle measure and arc length. For example, one exercise asks students to calculate the degree measure of several angles given only the proportion each subtends on the circumference of a circle of unspecified radius (see Figure 2 below). The question is phrased so as to foreground the quantities involved and the relationship between them. The radius is eliminated so students can focus on the connection between arc length and angle measure and can come to understand the invariability of this relationship across circles of differing radii. Another exercise builds on this last point by having students write the equation of a circle when its radius is first measured in feet, then writing the equation of the same circle when its radius is measured in yards. Once both equations are written, students are asked to explain how both equations model the same circle.

Other exercises build quantitative reasoning in the context of the sine and cosine functions. In one, students evaluate the sine and cosine of several (nonstandard) arc lengths by drawing each arc on an animated graph and estimating the position of its terminal point. Since the coordinates of this point are not given, students are encouraged to see the values as quantities, namely, the vertical and horizontal displacements of the point from the appropriate diameters. In a later exercise, students explore this relationship

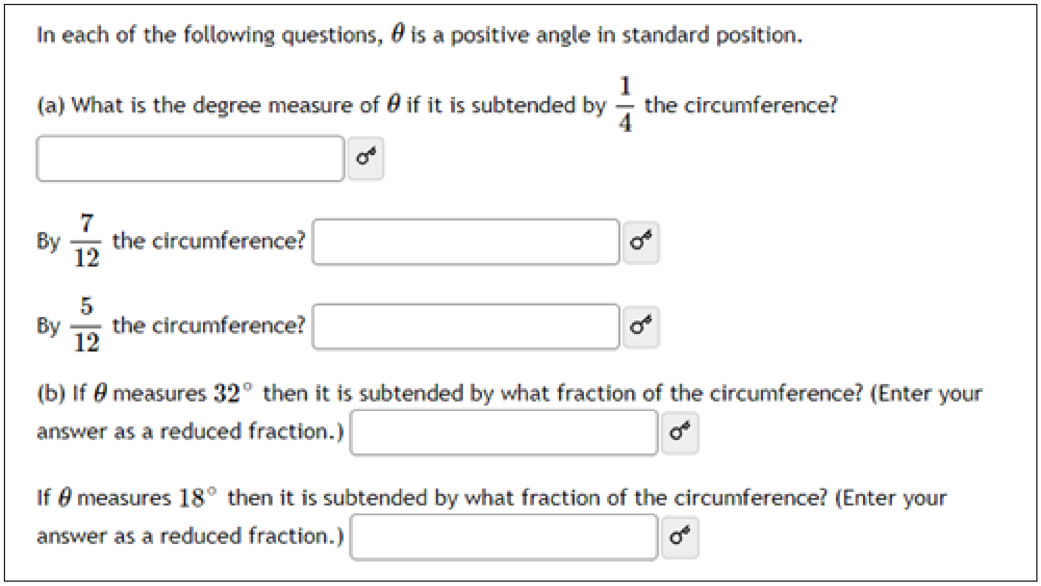
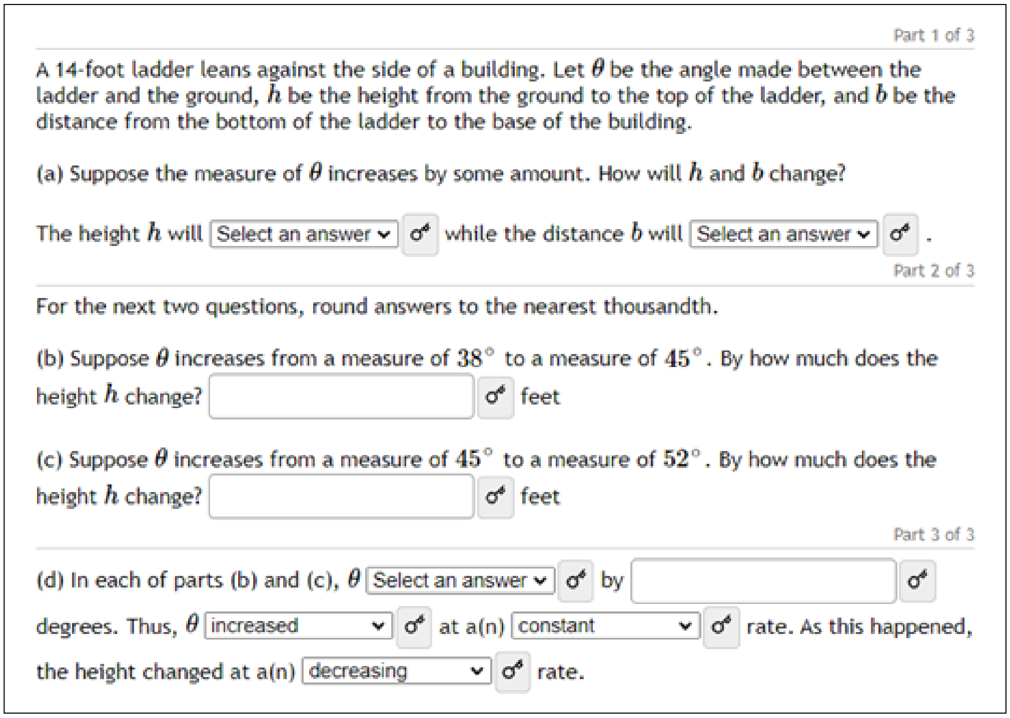


