Discordium Mathematica - A Symphony in Aleph Minor

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Discordium Mathematica

A Symphony in Aleph-minor

-Or, a Vision in a Dream, fragmented...

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“\textit{The heavenly geometries are nothing...}
\textit{but a continuous cosmic symphony of manifold voices,}
\textit{to be perceived by the intellect, not by the ear;}
\textit{a river of inferences and deductions,}
\textit{which, through discordant tensions,}
\textit{through syncopations and cadenzas, as it were,}
\textit{progresses toward certain pre-designed lemmic cadences,}
\textit{and thereby sets landmarks of proofs and theorems}
\textit{in the immeasurable flow of Mathematics}”

- Johannes Kepler

“\textit{Harmonice Mundi}”

(modified) \footnote{The original reads: “\textit{The heavenly bodies are nothing but a continuous song for several voices, perceived by the intellect, not by the ear, a music which, through discordant tensions, through syncopations and cadenzas, as it were, progress toward certain pre-designed six-voiced cadences, and thereby sets landmarks in the immeasurable flow of time}.”}
Overture

Votive Offering

Concentu canticum mundi

The Harmonious Song of the World

Lead us beyond this Reality to the higher Divine
Lead us from our Darkness to the Light of Mathematics
Move our Wonder from Myth to Myth
Take our Ignorance from Enlightenment to Enlightenment

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My heartfelt gratitude toward Aniketh Ganesh of Indian Institute of Technology, Madras, and Prof. Pavan Kumar Hari of Indian Institute of Technology, Bombay, for crafting this very well-balanced Sanskrit verse to capture the spirit of the votive offering in English. I am also indebted to Prof. Mythili Vutukuru of Indian Institute of Technology, Bombay, for facilitating and checking these translations, creating the other Sanskrit verses in this poem, and meticulously reading the manuscript to suggest changes and corrections.
Chapter 1

Scriptum : The Book of Calligraphy

Mathematica scriptum est in caelo

Mathematics is Written onto the Heavens

(wherein is described how Mathamos, the immortal and irascible God, fashioned Mathematics out of Nothingness to satisfy his desire for perfection by churning the primordial Void . . .)

Let us dwell on an ancient lore, my friend,
for the crackling fire burns strong against salted air,
a penny - if that - for this arcane yarn,
an hour but quarter if you spare.

Upon a time once,
long before the faery broods drove \(^3\)
    Nymph and Satyr from the prosperous woods;
long before King Oberon’s bright diadem, Sceptre, and mantle,    clasp’d with dewy gem,
    frighted away the Dryads and the Fauns;
longer even before

\(^3\)The lines from “Upon a time...” to “Dryads and the Fauns” are from John Keats’ epic poem, “Lamia”. The two lines from “the holy lamb...” to “clouded hills...” are from William Blake’s poem, “And Did Those Feet in Ancient Times”
the holy lamb of God on England’s pleasant pastures was seen,
or the Divine Countenance shone forth upon our clouded hills...

Longing for the harmonies of the yearning heavens,
restless in the absence of the Inevitable,
in Xanadu then did Mathamos a stately pleasure-dome decree:
  a resplendent cathedral of runes and siren strain,
  a celestial palace of mysteries and logic’s mien,
  a verdant garden of layered thoughts;
  a decadent library of deduced *bon mots*;

For *Mathamos, The Risible*, reigned
as the immortal God of Mathematiks;
as a gentle tyrant of Ideas,
as a giver of Epiphany’s burning wicks.

Of visage wise, and temper poised,
of silken touch, a pattern-weaving artiste;
a luminous mind ‘mongst equals, please bow,
to wit: the finest of celestial *Platonists*.  

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4Samuel Coleridge’s poem, *“Kubla Khan - Or, a vision in a dream. A Fragment.”*, has the line: *“in Xanadu then did Kubla Khan a stately pleasure-dome decree”*

5The Greek philosopher, Plato, had postulated a realm of perfect, timeless, unchanging forms; idealized and flawless blueprints of everything we see in our own reality. A belief in this idea is termed, “Platonism” and its believers, particularly mathematicians, are called “Platonists”.
A monument He desired for His sacred craft,
a temple for worship by the seeker, the haunted;
a fertile land of the siren’s honeyed calls,
a savage place! As holy and enchanted!  6

And yet! What is mere desire . . . without the bone of industry?!
But a pauper paper fiat . . . without the royal seal . . .
As a crack of silent lightning . . . with thunder buried in its jagged belly,
the Desire’s insistent agony . . . held His thrall without appeal . . .

Hark! Over bottomless, green fire-pits of Atashgah then,7
Mathamos did bake the preternal themes of nescience;
as Deep calling Deep,8 from Hadal bottoms of Abaddon,9
He crystallized Quintessence’s quivering prescience.

In Unobtanium 10 vats of Vulcania He stirred
the irreverent, pristine Axioms of the Naught;
of Ideograms 11 did He cast a gossamer web,
hypotheses ‘brodered in the filigree He wrought.

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6 This line taken from Coleridge, ibid.
7 In our reality, an “Atashgah” or “Ateshgah” is an ancient Temple of Fire, revered in Zoroastrianism.
8 Psalm 42:7, the Bible, has the phrase: “Deep calling to Deep”
9 “Abaddon” is the angel and guardian of the Biblical Abyss.
10 “Unobtanium”, also spelled, “Unobtainium”, generally refers to an element which has all the desired characteristics and properties one needs, but one which is not accessible in our reality.
11 An “ideogram” is a pictorial representation of an idea or a concept.
For a preternity He churned the Ocean of the Void, (as Hindoo gods and demons craving Elixir from sacred sea),\textsuperscript{12} and toiled! How! in the Stygian Depths of light devoid, (as another god later, in his own Herculean spree).

In the abysmal black of Powehi’s \textsuperscript{13} heart then stirred an ache of artistry, a benevolence of beatific grace, a pregnant instinct brushing watercolors of uncounted hues, to efface the glooming sighs plumbing its restless scarface.

Lo! At long last then, on wings of burning tempest arose a brilliant anadem too blinding on the eye; of dimensions unfathomed by the mightiest scale of stature such to put all Beauty to Lie.

In blazing ardor rose gleaming the ‘Order of the Symbols’; in the incommensurable Light of Zohar \textsuperscript{14} was it bathed; a ringing runestone of the Void’s unvoiced needs,

\textsuperscript{12}In Hindu mythology, the gods and the demons once formed an alliance to churn the “Ocean of Milk” to obtain the elixir of immortality, called “Amrut”. The fascinating story is generally referred to as “Samudra Manthana” and is well worth a deep reading.

\textsuperscript{13}The Hawaiian mythology speaks of “Po”, which can variously mean, “formless, fathomless chaos before creation” to “the utterly dark, obscure realm of gods”. “\textit{wehi}” is the Hawaiian word which means “to embellish or adorn”. Hence, “Powehi” means “an ornamented, abyssal place of ceaseless chaos and creation”. It appears in the Hawaiian creation-chant called “\textit{Kumulipo}”. Notably, in 2019, the supermassive black hole in the M87 galaxy 54 million light-years away became the first black hole to be imaged visually and was named, “\textit{Powehi}” by University of Hawaii-Hilo professor, Larry Kimura.

\textsuperscript{14}“Zohar”, meaning “luminescence” or “radiance”, is a fundamental text of \textit{Kabbalah}, the system of mystical thought of the Jewish people.
unplumbed, it was, uncontained, unscathed…

As ancient Indra’s jeweled net of pretty fables;\(^\text{15}\) the oeuvre of curving Geometry loosed its serpentine weave; an infinitely fluttering scroll of verities unveiled, meandering proofs adorning its touchstone sieve.

A labyrinthine pool of symmetries, stratagem, forms, and claims; self-reflecting, self-observing, ‘twas a tiara of sequined light; a paradigm unblemished, nor touched by calloused flame; it was… truth be told… an albatross in trans-luminal flight.

In endless, completed perfection, the Provable and the False rippled; the haunting majesty of its Olympian grandeur casting Shadow, across the Continuum spread its scriptured calligraphy, in thrall the souls and shepherds of Elysian meadows.\(^\text{16}\)

The Shape of every form it was, the Form of every shape, of formulae and fertile formulations, of soliloquies and singular solutions.

Interlocked, interconnected… no beginning nor of end,

\(^{15}\)“Indra” is the king of gods in Hindu mythology. “Indra’s Net” is a wonderfully mystical object of infinite extent, fully reminiscent of the modern concept of fractals, with its infinitely repetitive, involuted self-symmetry.

\(^{16}\)In Greek mythology, “Elysian Fields” refer to the fresh, peaceful resting place for the souls of gods and mortal heroes after their deaths.
of harmony and rhythm it was, unmarred by tear or rend;
an intricate melody of emanescence,\textsuperscript{17} unrestrained in Heaven’s vault;
limitless its fount of \textit{metaphora},\textsuperscript{18} ungraspable, irradiant \textit{gestalt}.

Thus did \textit{All-Mathematiks} repose in its Platonic haunt,
an artisanal beacon of riddles, this \textit{Jeweled Branch of Horai};\textsuperscript{20}
a Philosopher’s puzzled Dream, a Seer’s limpid Charm,
‘ \textit{The Book of Supreme Fascist} ’,\textsuperscript{21} scribed for the worthy magi.