Figure 2: Supplemental exercise created in MyOpenMath emphasizing the role of proportionality in the concept of angle measure.

further using an animated graph similar to the one in Figure 1. They are also asked to verbalize their understanding of sine and cosine in terms of quantitative relationships throughout these exercises.

I designed several exercises to develop covariational reasoning in the contexts of arc length, angle measure, and the sinusoid functions. In doing this I followed the hierarchy of mental actions (MA) forming the covariation framework set out by Oehrtman, Carlson, and Thompson [20]. My implementation of these actions is illustrated in a homework exercise (seen in Figure 3) relating the angle measure θ that a ladder makes when it leans against a wall with the height h of the top of the ladder from the ground (a relationship modeled using the sine function). The dependence of h on θ (MA1) is implicit throughout the problem. In part (a), students coordinate the direction of change in h with changes in θ (MA2). In parts (b) and (c), they calculate amounts of change in h as θ changes over subsequent intervals (MA3). Finally, in part (d) they consider how these changes themselves change (MA4) and use their answer to infer the shape of the graph of the relationship (MA5).



Part 1 of 3

A 14-foot ladder leans against the side of a building. Let θ be the angle made between the ladder and the ground, h be the height from the ground to the top of the ladder, and b be the distance from the bottom of the ladder to the base of the building.

(a) Suppose the measure of θ increases by some amount. How will h and b change?

The height h will [Select an answer] while the distance b will [Select an answer] .

Part 2 of 3

For the next two questions, round answers to the nearest thousandth.

(b) Suppose θ increases from a measure of 38° to a measure of 45°. By how much does the height h change? [ ] feet

(c) Suppose θ increases from a measure of 45° to a measure of 52°. By how much does the height h change? [ ] feet

Part 3 of 3

(d) In each of parts (b) and (c), θ [Select an answer] by [ ] degrees. Thus, θ [increased] at a(n) [constant] rate. As this happened, the height changed at a(n) [decreasing] rate.

Figure 3: Supplemental exercise created in MyOpenMath emphasizing the role of covariation in the sine function.

I also designed my exercises with the goal of helping students to see the sine and cosine as processes. The examples already provided illustrate some of the ways I attempted to meet this goal. The animated graphs allow students to explore how sine (or cosine) values change alongside changes in the angle measure, in real time, and without the need to perform calculations. Direct questions on how a function behaves over an interval (e.g., increases or decreases) prompt students to think about those functions acting independently of themselves. The frequent use of nonstandard angles (seen in Figures 2 and 3) encourages them to understand that these functions can take any number as input, as opposed to merely correspondences with "special angles" such as 30°, 60°, and 90°.