Surveying His work sublime, carved out on ivory tower’s frame,
the sage scholar in Mathamos, with tearing eyes proclaimed:

\begin{quote}
\textit{She who treads the waters of this Mathematiks}
\textit{shall don the mask of Wisdom’s tranquil face};
\textit{He who drinks from its eternal spring,}
\textit{Inner pool of halcyon light shall grace…} \\
\end{quote}

\begin{flushright}
\textsuperscript{17}Emanescence = a refulgent, inexplicable, \textit{sui generis} glow emanating from within, with no external cause.
\textsuperscript{18}“metaphora” is the original Greek word for “metaphor”.
\textsuperscript{19}“Gestalt” means a structure, an experience, a concept which transcends the totality of its individual parts and therefore, means much more as a whole than when deconstructed into its components.
\textsuperscript{20}In Japanese folktales, moon-Princes \textit{Kaguya} sets impossible tasks for her five suitors. The second suitor is asked to obtain a jeweled branch from the mystical trees growing on the floating mountain-island of \textit{Horai}.
\textsuperscript{21}The legendary, extremely quirky mathematician, Paul Erdos, used to make references to an imaginary book containing the best, most elegant proofs of every mathematical theorem, \textit{“The Book”}. This Book, he maintained, was in the care of a very secretive, miserly Creator who gave humans a glimpse into some of the proofs only once in a while, thus earning the sobriquet, “the Supreme Fascist”…
Chapter 2

Jal-Paatram\textsuperscript{22} : The Book of Waters

fluxus geometricus de mathematicus

The Geometric Flow of Mathematics

\textit{(wherein we read that Mathamos finds in his creation of perfection some signs of decay, and ameliorates it by fashioning the nourishing river of Logos to constantly wash away the moss of entropy...)}

Having builded this marvel of \textit{Ein Sof} \textsuperscript{23} resplendent, the decadent, enraptured hymn of timeless seal, \textit{Mathamos} stood at one end of the heavens for untold time, sated, watching as proud parent, that emblem \textit{non pareil}... 

A Trinity of \textit{Essence, Unity, and Perfection} it was, its marbled philosophy stretched out beyond the ken, blooming as the radiance of a hundred thousand suns, as watch-work supreme... the flowering splendor of that mighty Zen.

Such majesty! Such unbearable aura! Such a flawless cryptogram! A supernal flux in paradigm, rewriting Heaven’s coded engrams... 

\textsuperscript{22}In Sanskrit, “jal” = water; “paatram” = vessel. The “t” sound is soft, as in “thread”.

\textsuperscript{23}“Ein Sof” is an extremely rich, holy phrase in Hebrew signifying varied meanings; eternal/endless light; divine essence without limit, and so on.
Then it was that Mathamos espied
in His Mathematiks, in the passing flow of meta-time:
a curious diffidence in its sterling composition;
a quivering hesitation in the proclamations sublime;

    a quiet murmur displacing its spectral incantation;
    a shimmering in its rigid curtains of disputation;
    some smudged marring of fogging illusions;
    some shadows cast as uncertain allusions;
    a grey fading in pearly, lustrous principles;
    a clouding in its crystalline flow barely visible. . .

    a dark stain swirled in that elixir-ed chalice;
    minor creaks in the staircase of its deductive chains
    an *ignis fatuus* 24 afflicted that rainbow borealis,
    as scratches sandpapered in the smooth willow’s grain. . .

Perplexed He was of this skewness in His artwork,
a ripple of annoyance disturbed His calm,
in what manner - and *why?* - did this infection lurk,
a syncopation profaning His hymnal psalm?

Believed He not that beauty excellent

---

24 “*Ignis fatuus*” is a Latin phrase which refers to a deceptive light typically found over marshes and swamplands.
had some strangeness in proportion.  

Believed He not an approximation

was in the graces of Platonic Form.

Of this marbled Index of Thought,
an ageless brooding did He demand,
would that this Confusion of Mathematiks bent to His Will
if an iota’s mote did He command!

So Mathamos, in abstracted introspection, mensurated
in totality the edifice of Mathematiks with analytic mind,
twirled facets and categories of its gnarled topology,
un-sheaving philosophic topoi in queries of gedanken kind;

“Does a Canon sustain ad perpetuum in Isolation?” (he asked)

“Does a Faith call for recurring Prophets of Persuasion?

Does a Creed demand Reassurance’s comforting coil?

Does a flickering Lamp importune a flow of replenishing oil?”

\[25\text{Francis Bacon: “There is no excellent beauty that hath not some strangeness in the proportion.”}\]

\[26\text{A “topos” (plural, “topoi”) generally means a specific theme or formulaic structure in a given form of art. In the context of mathematics, some of the most abstract structures involve “Topos theory”, “Sheaf theory” and “Categories”. “Topology” is a branch of mathematics which treats geometrical objects and mathematical spaces as malleable constructs, without giving regard to distances or shapes, per se. This is why it is sometimes referred to as “Rubber-sheet Geometry”. A “Gedankenexperiment”, in German, refers to a thought imagining hypothetical situations where some theory, hypothesis, or principle is subject to analytic evaluation to determine its consequences. This concept featured heavily in the works of Ernst Mach and Albert Einstein, entering into popular lexicon through their celebrity.}\]
Does the Yggdrasil 27 crack and crinkle without motherly sap?

Does the Void but for the Eleven Rivers parch in the searing Orb’s lap? 28

...and there!

With its swaying curves of sensual enlightenment, Revelation unknotted His cogitation’s binds; the solution to slaking His thirsty Mathematiks unraveled to Him as a sinuous stream unwinds.

Then Mathamos did say unto Himself:

“Come, let us build ourselves a Mountain,
with an apex that reaches to the Absolute Halcyon,
so that We may establish
the dominion of Mathematiks
for Ourselves forever;
else It shall be scattered in the Great Void without Qualm.”29

---

27 “Yggdrasil” is the immense, sacred tree in Norse mythology. Around this tree do all the nine worlds and all of existence swirl, thus making it the centre of creation.

28 In Norse mythology, creation spouts from a place of fire and ice, where eleven mighty rivers pour their waters into the great Void.

29 The six lines from “Come, let us” to “without Qualm” are borrowed from Genesis 11:4. The Bible, with changes. The original verse reads as follows:
“Then they said, “Come, let us build ourselves a city, with a tower that reaches to the heavens, so that we may make a name for ourselves; otherwise we will be scattered over the face of the whole earth.”
And so saying,

*Mathamos,*

He builded for Mathematiks, He did,

a mountain soaring and high of the heavens.

*Mount Analogue*[^30] He named it, and thus it was known,

amongst the pious as amongst the heathens.

Its highest summit touched the empyrean Sphere

of self-proclaiming, untold axioms of Infinity,

Its base branching to manifold foothills

reaching the inaccessible rings of Eternity…

Dark was its immense form, forbidding!

as a Machiavellian Secret of inner palace intrigue;

baritone its rumbling, as *Voice of God on Judgment Day,*

daring intrepid logicians to death, O Fear!

On slippery logic of its ramparts oblique…

Its genteel faces and sheer walls,

how finely chiseled they were to functional forms!

Earthy and unworldly in the very same breath,

its flowering patterns exhausting all tilings and transforms…[^31]

A crystalline pool of limpid logic filled its high-caldera,

[^30]: “Mount Analogue” is the title of a mathematical novel written by Rene Daumal

[^31]: “Tilings”/“Tessellation” is a branch of geometry which studies non-overlapping covering of a surface with shapes.
with waters so blue they arrested every thought’s decree;  
o’er seventeen sides of forbidding precipice roared  
the *Waterfall of Induction*, spraying glacial-melt’s *esprit*. 

A myriad masks of abstract Number-systems  
grazed as surreal sheep in its basins real and complex;  
Geometries un-thought of in myths strafed and curled,  
illimited manifolds gushing forth, geyser from the low-simplex.  

Platonic solids not limited by five reclined as sculptures  
in immeasurable gorges without distancing metrics;  
Projective rays and tropical semi-rings dazzled  

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32: “Induction” is an interesting method of proving statements involving natural numbers, akin to the process which describes the falling of dominoes.

33: “Surreal numbers”, invented by the legendary mathematician John Conway, can be thought of as a “superset” of all types of numbers – Robinson infinitesimals, real numbers, and transfinite numbers, amongst others.

34: “Simplexes” are higher-dimensional generalizations of triangles. They are “manifolds with corners”. A “manifold” is any mathematical space which looks like the Euclidean space in the neighborhood of any point in that space.

35: A “Platonic solid” is a 3-dimensional object with identical, symmetrical polygons as faces. It is very easy to prove, as the ancient Greeks did, that there can be only five Platonic solids. In four dimensions, there can be six “Platonic Polytopes”. Beyond that, any higher dimension contains only three Platonic “solids”. Of course, since *Mount Analogue* in this myth is beyond “our” mathematics, and not limited by “our” logic, who is to say it must be limited to only five platonic solids?!

36: A “metric” in a given mathematical space is a concept which measures “distance” between the components of that space in some useful manner while satisfying some basic conditions (e.g. the metric “distance” cannot be negative).

37: Indeed, there are mathematical concepts with delightful names like “Tropical semirings” and “arctic algebra”! “Tropical geometry” arises when, in ordinary algebra, “addition” is replaced by “minimize / maximize” and multiplication is replaced by “addition”. “Rings” and “Semi-rings” are abstract algebraic structures. “Projective Geometry” is a study of specific types of extensions of Euclidean geometry, including “Perspective” / Shadow geometry, which (loosely) looks at behavior of higher-dimensional objects from...
alongside mappings transcendental and trigonometric.

Erudite theorems universal spanned across... suspended...
as misting bridges on the deepest chasms,
Mathematical morphology seeped in every crook and gully,\(^{38}\)
in every one of its speluncean\(^{39}\) spasms.

Filtered ice scattered clear in Analogue’s furrowed topology;
boulders of epsilon calculus lay bare on plateaux of homology,\(^{40}\)
form-changing Groups of Operators saddled its passes and ledges,
Differential algebra shaped its pinnacles, flowing on its ridges...

Model leaves! Fractal ferns! Nested Trees! Fine topoi! \(^{41}\)
Quivered mirrors of puzzled fevers! Axis Maximus Mundi! \(^{42}\)

Thus was the grandeur of Analogue,

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\(^{38}\)“Morphology” refers to the study of geometric structures using tools from disparate branches of mathematics.

\(^{39}\)“Spelunking” refers to the exploration of caves and underground caverns. “Speluncean” is its adjective form.

\(^{40}\)“Epsilon Calculus” is a technical concept in Mathematical Logic. “Homology”, in mathematics, is a complex topic referring to a branch of topology. In its simplest form, it refers to a study of geometric spaces in terms of their distinct components and the various manners in which they relate to each other.

\(^{41}\)“Model Theory” is another very abstract branch of mathematical logic. “Fractals” are complex geometric figures boasting “fractional dimensions”, unlike regular geometric figures which fit in one, two, three, or higher integral dimensions. In network/graph theory, a “tree” is a combination of nodes where any two are connected by exactly one path. This leads to esoteric topics like Kruskal’s Tree Theorem and the fearsome TREE and SSCG functions...

\(^{42}\)“Axis Mundi” is a literary concept referring to an imaginary axis which connects the opposite poles of reality around which the universe revolves.
the mountain-father of Mathematiks,
fountainhead of the mighty river, Oh Logos!
her brooks to refresh its salubrious magic.

Then Mathamos once again did say unto Himself: 43

“Whomever believes in Me he knows,
as the Scriptures of the Void have proclaimed,
I am the Zero and I am the Omega,
the Beginning and the End.

Out of Mount Analogue’s heart will flow
a river from Living Logic’s lake;
to this thirsty Mathematiks I shall hence give
from the well-spring of primum mobile sans break; 44

and wherever shall this river go in its bends,
every theorem shall swim again as a vibrant proof,
and there will be very many theorems without end;
nary a deduction shall escape, or false ears lend.

43 The lines from “whomever believes in Me . . .” to “river goes; amen!” have been transpositioned from Ezekiel 47:9, John 7:38, and Revelation 21:6, The Bible, with significant changes.

44 Primum Mobile was a concept espoused by the ancient philosopher, Ptolemy, who postulated this 10th sphere of heaven, embedded in the Empyrean, as the one providing impetus to the entire universe in its rotation around earth.
This water, it travels of its volition just so
the souls of infinite algebras may become fresh stamen; ⁴⁵
and every omen of logic shall prove itself ever and anon
where ever this holy wine spills its grace; amen."

And so saying,
from this mount of Meru,⁴⁶ with a single strike of thunder,
Mathamos provenanced the thousand-headed River of Logos;
to irrigate the sweeping reaches of sweet Logic,
buttressing the topology of Geometry’s wavering cosmos.

In a hundred directions, down Alexander’s Infinite Horns,⁴⁷
the crystal-bright Logos fled from Analogue to establish its prominence,
as the frightened Satadru had scattered once;⁴⁸
as the holy Ganges had drowned Shiva’s countenance;⁴⁹

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⁴⁵“Stamen” is the fertile part of a flower producing pollen grain involved in the fertilization of the flower.
⁴⁶In many Indian and far-East mythologies, “Mount Meru” is the sacred mountain at the centre of the universe.
⁴⁷“Alexander’s Horned Sphere”, discovered by J. W. Alexander in 1924, is a mind-tickling topological entity, quite bizarre to look at and understand. It is formed by repeatedly splicing and reconnecting a torus, which is a donut-shaped geometrical object.
⁴⁸“Satadru” (modern day Sutlej river in the Himalayas) is the hundred-branched river of Hindu mythology, arising from the sacred waters of river Haimavati. The myth recounts how one of the greatest sages, Sage Vasishtha, lost a hundred sons to a cannibal, and in grief, tried to kill himself by jumping in the river Haimavati. Given the divinity of the sage, the river wished him no harm and split itself into a hundred branches, fleeing from the scene, thus getting christened, “Satadru – the hundredfold”.
⁴⁹In Hindu mythology, the great king, Bhagiratha, after intense penance, was granted a boon by the heavenly river, Ganga, that she would descend on Earth to wash off the sins of all his ancestors. However, the force of her descent would have drowned and destroyed earth so Lord Shiva had to use his head as a breaker and let her powerful waters flow down
Across the intricate firmament of Logic did Logos flow, ceaselessly carrying the messages of its unfounded Axioms, and all the theorems of their own dialectic domains, were established and cradled in its lexicon of Idioms.

The rational currents of Logos they were which watered the fertile lands of Mathematiks, with nourishing silt on inferencing waves, and deductions of its diktats dogmatic.

A fluid covenant of faith did that Okeanos of Mathematiks carry, encircling, binding the mercurial threads in a whole-cloth fabric, heralding from crescent to crescent, across the Kingdom; reasserting - over and over - the vast Dominion of Arith-metik.

Of foaming tides of ratiocination - and high winds! the flooding edge of that quicksilver river was, in secret tongues it spoke with the lemmas, and with profound theorems, espousing their cause.

This all-knowing Saraswati was the ether in which Logic pulled its oars, ferrying the seeds of the axioms to new-found-lands in their wake, its flooding sway was the missionaries, the prophets, the apostles, through his hair-locks as the earth-river, the Ganges.

50 In Greek mythology, Okeanos was the river which circled the entire earth, becoming the well-spring of all the nourishing, life-giving water bodies in the world.

51 Saraswati is the hidden river of knowledge of Hindu mythology, named after the goddess of wisdom & learning.
and the shepherd dogs keeping Algebra’s errant sheep in check.

Vectors of abstract spaces, and solitons of pure logic,\textsuperscript{52} as compact argosies of magic sails sailed in Logos’s veins, filling the heavens with the commerce of Mathematiks, pilots of purple twilight, captains of Reason’s alluvial plains. \textsuperscript{53}

It was that Soma \textsuperscript{54} from a bottomless jug, which kept intoxicated the drunken Geometries; it was that nectar of alluring Mohini \textsuperscript{55} which brought stillness to its eternal Symmetries.

Thus did Mathamos establish His iron-jacket empire of ‘that which is proven’, from ‘that which also has been proven’, no decree nor divine pronouncement brooked as The Word, each truth a proven commandment, every falsity a branded scarlet sin.

The Holy Ghost in the Machine hummed with exquisite fidelity that fathomless Logic’s musical hymnals, and in sympathy, as a buxom highland sprite,

\textsuperscript{52}“Solitons” are self-sustaining, self-reinforcing structures which can move about in a fluid without breaking apart.

\textsuperscript{53}Lord Tennyson, in “Locksley Hall”: “Saw the heavens fill with commerce / argosies of magic sails / Pilots of the purple twilight / dropping down with costly bales”

\textsuperscript{54}In Hindu mythology, “soma” is a drink with diverse interpretations, but generally associated with either health-giving qualities, or one which gives a lift to the spirit through inebriation.

\textsuperscript{55}“Mohini” was an avatar of “Lord Vishnu” who stole the nectar of immortality which arose from the churning of the Ocean of Milk. See earlier footnote 12 on “Samudra Manthan”.

danced the well-ordered ordinals and cardinals.\footnote{Ordinals and Cardinals are counting concepts which enable mathematicians to assign sizes to collections of mathematical objects. For finite collections, ordinals and cardinals are the same and behave like the familiar counting numbers. Their full utility is seen once you start considering transfinite collections, like sets of real numbers. Well-ordering is a powerful technical concept in Set Theory.}

The tearing eyes of Mathamos looked upon all He had made, and indeed, it was very good, across \textit{Nihil} let it be said; \footnote{\textit{Genesis} 1:31, The Bible: \textit{“And God looked upon all that He had made, and indeed, it was very good.”}} on a bed of weariness then did Mathamos rest in seventh heaven, His Canon buil ded to His decreed pleasure; so it is said, amen!
Chapter 3

_Somnium : The Book of Dreams_

_illa desiderat caelestia velamina_

She Wishes for the Cloths of Heaven

_(in which we are told how Euclidia, our Good Lord, is seduced by the beauty of All-Mathematiks, and has visions of deploying it to build a Reality with more wonders than dreamed in heavenly philosophies...)_

Now our Lord God, _Euclidia_,
She was of the Exalted Holy Order,
a Dream Catcher, a Pattern-Matcher,
with long dominion over Disorder.

From the fiery throne of Judgment,
She presided o’er the Face of Waters Chaos,
when All was Nothing, no blessed rosary,
nor devil’s nails nor curse-ed Cross.

In a shadowy dream of _Platonia_ then,
in the hour of awakening, She descried
that Tree of the Knowledge of Good and Evil
_carved by _Mathamos_, a Tablet sanctified.

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58 Title of a poem by W B Yeats, gender modified.
59 Genesis 2-9, The Bible, refers to such a tree.
In a flash of theophany She divined:
“Thou Art That eternal truth... unknown... un-premised!”,
and desired, She did, in a smiling sleep,
to loose that ocean of truth upon Her unruly Abyss.

Begged! How She begged of Mathamos in penance!
To open the Gate of Horn in generous temperament;
let Logic sluice and shine Her disordered domain,
and release for a commoner the sermon of Sacrament...