5. Some Final Matters

Due to the COVID-19 pandemic, all instruction was moved online beginning March 30. From this point until the end of the semester I conducted lectures in real time using Zoom videoconferencing with lesson slides created in Microsoft PowerPoint. In the spirit of openness, perhaps I should have used LibreOffice Impress instead. However, the use of open materials did continue through our use of MyOpenMath, which now hosted the remaining exams in addition to homework assignments. (I proctored these exams myself, by requiring my students to log into Zoom with their webcams enabled while they took the exams.)

At the end of the semester, I asked all students to participate in a survey on the implementation of OER in the course. I received eight responses from the fifteen students then enrolled, of which 1 was removed for not providing a fair assessment. The survey consisted of nine questions; the first seven asked students to rate their satisfaction with various aspects of the OER used on a scale of 1 to 5, with 5 meaning "very satisfied" and 1 meaning "very dissatisfied;" and the last two asked their opinions on statements regarding OER, with 5 meaning "strongly agree" and 1 meaning "strongly disagree." (The results of these questions are summarized in Tables 1 and 2.) Additionally, at the end of the survey students were asked whether they would consider enrolling in another course that uses OER, and all seven responded "yes."

Following the successful implementation of OER into one section of MATH 128, in the fall of 2020 our department transitioned most introductory math courses, including all sections of trigonometry, college algebra, and the calculus sequence, to open materials. To promote continuity between the two classes, we adopted *Algebra and Trigonometry* by Jay Abramson [1], available at OpenStax, for both college algebra and trigonometry. We continue to use *Trigonometry* by Sundstrom and Schlicker [25] as a supplementary text for the latter course.

Question	Mean	S.D.
Overall textbook quality	4.1	0.7
Quality of discussion/explanations	3.9	0.7
Quality of examples	3.9	0.7
Quality of exercises	3.6	1.0
Quality of Geogebra applets	4.1	0.7
Quality of the MyOpenMath platform	3.7	1.0
Quality of the MyOpenMath exercises	4.3	0.5

Table 1: Student satisfaction with aspects of OER

Statement	Mean	S.D.
Open textbooks offer resources comparable to traditional textbooks without the hight cost.	4.4	0.8
Open textbooks have the potential to largely replace traditional textbooks in the college classroom.	4.3	1.0

Table 2: Student agreement with statements regarding OER

6. Discussion

As should be evident by now, adoption of OER can take a substantial amount of work. The exact amount depends to an extent on the goals of the instructor. I could have saved a lot of time and effort by compiling pre-existing exercises into homework assignments instead of spending months writing my own. However, the resulting assignments would have been less coherent and often would have strayed from the book in matters of terminology and notation. Additionally, I would not have developed the supplemental exercises that I considered so important. The customizability of OER is one of their greatest strengths, but adopters should be aware of how much work creation and modification of materials can require.

(I should also note that I could not have afforded to spend so much time on this process had I not received a grant for my work. Thus not only time, but in some cases financial, considerations must be a part of OER adoption.)

Related to this is the matter of technical skills that may be required in OER creation. My work required some familiarity with the MyOpenMath scripting language, which I had previously developed over a period of two years writing questions for IMathAS. Before that, I had spent some time working with other scripting and programming languages, including C++. Obviously, other forms of OER do not require coding skills, and to an extent the skills, much like the time, required are commensurate with the desires of the instructor. That said, OER are largely technology-dependent, and so familiarity with some technology is helpful for practically any OER.

I had specific pedagogical goals in mind when implementing OER in my class. Had I wished merely to "cover the material," I could have selected practically any open textbook available. Instead, I tried to select the book I felt was best compatible with current research on the learning of trigonometry.