For She visioned the ironed logic of Math would tame,
the mightiest dragons of Her fearsome Pandemonia,
bring order to the babel of malice and malarkey,
under the ethereal dominion of Platonia.

Of Laws and decrees would the new realm be;
of immutable dominoes of assertions and demonstrandum’s;
in austerity forged monastic its steeples and spires,

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60. “Thou Art That” is a translation of the Sanskrit phrase, “Tat Tvam Asi”, which is one of the “Mahavakyas” (“The Great Truths”) articulated in the Vedic canon of sacred texts called “Upanishads”. This specific phrase appears in “Chandogya Upanishad”. It is generally taken as an axiomatic truth to mean, amidst a wide variety of philosophical interpretations, that the self – you - is an undivided, even if differentiated, part of the Supreme Oversoul, the “Brahman”.

61. In Greek mythology, dreams are said to pass through one of two portals – The Gate of Ivory and The Gate of Horn. Those passing through Ivory are false dreams, those going through Horn are factual.

62. The word, “Pandemonium” was coined by the great poet, John Milton, to refer to the capital city of Hell. It literally means “All Demons”. The word has come to signify a state of complete chaos, confusion, and disorder.
a cathedral of order rising stratum by latticed stratum.

Basilic orbs in burning desire She saw, midst darkness and trembling fright; there would be Light! **There would be Light!** Blooming miracles from the depthless Maw.

A soaring *Shangri-la*\(^{63}\) of Nothingness would It sculpt, of grimoires unimagined would It be in heavens mundane; out of Zero would It calve the *Yin* and the *Yang*, shaped from uncommon legerdemain.

In her vision she saw Her Children of Math rising from primal ash to strike fire, treading fearlessly the thickets of blundered bowers in ceaseless reverence strung on Algebra’s lyre.

Seek Her they would as *Painter of Dreams*, *Mother of Destiny! Maker of Oars!* and in high homage would proclaim Her deeds, in their dreamy, philosophic lore:

> “*All things began in Order*, (they would say)
> “*and only so shall they end,*

\(^{63}\) “Shangri-La” is a mythical place set high in the Tibetan Himalayas, as described in James Hinton’s 1933 novel, “Lost Horizon”. Over the years, it has come to signify an earthly paradise, an idyllic, utopian fantasy.
and so shall they begin again,
according to the Ordainer of Order,
the mystical Mathematiks of the City of Heaven.” 64

A mirage to whet the thirst,
a distant song to lure shrewd minds,
Mathematiks letting them rest not,
beyond their kith and kind;
of wondrous, thrilling sciences,
She foresaw them building tools,
unshackling their Nature’s laws,
transcending binds of body and soul.
Unravel they would Her secrets,
unknot the puzzles woven in that Reality,
unceasing their itches, inflaming their scratches,
an apocalypse to reveal Nature’s deepest dualities . . .

And then . . . and then . . .

when Her children would explore all,
and find Her imaged in every pore of Virtue and Sinning,
they would arrive, from their travels of eighteen circles,65
. . . back at the very Beginning . . . 66

64 The five lines from, “All things began” to “City of Heaven” are borrowed from Thomas Browne’s poem, “The Garden of Cyrus”, with minor additions.
65 Dante’s nine circles of Hell and nine circles of Paradise.
66 T. S. Eliot, “and the end of all our exploring . . . will be to arrive where we started”, in “Little Gidding”
Chapter 4

Stealthium: The Book of Thieves

somniat in tenebris somnia somniat ignis

Dreams in darkness dream dreams of fire

(in which we read how Euclidia is denied her wish by an angry Mathamos, and how she steals the waters of Logos to bring order to her domain of Chaos, to build a new cosmos of wonders...)

But what is adversity in a granted wish?
Where have gods seen reason in prayer?
T’is the Lord who bringeth the cup close-er to the lips;
t’is the Lord that taketh it away-er.

Euclidia, She desired a bite of that forbidden apple
from the bountiful Orchard of Platonia’s Eden,
But Mathamos, He was a jealous god,
in wrath denied Her the divine spark’s allegiance.

Intransigent, implacable, of primal rage He was,
Mathamos the Antaean protagonist;
no apostle He brooked for his ‘rithmetik,
not even a Goddess for an evangelist.

Powerless She was, though Almighty,
Her blueprint at the precipice of ruin,
rescue her creation She would, no doubt,
with thoughts of truancy a’ brewin’…

In defiant resolution She uttered, “Thou Shalt Steal!”, and so it turned out, a commandment to the letter!
For stolen water is sweet (no doubt),
and food eaten in secret tastes better…

Ere long, Euclidia had planned Her thieving
of the analytic waters of Logos,
as Prometheus had with Zeus’s flaming phosphor
as Maui, the demigod, from Mahuika’s fingernails;
as bewitching “Mohini” with Immortality’s Elixir.

In cold, resolute light She said of Herself:
“I have dreamed a mesmerizing Vision,
and my spirit in its spring hath doubled,
Dream I shall again of that sorcerous Enchantment
and bring its fruits to the lower realms,
for that Fire belongs not just to the Ennobled!”

In a drugged phantasm She rode up to the zenith of Mount Analogue,

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67 The 8th Commandment, of course, says, “Thou Shalt Not Steal”…
69 In Polynesian folklore, “Maui” was a demigod who stole fire from the fingernails of “Mahuika”, the Maori goddess of Fire.
70 See footnote 55
71 Daniel 2:3 of The Bible, in a different context of King Nebuchadnezzar, says: “I have dreamed a dream, and my spirit is troubled”
and descried the yawning fountainhead of the mighty Logos;
in thunder-struck awe she stared into the womb of Mathematiks,
and with severe serenity the Mathematiks stared back.\textsuperscript{72}

From that abyssal chasm, with ceaseless turmoil seething,\textsuperscript{73}
as if the mountain in fast thick pants were breathing,
a mighty fountain momentely was forced,
amic whose swift half-intermitted burst,
huge fragments of mathematicons\textsuperscript{74} vaulted like rebounding hail,
or chaffy grain beneath the thresher’s flail,
and mid these dancing logicons at once and ever,
it flung up momently the sacred river. . .
Five realms it meandered with a mazy motion,
through the ridges of Analogue the sacred river ran,
then reached the caverns measureless to gods,
and sank in tumult to a booming Ocean of Nod;

Here was the fountain of youth, if there e’er existed one,
a \textit{Pierian Spring}\textsuperscript{75} Euclidia coveted to carry in Her Holy Grail,

\textsuperscript{72}Nietzsche: “\textit{stare into the Abyss for long, and the Abyss stares right back at you}”
\textsuperscript{73}The twelve lines from “\textit{from that abyssal chasm...}” to “\textit{Ocean of Nod}” are directly from
Samuel Taylor Coleridge’s poem, “Kubla Khan - Or, a vision in a dream. A Fragment.”,
with some contextual modifications.
\textsuperscript{74}“Logicons” and “mathematicons” are neologisms, expressing fundamental particles of
logic and mathematics, resp.
\textsuperscript{75}In Greek mythology, the \textit{Pierian Spring} are a metaphorical stream of knowledge of
arts and sciences. The famously witty poet, Alexander Pope, wrote, “\textit{A little learning is a dang’rous thing / Drink deep, or taste not the Pierian spring.”. Indeed, as our tale
unfolds, the reader shall see that there was more truth to this than just a pretty little
literary reference...
no two ways for her oneiric vision to bloom flower,
these waters she had to steal, in some hollow stalk of fennel...76

And so...

A cataract – the Dragon Gate77 - Euclidia set high up in the Void,
a tributary to Logos She cut to funnel into its gorge,
with a glass-blower’s patience, in mirroriam,78
a reflected copy of Mathematiks She hoped to forge.

In arcane incantations of symbology she invoked Logos
to descend in secret from its empyrean cleeme:79

“Let rejuvenation in thought roll down like crystal waters,
let rational catechism gurgle like an ever-flowing stream!

Drip down, O River Mine, from your luminous welkin!
And let the clouds pour down their copious analytics!
Let Chaos surrender; let mathematikal salvation bear fruit!
And let a rational cosmos spring, for I, the Lord, have decreed it...”80

76In the famous Greek myth, the Titan god, Prometheus, took a single spark of fire from
the Olympian god, Hephaestus's workshop and stole it by storing it in a vessel hidden in
the hollow stalk of fennel...
77In Chinese mythology, “Dragon Gate” is the legendary waterfall formed by the Yellow
River cascading down Longmen Mountains.
78in mirroriam” = in the endless memory of parallel mirrors
79cleem = clime = climate = abode
80The four lines from “Drip down...” to “spring up from it” are from Isaiah 45:8, The
Bible, with changes. The original reads: “Rain down, you heavens, from above / And let
the skies pour down righteousness; / Let the earth open, let them bring forth salvation / And let
righteousness spring up together. I, the LORD, have created it.”
Behold then!

*Logos, all-giving, all-asserting Logos did act on this Command!*

And irrigated *Dragon’s Gate* as a roaring torrent,
from where *Aleph*, the sacred river, trod
through catacombs unplumbed by gods,
down to the sunless sea of Chaos...

So twice the geometrics of Chaos’ fertile ground,
with propositions and assertion were girdled round;
and there were manifolds bright with sinuous thrills,
where blossomed many a lemma-bearing tree;
and here were thickets of thought ancient as the hills,
enfolding sunny verity quenching spots of sophistry.
But oh! So deep formed the mathematik chasm which slanted
down the infinite hill athwart paradigms supplanted!

For as the rain and the snow come down from heaven,
and do not return there without watering the earth,
and making it bear and sprout,
and furnish seed to the sower and bread to the eater.\(^2\)

\(^1\)The eleven lines from “where *Aleph*, the sacred river...” to “chasm which slanted” are borrowed from Coleridge’s poem, *ibid*, with significant changes.

\(^2\)The four lines from “for as the rain...” to “...bread to the eater” are from *Isiah 55:10*, The Bible, with changes. The original reads: “The rain and snow come down from the
So did the awakened principles of rationality flood
in profusion across the new channels of Expression,
cutting the confutations Chaos hid in her bosom,
forging lawful symbols into embossed Impressions.

_Aleph! Sweet Aleph!_ She had brought back in glee
the holy tablets of heavenly metamorphosis!
In two score years would the deserts be Promised Land,
Deliverance for Disorder, as striven by Abraham’s Moses... 83

Lo! The hour of _Euclidia’s_ desires was nigh; soon,
Her _fata morgana_ 84 would alight in Cosmos’s dusty dreams,
a sculptor’s chisel shaping the micro and the mega,
in photonic leaps would Reality rise on lux’s beams.

The shadow of the dome of pleasure 85
floated midway on the _Aleph’s_ waves
where was heard the mingled measure
from the fountain and the caves.
It was a miracle of rare device,
a sunny pleasure-dome with caves of ice!

---

83 Moses did spend 40 years wandering the desert before passing away within eyesight of the Promised Land for the Israelites...

84 A “_fata morgana_” is a very complex mirage giving seeming reality in the sky to distant illusions.

85 The lines from “_the shadow of_” to “_caves of ice_” are from Coleridge’s poem, _ibid_, modified suitably.
A child... a new mathematiks... was taking birth...

soon, the bird... a new universe... would take its maiden flight...
Chapter 5

*Kopamudra* 86 : The Book of Wrath

दीघौ हि कृूरदेवतानां बाहू
(deerghau hi krooradavataanaam baahoo)
For Long are the Arms of Cruel Gods...

(in which Euclidia’s theft is caught midstream by a thundering
Mathamos in high umbrage, who then proceeds to deny all her pleas and
lays severe curses on her utopian Platonia...)

But every thief’s misfortune!
The law has long eyes!
Rumor travels swift, it does!
A modicum of truth, a bushelful of lies!

Ere long the disturbance of affront
did a vengeful *Mathamos* reach,
in *Euclidia’s* brazen pilfer...
a sacred trust had breached.

A covenant broken, an ethic bent,
that soiling blasphemy was not to brook,
‘What Heaven hath to the Plebeian gift not! ’,

86 In Sanskrit, “kopa” = wrath; “mudra” = pose. So “kopamudra” implies a severe
demeanor of intense anger.
that unwritten code had *Euclidia* forsok.

Oh mercy! The pawn sees not the darkening shadow, the rook has raised the king in his castled reverie, of ill ague the Regent, with his kingdom in plunder, of drunken rage, the Hatter mad in the brewery!

Amidst the tumult of *Aleph’s cascade* ⁸⁷

*Euclidia* heard from distance afar
ancestral voices prophesying war!
A cavalry’s siege arriving in cavalcade.

As *Calypso*, ⁸⁸ *She* sailed faster through the warren of caverns
to outrace the Fiend on Her *Canoe of Sixty Wings*,
but on a slow morrow did Her dream-waters move, avoid not could She, alas! the angry scorpion’s sting…

As a *harpy* ⁸⁹ in high dudgeon,
*Mathamos* swooped upon Her skiff;
with a maelstrom as clasping hands,
He arrested *Aleph* by midriff.
Had stopped Euclidia for the nonce, He had,

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⁸⁷ The three lines from “Amidst the tumult” to “prophesying war” are from Coleridge’s poem, *ibid*

⁸⁸ Calypso is the sea nymph / goddess of Greek mythology.

⁸⁹ In Greek and Roman mythologies, Harpies were half-human, half-bird creatures, variously considered to be personifications of storm winds and angry gales. They were deployed by Zeus and others for vengeful purposes, earning the monikers, “Hounds of mighty Zeus”, “Ministers of the Thunderer”, and “Human-Vultures”.
and a welling of calm in him reared,
with dark reins of nightmares he polluted,
Euclidia’s dream amidst His burning tears...

And then Mathamos wept... as Jeremiah ⁹⁰ had,
for Euclidia was His favorite acolyte once;
a nettlesome transgression too far this time,
a cheating hand’s sleight in the shadows of a trance.

In stinging hurt He snarled at poor Aleph a vicious curse,
let go not He would this affront ‘gainst His obsidian shrine;
a Curse of Babel He injected into Euclidia’s ivory tower...
susurrating a dozen confused logics, her waters be swirling brine.

“I have heard your vision, Euclidia,⁹¹
which prophesies lies in my holy Name;
My anger burns, O Shepherd who misleads!
and thou children shall I punish in fair game.”

“You have dreamed, you have dreamed; Oh woe!
In desecration decamping with this sacral flame...
Woe again! If only you had alone left
on this signal Geometry your stealing claim...”

⁹⁰One of the prophets in the Abrahamic religions, sometimes called “the Weeping Prophet”.
⁹¹The lines from “I have heard” to “tiny minds” are inspired by Jeremiah 25:26 and Zechariah 10:3, The Bible.
“Though thou shalt build thy towers and thoughts, 
long will they last ‘gainst your lying prophets’ grinds?
With their pretty clay-idol gods and petty, bloodshot lust, 
prophesying delusions in their tiny minds...?”

“Alas! Thou dark deed is done, Wicked One!
own rein in the waters I must, and tarnish its flood’s aftermath;
for what thou has stolen is lost to Platonia’s exaltation, 
and invoketh, to boot, my vengeful wrath!”

I shall offend your cause and avenge Us, watch and weep!
I will boil up Your seas, and make Aleph’s springs asleep! 92
In all thy ways you know me; I shall direct your paths rough and splayed;
innumerable shall be thy thorns, pernicious obstacles to block your ways.93

“In your idle Idyll you have visioned your Vanity, O Diviner, 
thou hast seen Lie’s algebra in Time’s grain,94
Mark these words! Your people shall wander like sheep, 
in fields of falsehoods, stumbling in enterprise Vain.”

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92 The 2 lines from “I will offend...” to “springs asleep” are from Jeremiah 51, line 36, suitably modified. The original reads: “Therefore, this is what the LORD says: “See, I will defend your cause and avenge you; I will dry up her sea and make her springs dry.””

93 Proverbs 3:6 from the Book of Proverbs, The Bible says: “In all thy ways acknowledge him, and he shall direct thy paths [straight].”

94 As an aside, there is a field of Mathematics called “Lie Algebras”, introduced by the Norwegian mathematician, Sophus Lie (pronounced as “Lee”) in late 19th century...
The jeremiad of hurled invectives hurt Euclidia; surely in Her act Mathamos could charity see ingrained! The high and mighty gods in their cold, superior realms... was it petty indignation, casting off Her art in disdain?

Then Euclidia implored Him with defiance, To His quality of mercy She wished appeal, surely the heaven’s light would not dim or flicker!, if a divine spark turned Chaos’s eternal night genteel...

“Let those who seek find what they seek, Mathamos!” (She exclaimed)
“let them receive the treasure, O Great Geometer!
Let Her deduce what was lost before;
let Her push forward the Cause of Truth....

We have drunk the Soma; We who are the Architect;
We have come to the Light; We have found the Logic.
What can hostility do to us now, My Lord,
and what the malice of a vengeful thought, O Immortal One?! ”

In cold fury, Mathamos gathered the storms of ether, and blistering anarchy in infernal rendition, proclaimed a curse pregnant with blight and spleen,

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95 Rigveda (8.48.3): “We have drunk the soma; we have become immortal; we have gone to the light; we have found the gods. What can hostility do to us now, and what the malice of a mortal, o immortal one?!” translated by Stephanie W. Jamison and Joel P. Brereton in “The Rigveda — Earliest Religious Poetry of India”
of omen sepulchral, of charnel perdition...

“Euclidia, grand-daughter of The Caliph of Time,
surpassing all in craft, cunning, and guile,
glad you are you have outwitted Us
and stolen Our Mathematiks for a while;
but hear this and weep, the truant god that you are,
for the ugly mischief which you have recklessly wrought,
a great plague upon yourself and your mathematic prophets I shall loose...

and THAT! – THAT shall be your Job’s lot…” 96

“I shall give men as the price for mathematiks
an evil thing in which they may all revel, be glad of heart;
in their blind Physiks and in their profligate Arithmetik,
embrace Uncertain Incompleteness in their ramparts.” 97

“For surely I will command all of Logik,” 98
and will sift the waters of Aleph through false crevices,

96 Modified from Homer’s epic poem, “Hesiod: Works and Days”, which has Zeus cursing Prometheus for stealing fire: “Son of Iapetus, surpassing all in cunning, you are glad that you have outwitted me and stolen fire... a great plague to you yourself and to men that shall be. But I will give men as the price for fire an evil thing in which they may all be glad of heart while they embrace their own destruction.”.” “Job’s Lot” refers to all the various calamities which befell Job, a God-fearing man (and future prophet), as depicted in “The Book of Job” in the Bible.

97 We shall see what Incompleteness and Uncertainty mean in Mathematics later in this saga...

98 In Amos 9:9, The Bible, the Lord says: “For surely I will command, and will sift the house of Israel among all nations, as grain is sifted in a sieve; yet not the smallest grain shall fall to the ground.”
as grain is sifted in a sieve of crooked grain;
not the truest of grain alone shall follow from her premises.”