Similarly, the supplemental exercises were developed with the aim of strengthening important ways of mathematical thinking that are often underemphasized. Pedagogy should always be a consideration when selecting course materials, whether open or commercial. The customizability of OER provides an opportunity for instructors to adapt and create materials to be aligned with current best practice, even as many popular curricula lag behind.

I have said little here about the third criterion of critical pedagogy in McDermott's [15] paper: cost and access to education. As McDermott points out, there is more to expanding access to education than replacing a costly textbook with a free one. My own initiative to implement OER was much more concerned with pedagogical foundations than with expanding access beyond the matter of cost, and thus I have little to say about this last matter. Additionally, McDermott also notes that relatively little has been written about the pedagogical practice and academic labor involved in implementing OER, which is the reason for the focus of this paper.

The COVID-19 pandemic forced a sudden shift in the way we used OER, and the materials proved up to the task. That is not to say that traditional materials would not have adapted to the change (I imagine WebAssign would have weathered the transition about as well as MyOpenMath did), nor did we use open resources exclusively. I believe there was, however, a benefit to using an open textbook during the pandemic. A student that, say, could not leave his or her room due to quarantine might not have immediate access to a physical textbook. Because our text was free to distribute, every student in my class could, given internet access, view the PDF copy I had uploaded to MyOpenMath, and download a copy to a personal device for when access was unavailable.

The survey indicated that students were generally favorable towards the OER that we used. Although there were some specific complaints about the textbook, student opinion of it was high (4.1). Their opinion of its textbook exercises was lower (3.6), but this number is suspect given that I never assigned exercises directly from the textbook. Surprising to me was the high rating (4.3) students gave to the MyOpenMath exercises, considering how often they reported bugs in these problems. Perhaps their direct involvement in the debugging process led them to a more sympathetic view of the problems inherent in implementing a vast batch of exercises for the first time.

7. Conclusion

For years, the conversation around OER has been largely concerned with reducing the cost of education and student success using OER versus commercial materials. In this paper I sought to shed light on two topics that have received much less attention: the labor required to implement OER in a college class, and the pedagogical considerations that go into such an implementation.

My own implementation of OER involved a great deal of work creating new material and required technical skills that the average instructor may not possess. This should not put anyone off from adopting OER, though. I had specific goals that I felt could best be met by creating new materials, and funding to help accomplish those goals. An instructor who wishes merely to adopt an open textbook for a course will find an abundance of options already available.

I used my move to OER as an opportunity to better develop important ways of mathematical thinking that are often underemphasized. I chose a book that I felt best suited this goal and developed numerous homework problems designed to exercise these ways of thinking. I hope that, as more courses transition to OER, instructors will use the flexibility of the medium to enhance and improve their teaching.

Open educational resources have the potential to improve education in ways that go well beyond providing students with free textbooks. I hope that by better understanding the process involved in their implementation and by applying best practice in their use, we can begin to realize that potential.

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References

[1] Abramson, Jay, *Algebra and Trigonometry*, online textbook available at https://openstax.org/details/books/algebra-and-trigonometry, 2015.

[2] Blackett, Norman, and David Tall, "Gender and the versatile learning of trigonometry using computer software," pages 144–151 in *The Proceedings of the International Group for the Psychology of Mathematics Education XV*, edited by F. Furinghetti (International Group for the Psychology of Mathematics Education, Assisa, Italy, 1991).

[3] Blomgren, Constance, "OER awareness and use: The affinity between higher education and K-12," *International Review of Research in Open and Distributed Learning*, Volume **19** Number 2 (2018), pages 55–70. doi:10.19173/irrodl.v19i2.3431

[4] Breidenbach, Daniel, Ed Dubinsky, Julie Hawks, and Devilyna Nichols, "Development of the process conception of function," *Educational Studies in Mathematics*, Volume **23** Number 3 (1992), pages 247–285. doi:10.1007/bf02309532

[5] Bressoud, David. M, "Historical reflections on teaching trigonometry," *The Mathematics Teacher*, Volume **104** Number 2 (2010), pages 106–112.