“And mathematicians who would easily do work enough in a day to supply theorems and magiks for a full year even without working in haste;
soon shall they put their rudders over the smoke,
and work pencils on paper as oxen and sturdy mules running to waste!”

“Your Logicians shall speak not the same language as one,
what they begin to do shall prove for them impossible;
Confuse their language, I shall, so each other they will not understand
and stop building their Mathematiks... from axioms of leaking Crucible...
That is why your “Mathematiks” shall be called “Babel”,
and your mathematical minds scattered over the face of restless do-

mains,
as forty tribes they shall wander the deserts of tangled liturgies;
forfeit this game, they shall... for I have Decreed so... amen!

99 The four lines from “and mathematicians who would” to “running to waste” are reflective of the curse laid by Zeus on Prometheus, modified in context.
100 The lines from “Your Logicians...” to “decreed so...amen!” are from Genesis 11:1-9, The Bible, modified suitably. The original reads: “But the Lord came down to see the city and the tower the people were building. The Lord said, “If as one people speaking the same language they have begun to do this, then nothing they plan to do will be impossible for them. Come, let us go down and confuse their language so they will not understand each other.” So the Lord scattered them from there over all the earth, and they stopped building the city. That is why it was called Babel because there the Lord confused the language of the whole world. From there the Lord scattered them over the face of the whole earth.”
Chapter 6

Antardhyanum\textsuperscript{101}: The Book of Metaspection\textsuperscript{102}

\begin{quote}
“I shall be the first un-driven to enter…
the entangling vortex of the maze
the twists which stable at their centre…
the beastly ghost who stains our days.” \textsuperscript{103}
\end{quote}

(in which we take a slight detour to delve into extremely important questions about the fundamental nature of Mathematiks, facilitated by the thoughts of Lord El… and hark! Close does Lord El get to proving through his cogitations that All-Mathematiks is fully Consistent and Complete – unlike the mathematics which governs the reality of humans… but He starts realizing that He needs help from another great Mind on points of nuance before the nightingale can return home…)

Pause a moment our lyric of Mathamos waxing wroth;
ponder for a moment the reflections of a deeper lake:
in all our eulogy of Supreme Algebra, have we asked:
“What stock of its Foundations did the Platonists take?”

Lord EL did, of course,

\textsuperscript{101}In Sanskrit, \textit{“antar”} = inner space; \textit{“dhyanum”} = focus. So \textit{“antardhyanum”} = introspection
\textsuperscript{102}“Metaspection” = an inspection of a field from a “meta” perspective; a “meta-analysis”; an analysis which stands “outside” of the field and analyzes its various aspects, like its internal structure, characteristics, syntax, vocabulary, relationships between its different parts, hierarchy of its structures, the kinds of analysis the field itself admits or supports, the nature of its mapping into other known fields of inquiry, and so on.
\textsuperscript{103}Jack Lindsay, \textit{“Clue of Darkness”}
Lord EL, of the Region of Porlock,
the greatest logician of Platonia was He,
unfortunate His Query’s aftershocks...

Lord EL, too, dreamt numerously of these Logiks,
and a peculiar madness of inquiry he had breathed,
rest His mind would not till it had divined,
the very nature of Mathematiks Mathamos had bequeathed.

And asked with gravity, He did,
as a Delphic mystic in brood:

"Of what fabric, this lustrous Mathematiks?
What silken threads has it imbued?"

“What fibers bind its being?
Of what structure its sprawling theme?
What woof and weft do its Needles blend?
What embroidery adorns its seams?

Does it end and close upon itself, a sphere-d continuum?
Or endless is its flaring, as the sails of Lord Ethereum?
Complete\(^\text{104}\) is it in its own image, or frays it at its weave?
Sports it holes of glaring faults, a stitched-up, leaking sieve?

\(^{104}\)A system of thought is said to be “Complete” if any statement expressible in the language of that system can be proved unambiguously to be either true or false using the accepted procedures and rules of inference and deductions in that system. It should be noted that a Complete system does not have to be of finite size – it can contain infinitely many statements which can be provably true or false.
Is its existence to Omega Consistent still?
Or does contradiction cut it crosswise?
Are its pronouncements inviolate as the Three Fates’ thread and wheel?
Or is confusion sowed in its Propositions’ unmatched ties?”

Close to meta-mathematical proof was He,
that the intricacies of Mathematiks were All they could be;
fully Complete, no orphans of proof in its infinitary spread;
fully Consistent, no contradiction’s blight in its inductive tree;

for its axioms were turgid and fulsome,
not a base thought stray or missing;
an a-tal-paatram brimming with self-evidence,
an anantapaatram flawless in its filling.

Its deductions flowing without blame or blemish,
the structure a baroque concerto for Logos’s sweet flute;
faultless to a proof the edifice stood, He thought,

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105 A system of thought is said to be “Consistent” if no statement expressible in the language of that system can be proved to be both true and false using the accepted procedures and rules of deductions in that system.

106 In Greek mythology, the three “Moirai” were 3 sisters who were in charge of ensuring that all creatures, both mortal and divine, lived out the destinies assigned to them by the governing code of reality. For this reason, they were also known as “Fates”. “Clotho” turns the spindle / wheel on which the thread of life spun, “Lachesis” measures out the length allotted to a being, while “Atropos” cuts the thread (thus marking death).

107 In Sanskrit, “a-talpaatram” = a vessel (“paatram”) without a bottom (“tal”). The word, “a-tal” rhymes with “a gull”. The “t” sound must be soft, as at the beginning of the word, “thread”. Similarly, the “t” in “paatram” is soft.

108 In Sanskrit, “anantapaatram” = a vessel of infinite, endless capacity. Again, the “t” must be soft throughout.
a seeker’s paradise in its commandments absolute!

... and yet... and yet...

The final piece of the puzzle would dislodge not from the Library of Intellect, from the Book of Epiphanies to bridge the gap... between proof... and intuition; stuttering... a soulful hymn in its moving symphony...

...and so in mumbling dream, Lord EL moaned his impassioned grief; a thwarted thought, quicksand in its tracks, of dead ends a burning quest’s leitmotif.¹⁰⁹

“Tis true, I feel! In vain have I amass’d
within me all the treasures of Platonia’s mind,
for at pause I am, and sit me down this Logic has,
no new power welling inwardly I find;
a breath of divine touches not this puzzle I beseech,
No closer, alas! No closer to Infinity’s reach...”¹¹⁰

¹⁰⁹ “Leitmotif” = an overarching, recurrent theme in a specific body of thought.
¹¹⁰ The six lines from “’tis true...” to “Infinity’s reach” are borrowed and modified from Goethe’s “Faust”. Specifically, in Theodore Martin’s translation, an anguished Faust says to Mephistopheles:
“Tis true, I feel! In vain have I amass’d / Within me all the treasures of man’s mind,
And when I pause, and sit me down at last, / No new power welling inwardly I find;
A hairbreadth is not added to my height, / I am no nearer to the Infinite.”
Chapter 7

Vichchhinnum 111: The Book of Sundering

पुण्यपात्रे नित्यदोष: 112
(punya-paatre nityadoshaha)

Girl, Interrupted...

(in which a devious Mathamos first encourages Lord El’s inquiry into the foundations of Mathematiks, and then lures him to disturb Euclidia’s sleep with his burning queries, thus obstructing her in her thieving dream... and how, as Euclidia tries to regain traction in her dream after the interruption, Mathamos pollutes her entire philosophy, pouring adulterating grains of mistrust in her granary... thus rendering her new mathematics full of Uncertainty and Undecidability...)

Seeing that vision, in His hectoring animus rose Mathamos, in Lord El did He foresee His instrument of sabotage; stop Euclidia He could not, Her Almighty power too strong, Lord EL would He use to wreck retribution for Her persiflage.

The God of Mathematiks, in earnest demeanor,
He exhorted Lord EL ‘s scholarship in erudite phantasm,
Truth He did speak, to be fair, though ever the schemer,
of Logik’s clarion call to bridge a metaphysic chasm...

A singing madness He breathed deep in Lord El ‘s fancy,

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111 In Sanskrit, “vichchhinna” = broken; sundered; sundering
112 In Sanskrit, this phrase means “an eternal flaw in the sacred vessel”
a manic madness questioning the soul of Mathematiks, surely that lofty essence of *Mecanique Celeste*\(^\text{113}\) hath not cracks unseen in its foundation’s burly bricks!

> “Spread the great knowledge of Metaspection, O First Amongst Equals!\(^\text{114}\)

*Blessed is this search for the Nature of Order and Meaning of Forms;
Spread thy querying wisdom ‘midst the Learners of our Realm;
of higher echelons, this exegesis of mathematik’s norms. . .”\(^\text{115}\)

*Mathmos* shewed\(^\text{116}\) Lord EL a vision of Euclidia’s virtuosity;
She, in heady bravura, sating queries Lord EL would inquire;
urged Him, *Mathamos* did, to seek Her counsel immediate,
a transcendent Scholar courted by the genius Squire.

And so it came to be that the Great Logician arrived at the steps of a dreaming Euclidia,
The *Tragedy!* Oh, the *Tragedy*!
Unawares was He of *Mathamos*’s simple *insidia*.\(^\text{117}\)


\(^\text{114}\) “First Amongst Equals” is the title bestowed upon the Hindu god, *Ganesha*, son of *Lord Shiva* and Goddess *Parvati*. It traces back to a fascinating myth about his wisdom in winning a race, when all the gods were asked to circumnavigate the entire universe in a test of the speeds of their respective animal-carriers.

\(^\text{115}\) A “norm”, as a general matter, refers to a protocol or accepted practice. In mathematics, it also refers to the broad aspect of measuring, re-scaling, or sizing up a given geometric or abstract space under consideration.

\(^\text{116}\) *Shew* is the archaic form of “showed”, showcased so adorably in S L. Loney’s classic textbook, “*Plane Geometry*”

\(^\text{117}\) *insidia* = a treacherous plot or snare (in Latin)
Arouse *Euclidia* from Her ethereal dream, *Lord EL* did, ere the reflection of Her enchantment found completion, the bough of sorcery cracked and shattered asunder, for when the Dreamer awakes, the vision of yonder slips volition . . .

For a great while, *Euclidia* humored *Lord EL*, all His probing thoughts of import paramount, and yet, half her mind ached for slumber deep, channel again Her dear *Aleph*, Her *gyaan sarita*’s fount.\(^{118}\)

In that celestial witching hour *Lord EL* had Her sapient guidance in His meta-thesis found, bid He His adieu with a tip of *fedora* . . . but Oh! In innocence . . . Logos’ *Argo*\(^{119}\) He had run aground . . .

In disturbed sleep *Euclidia* set sail on *Aleph* again, but Alas! Its viscous currents no more in laminar\(^{120}\) state; choppy were her waters, inky swirls churned her geometry, her *logicons* but flying surf, smudged symbols of grey on grey slate . . .

Bewildered She was, chasing Her muse in rising bedlam, the prophetic Revelation all fractured, in frisson; in venomous fury *Mathamos* poured more dreams

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\(^{118}\)In Sanskrit / Hindi, “*gyaan*” = knowledge and “*sarita*” = a flowing river.

\(^{119}\)In Greek myth, *Argo* was the celestial ship which Jason and his Argonauts sailed to retrieve the Golden Fleece.

\(^{120}\)In the study of fluid dynamics, a laminar flow characterizes an orderly movement of a liquid, as if it comprised layers upon sliding layers of almost frictionless particles.
from the *Gate of Ivory* in Her disturbed Vision.

In waning hours these vile chimera descended
their muddied spectrum polluting Truth’s prism,
and what was pure in Logic... and pristine,
suffered in that river’s eddies a yawning, ghostly schism.

All of Her Mind cried, *Beware! Beware!* \(^{121}\)
*Mathamos’* flashing eyes, His floating hair!
Wove a circle round Her thrice,
Her sluiced *Aleph* in condemned splice!
Closed Her eyes with holy dread, She did,
For He on poison-ivy-dew hath fed,
and made Her drink the hemlock of Paradise,
proof and lies dissolved in *Undecidability’s* \(^{122}\)

\(^{121}\)The seven lines from “All of Her mind...” to “hemlock of Paradise” are borrowed from Coleridge’s poem, *ibid*, with significant changes.

\(^{122}\)“Undecidability”, with respect to a specific assertion / proposition in a given logic system, means that neither the proposition, nor its opposite assertion, can be indisputably established using the accepted logical methodology of proving assertions in that system. The observant reader should be startled that one can prove that something is *Undecidable* in a system of discourse. In Mathematical Logic, such proofs of meta-analysis almost invariably trace back to self-referential statements like “I always lie”, whose truth-value is undecidable when uttered by someone who can only *always* lie or *always* be truthful... (in this example, what becomes meta-clear is that such a statement cannot be uttered by anyone in a world where a person is either a *consistent* liar or a *consistent* truth-teller)
Then *Mathamos*, ever the ascendant
stanchèd the waters burgled from *Logos’* run,
and closed the adamantine gates to *Mathematica*,
as Lord Kafka’s Gatekeeper had done.¹²³

A woe! A hundred woes! A hundred thousand woes!
And curses to the ends of Omega Absolute!
Constrained in *Aleph’s* dry throes, God *Euclidia’s* stranded math...
 parchèd islands of probity... trapped and tangled in Logic convolute...
Not could She complete Her geometry, She couldn’t,
O Fate! Forever disturbed... Her circles in the sand,
Slain, you have, the *Briareus Geometrical*’s dream!¹²⁴
O Soldier! Your blow to the back staining Archimedes’s land...
Punctured now our reality’s billowing, manifold sail,
many a Logic’s claims now as *Buridan*-donkey’s intel...¹²⁵
What hath you wrought, O *Person from Porlock*!²⁶
Rendered Incomplete our ‘rithmetic... did He our good Lord... *God EL*...
Chapter 8

Axiomata : The Book of Formality

स्वयंभूताः स्वतःसिद्धाः संहिताः 127

( swayambhutaaha svatahasiddhaaha samhitaaha)

All things are established from a non-abiding basis 128

(in which is described how, given no further choice by Mathamos, Euclidia forged wonders from her limited mathematics nourished by river Aleph, and the two-thousand-year journey of that logic through human ingenuity and foibles...)

With Heaven’s Gates shut to Euclidia, and barred the royal road to All-Mathematics,129 no more could Aleph drink from the watering-fount of Logos, nor reproduce in her flow All-Logic’s reflections enigmatic.

In determination resigned, Lord Euclidia sighed:

“Even Gods must be takers of blameless Fate.

Verily... not till our kinship with the stars dies

127 In Sanskrit, the phrase means: “The Self-existing, Self-evident Collection (of analytic thoughts)”: “Swayambhutaaha” means “sui generis” i.e. “something that comes to existence on its own”. “Svatah siddhaaha” means “self evident”. “Samhitaaha” means “a collection”, typically applied to a collection of prayers, hymns, philosophical thoughts, etc.

128 From the Buddhist text, “Vimalakirti Sutra”

129 Euclid, of course, very famously had told King Ptolemy I (the 1st Egyptian king after Alexander the Great’s death) when the regent had asked him if there weren’t an easier way to understand Euclid’s books: “Sir, there is no royal road to Geometry!”
do we look for the fire in the grate...” 130

The geometries swirling in the pilfered Aleph,
they were rich enough yet to blossom a myriad realities,
with simple commandments, and “Let there be Light!”
Euclidia breathed Aleph’s magic into blooming dualities.

Motifs & myths raced on unbounded tides,
rumored from vaulting skies to nadir’s nihilility;
as artists chiseling imagination on unruly waves,
multiplied Essences out of necessity.131

Analytic Yang separated from diffuse Yin,
submerged artha became dvi-artha’s kin; 132
Reason sculpted the marble of Chaos to Cosmos,
ordering Laws wove meaning from logic’s skein.

Euclidia’s creation took its universal shape;
many a scripture waxed eloquent Her celestial reign;
Time, it took its measured course down uncounted eons,
rewriting Tedium’s endless screed with stellar paean.

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130 George Meredith’s original lines in “Modern Love – IV” say:
“Not till the fire is dying in the grate / Look we for the kinship of the stars.”
131 William of Ockham, famous for his principle called “Occam’s Razor”, had said: “Multiply not essences without necessity”, a pithier form of his original statement, “It is vain to do with more what can be done with fewer”.
132 In Sanskrit, “artha” = “meaning”, “dvi-artha” = “two meanings”; thus, the line implies a duality of opposites emerging from a conjoined whole.
The earth floated from the abyss on Varaha’s tusks,\textsuperscript{133} under Eden’s bowers, Adam multiplied his symbols with Eve; Moses and Mohammad they had their own holy says, luminous looms of Vedas\textsuperscript{134} founded wisdom’s ancient weave.

Noah and Manu,\textsuperscript{135} their trysts with the manic Flood had, but there the antediluvian myths we must leave, knowing the dark ages of zealotry never lagged, in imaginary algebra of religion did the irrational world grieve.

In the anon, The Great Philosopher arose, Euclid of Alexandria he was, of deep antiquity, amid a spray and kilter of geometric theorems, devised he a scheme of austere sublimity.

Inherited he had many a mathematical hymn and hum, some syntheses, some analyses, some arcana disjointed; a smorgasbord of glittering \textit{erat demonstrandum},\textsuperscript{136} in part a litany of results intuited, anointed.

Theorems from far flung lands they were,

\textsuperscript{133}In Hindu mythology, “Varaha” was an avatar of Lord Vishnu, one in which He took the form of a boar which raised earth on its tusks from the depths of primordial waters back into light.

\textsuperscript{134}The “Vedas” are the oldest of Hindu scriptural texts.

\textsuperscript{135}“Manu” is a Noah-like figure from Hindu mythology, also associated with the universal flood.

\textsuperscript{136}In Latin, “\textit{quod erat demonstrandum}” (Q.E.D.) = “that which was to be demonstrated”
of earthly mensuration\textsuperscript{137} which mattered,
each, by itself, a glowing scintillation,
but not of a main vein, all disparate, all scattered.

Took \textit{Euclid} that morass in his sagely, steadfast stride,
and with \textit{definitions, postulates} and \textit{inferences} did he arm,
the Method of Axiomatics, which in common thread elide,
the myriad geometric truths; proofs of all shades did he charm.

In thirteen books of “\textit{The Elements} ” he laid out
deductive paths of implicative, step-wise thought,
irrefragable demonstrations for figures laid flat,
a masterpiece of parsimony he had sought.

Of unrivalled power, this rigid conception mathematic,
\textit{Euclid’s} influence rang down the corridors of two millennia,
all math forthwith an acolyte of structured analysis,
its influence reigning wide, from \textit{Samarra} to \textit{Symzonia}\textsuperscript{138}...

\textit{And then}

\textsuperscript{137}“Mensuration” = a part of geometric studies related to concepts of lengths, areas, volumes etc of various shapes.