[6] Clinton, Virginia, and Shafiq Khan, "Efficacy of open textbook adoption on learning performance and course withdrawal rates: A meta-analysis," *AERA Open*, Volume **5** Number 3 (2019), pages 1–20. doi:10.1177/2332858419872212

[7] Coffman-Wolph, Stephany, and Kimberlyn Gray, "Exploring issues faced by students in STEM fields: First year focus and first generation focus," 2018 FYEE Conference (Glassboro, New Jersey, July 25, 2018). Full paper available at `https://peer.asee.org/full-paper-exploring-issues-faced-by-students-in-stem-fields-first-year-focus-and-first-generation-focus.pdf`, last accessed on July 27, 2023.

[8] Confrey, Jere, and Erick Smith, "Splitting, covariation, and their role in the development of exponential functions," *Journal for Research in Mathematics Education*, Volume **26** Number 1 (1995), pages 66–86. doi:10.2307/749228

[9] DeJarnette, Anna. F. "Students' conceptions of sine and cosine functions when representing periodic motion in a visual programming environment," *Journal for Research in Mathematics Education*, Volume **49** Number 4 (2018), pages 390–423. doi:10.5951/jresematheduc.49.4.0390

[10] Edwards, Barbara S., and Michael B. Ward, "Surprises from mathematics education research: Student (mis)use of mathematical definitions," *The American Mathematical Monthly*, Volume **111** Number 5 (2004), pages 411–424. doi:10.5948/UPO9780883859759.018

[11] Florida Virtual Campus, *2016 Student Textbook and Course Materials Survey*, Tallahassee, FL, 2016. Available at https://www.flvc.org/documents/96858/931951/2016+Student+Textbook+Survey.pdf/591cf5b0-bbe8-406d-acd8-b23d89b8577f, last accessed on July 27, 2023.

[12] Griggs, Richard. A., and Sherri L. Jackson, "Studying open versus traditional textbook effects on students' course performance: Confounds abound," *Teaching of Psychology*, Volume **44** Number 4 (2017), pages 306–312. doi:10.1177/009862317727641

[13] Hilton III, John, "Open educational resources and college textbook choices: A review of research on efficacy and perceptions," *Educational Technology Research and Development*, Volume **64** Number 4 (2016), pages 573–590. doi:10.1007/s11423-016-9434-9

[14] Krüger, Katja, "Functional thinking: The history of a didactical principle," pages 35–53 in *The Legacy of Felix Klein*, edited by Hans-Georg Weigand, William McCallum, Marta Menghini, Michael Neubrand, and Gert Schubring (ICME-13 Monographs, 2019). doi:10.1007/978-3-319-99386-7_3

[15] McDermott, Ian, "Open to What? A Critical Evaluation of OER Efficacy Studies," *In The Library With The Lead Pipe*, (February 2020), available at http://www.inthelibrarywiththeleadpipe.org/2020/open-to-what/, last accessed on July 27, 2023.

[16] Moore, Kevin C., "Coherence, quantitative reasoning, and the trigonometry of students," pages 75–92 in *Quantitative Reasoning and Mathematical Modeling: A Driver for STEM Integrated Education and Teaching in Context*, edited by Robert L. Mayes, Larry L. Hatfield, and Shashidhar Belbase (University of Wyoming, Laramie, WY, 2012). Available at http://www.uwyo.edu/wisdome/_files/documents/Moore.pdf, last accessed on July 27, 2023.

[17] Moore, Kevin. C., "Making sense by measuring arcs: A teaching experiment in angle measure," *Educational Studies in Mathematics*, Volume **83** Number 2 (2013), pages 225–245. doi:10.1007/s10649-012-9450-6

[18] Moore, Kevin. C., Irma E. Stevens, Tao Paoletti, and Natalie L. F. Hobson, "Graphing habits: 'I just don't like that'," pages 16–30 in *Proceedings of the 19th Annual Conference on Research in Undergraduate Mathematics Education*, edited by Tim Fukawa-Connelly, Nicole Engelke Infante, Megan Wawro, and Stacy Brown, (Pittsburgh, PA, 2016).

[19] Moore, Kevin. C., Jason Silverman, Tao Paoletti., and Kevin LaForest, "Breaking conventions to support quantitative reasoning," *Mathematics Teacher Educator*, Volume **2** Issue 2 (2014), pages 141–157.

[20] Oehrtman, Michael, Marilyn Carlson, and Patrick W. Thompson, "Foundational reasoning abilities that promote coherence in students' function understanding," pages 27–41 in *Making the Connection: Research and Teaching in Undergraduate Mathematics Education*, edited by Marylin Carlson and Chris Rasmussen (*MAA Notes*, **73**, 2008).

[21] Oehrtman, Michael, "Collapsing dimensions, physical limitation, and other student metaphors for limit concepts," *Journal for Research in Mathematics Education*, Volume **40** Number 4 (2009), pages 396–426.

[22] Paoletti, Tao, Irma E. Stevens, Natalie L. F. Hobson, Kevin C. Moore, and Kevin R. LaForest, "Inverse function: Pre-service teachers' techniques and meanings," *Educational Studies in Mathematics*, Volume **97** (2018), pages 93–109. doi:10.1007/s10649-017-9787-y

[23] Seaman, Julia. E., and Jeff Seaman, *Freeing the Textbook: Educational Resources in U.S. Higher Education, 2018*, Babson Survey Research Group, 2018. Available at `https://www.bayviewanalytics.com/reports/freeingthetextbook2018.pdf`, last accessed on July 27, 2023.

[24] Stewart, James, Lothar Redlin, and Saleem Watson, *Algebra and Trigonometry*, Fourth Edition, Cengage Learning, 2016.

[25] Sundstrom, Ted, and Steven Schlicker, *Trigonometry*, online textbook available at `https://open.umn.edu/opentextbooks/textbooks/trigonometry`, 2016.

[26] Thompson, Patrick W., "The development of the concept of speed and its relationship to concepts of rate," pages 179–234 in *The Development of Multiplicative Reasoning in the Learning of Mathematics*, edited by Guershon Harel and Jere Confrey (SUNY Press, Albany, NY, 1994).

[27] Thompson, Patrick W., "Quantitative reasoning and mathematical modeling," pages 33–57 in *New Perspectives and Directions for Collaborative Research in Mathematics Education*, edited by Larry L. Hatfield, Scott Chamberlin, and Shashidhar Belbase (*WISDOMe Monographs*, **1**, University of Wyoming, Laramie, WY, 2011).

[28] Thompson, Patrick. W., and Marylin P. Carlson, "Variation, covariation, and functions: Foundational ways of thinking mathematically," pages 421–456 in *Compendium for Research in Mathematics Education*, edited by Jinfa Cai (National Council of Teachers of Mathematics, Reston,VA, 2017).

[29] Thompson, Patrick. W., and Guershon Harel, "Ideas foundational to calculus learning and their links to students' difficulties," *ZDM - Mathematics Education*, Volume **53** Number 3 (2021), pages 507–519. doi:10.1007/s11858-021-01270-1

[30] United Nations Educational, Scientific and Cultural Organization (UNESCO), *Open Educational Resources*, website available at https://www.unesco.org/en/open-educational-resources, last accessed on July 27, 2023.

[31] Wiley, David, "The Access Compromise and the 5th R," blog post at *improving learning*, posted on March 5, 2014, available at https://opencontent.org/blog/archives/3221, last accessed on July 27, 2023.