\textsuperscript{138}“Symzonia” is the imaginary region at the centre of a hollow earth, accessed through the south pole, as described in Jonathan Seymour’s novel, “\textit{Symzonia – A Voyage of Discovery}”. “\textit{Samarra}” is an ancient city in Iraq.
the road arrived at Messrs Whitehead and Russell...\footnote{139 Alfred North Whitehead and Bertrand Russell were two logicians who wrote the monumental work of symbolic logic, \textit{Principia Mathematica}, designed to capture the language and methods of all of mathematics and serve as its structured foundation.}
rested on P.M. and ZFC’s\textsuperscript{140} colonnades.

And thus the road... of years two thousand long,
from dark pitfalls did a luminous light wrest,
in meandering time led to the eighth day,
when the Mathematicians tried to rest... .

\textsuperscript{140}ZFC = “Zermelo-Fraenkel set theory with Choice”. This is an extremely powerful
axiomatic system resting on 9 axioms, vast enough in its coverage to encompass almost
all known mathematics (though there are many areas of inquiry e.g. certain transfinite
mathematics, which fall beyond its remit). The theory was formulated by celebrated
logicians, Ernst Zermelo and Abraham Fraenkel.
Chapter 9

Seemitaartha ⁴¹ : The Book of Limits

अगाधस्य उपरि सदा प्रभणम्
(agadhasya upari sada bhramaNam)

Forever Hovering Over an Abyss

“ He took the golden Compasses, prepar’d
In Gods Eternal store, to circumscribe
This Universe, and all created things:
One foot he center’d, and the other turn’d
Round through the vast profundity obscure,
And said, thus far extend, thus far thy bounds,
This be thy just Circumference, O World.” ⁴²

Mathematics, and mathematical laws,
lay Incomplete in the night:
Principia said, “Let Axioms Be! ”,
showing Hilbert its Consistency’s light!
It was not to last, oh no!
the Devil howling “Hark!”
“Let Godel Be!”, and signed

⁴¹ In Sanskrit, “seemita” = limited; “artha” = meaning. So “seemitaartha” = one which has its meaning constrained.
⁴² John Milton, “Paradise Lost, Book VII”
(in which we learn a little about the effects Mathamos’ obstruction of Euclidia’s thievery had on the mathematics which governs our reality, rendering it incomplete, and unable to prove that it does not contain any contradictions, as proved by Kurt Gödel (who, of course, was a human, different from the exalted Lord God El...))

Ask:

“When a symbol writes itself,
has it understood its own truth?
When meaning asserts its presence,
has it grounded its semantics, forsooth?”

T’was Hilbert who issued his sermoned call, exhorting formalists and logicians with foundational mission:

“Establish unshakeable faith in Arithmetic! There be no Ignoramus!
No Ignorabimus! Wir müssen wissen – wir werden wissen!”

143 Based on the original by Alexander Pope, “Nature and Nature’s laws lay hid in night:
/ God said, Let Newton be! and all was light!”, which was augmented by J C Squire as: “It did not last: the Devil howling “Ho!” “Let Einstein be!” restored the status quo.”.

144 “Forsooth” = “in reality”; “in truth”

145 In 1900, the legendary mathematician, David Hilbert, laid out 23 problems for the 20th century to solve. The 2nd problem on the list was an invitation to establish the consistency of ordinary Arithmetic beyond doubt. In Latin, the phrase “ignoramus et ignorabimus” means “we do not know and we shall never know”. The phrase was famously used by Emil du Bois-Reymond in his 1872 address in which he laid out 7 questions he deemed to be
Alas and woe! No rest for the weary soul, they say; how true this cruel apothegm!

The curse of Mathamos turned on purple vengeance, tarnishing Logic’s introspective stratagem...

Godel it was who proved ‘mongst men, those Incompleteness Theorems, celebrated legends, that a formal system which can count its hounds, in self-referential paradox of the liar descends.

“Any formal system of thought with basic ‘rithmetic”, (said Godel)
“contains unprovable propositions in its own remit, and every consistent system, as far as it can range, would forever, in its unfolding... stay incomplete...
Many a problem exists which no logician can ever prove or disprove in its lingua formal system, nor could she ever serve the system’s claim with a proof of its self-consistency’s adamant dictum...”  

Absolutely unanswerable. Hilbert powerfully used the negation of that phrase to rouse the mathematicians to his challenge.

Amongst the very basic questions about the foundations of mathematical logic, we can ask if the logic is “Complete”, and whether it is “Self-Consistent” (see Chapter 6 definitions for explanation on these). Logicians since Hilbert had expected to prove that the axiomatic system encompassing almost all of mathematics, the Zermelo-Fraenkel system, was Complete and Consistent. In 1931, Kurt Godel, showed in 2 startling theorems of logic that such an expectation is misplaced. In particular, any system of logic which defines how one should count objects inevitably becomes incomplete i.e. it contains assertions which cannot be proved or disproved in a finite number of logical steps while maintaining consistency. What’s more, one cannot prove that the system is self-consistent, either, by staying within the system. Unless we are ready to accept non-finitary arguments and tools in our logical discourse, we must be at peace with these facts. And accepting non-finitary
The Cause! Ahh... the merciless Cause!
Recall what you had a moment before read!
Did not an Almighty’s miserly heart condemn,
a fountainhead to an impoverished riverbed?

Aleph – dear Aleph,
interrupted by Mathamos in her measureless caverns,
lost many a channel to the theorems of All-Math,
into dry logic turned many of its myriad turns...

A victim of Mathamos’ uncouth whim,
questions now its own esteem does every hypothesis,
islands of propositions now as atolls do swim,
blocked are the proving ports for their common exegesis.

No bridge of finitary\textsuperscript{147} proof to span some orphaned claims;
some symbols of faith now in the canon’s uncertain treasure,
not every Assertion in the system be “Theorem” or “not ”,
not every contention be proved beyond reproach’s measure.

This geometry of dusk with its penumbra of shadows,
it meanders now as a wavering, culprit emotion,

\textsuperscript{147}The concept of “finitary proof” is not very precisely defined but is meant to capture the notion that the proof methods must not involve infinitely many steps, esoteric methods outside of ‘accepted’ mathematical practice, etc.

means is a bridge too far for many logicians since our intuition with transfinite concepts fails very badly. So it is not clear what we gain by trading one unknown dragon for another...
erasing many a proven lemma of *All-Mathematiks* in its wake, “Undecidable” have now become many a true Proposition.

And so it happens that our math is afflicted, by a certain incompleteness, a quivering diffidence, for were it Consistent, Incomplete it would be, and its Consistency an unprovable incidence...

Oh *Mathamos!* *Risible Mathamos!* How!

Stained is the worship of your reverent name!
Would that you were a touch more of munificence, far side of heaven’s light to redound on your acclaim!

Uncharitable, churlish, the exalted God, with *Euclidia* so petty Thy scrimmage, thou selfish wrath doth strand a cosmos midway upon its sacred pilgrimage...

And mourn! We mourn! Thy own, un-prism-ed light!

*Euclidia’s* Paradise bereaving its lost delight, for there was an ether’d rainbow once in heaven: ¹⁴⁸

we know her woof, her texture; she is given

¹⁴⁸The lines from “There was an ether’d rainbow...” to “...erewhile made” are borrowed from John Keats’ poem, “Lamia”, with changes. The original reads: “There was an awful rainbow once in heaven / We know her woof, her texture; she is given / In the dull catalogue of common things / Philosophy will clip an Angel’s wings / Conquer all mysteries by rule and line, / Empty the haunted air, and gnomed mine / Unweave a rainbow, as it erewhile made.”
in the dull catalogue now of common things,
a sundered dream clips an Angel’s wings,
unconquered remain some mysteries by rule and line,
empty the haunted air, and gnomed mine –
unweave a rainbow, as it were erewhile made, Thou did,
coded now some colored truths, proof-less their verity hid...

Oh Cassandra of Mathematica!  
Such irony!
What you can say of conviction
is the truth you cannot prove,
and the curse of Apollo!
When you can prove it
is verily too late...

\footnote{Cassandra, in Greek mythology, was the daughter of Priam, the last king of Troy, and Queen Hecuba. She was given a gift of prophecy by her suitor, the Olympian god, Apollo, but later cursed by him that no one would believe her prophecies, even though true.}
Chapter 10

Anantavidya\(^{150}\) : The Book of Hope

एकीक्रतं तव संकल्पम् एकीक्रता तव इच्छा
भवतु तव गणित अनाकुलं च स्थिरं सदा
चिरात् सहवसन्तु तव सूत्राणि स्वयंसिद्धानि
संयुक्तानि समाहितानि चातिस्मोहणानि

\((aekkeekrutam tava samkalpam aekkeekrutaa tava ichChaa
bhavatu tava gaNitam anaakulam cha sthiram sadaa
chiraat sahavasantu tava suutraaNi svayamsiddhaani
samyuktaani samaahitaani chaatisammohanaani)\)

“united your resolve, united your Will
may your mathematics find its Still
may your axioms long together dwell
in unity, in harmony, in deeper Spell”

\((\text{in which we know... we know in our deepest hearts... that adversity is}
\text{an opportunity; that we, who contain the divine spark, must forge our}
\text{own paths through the jungled thickets of Undecidability.. and ascend to}
\text{the mathematical zenith which cannot be denied the true seeker...})\)

Mighty is the chasm of these Abstractions
to a mind beset with uncertain images,

---

\(^{150}\)In Sanskrit, “ananta” = without end; “vidya” = knowledge. So “anantavidya” = infinite knowledge.
and haunted by herself, in spectral dreams,\textsuperscript{151}
of failures in arithmetic scrimmages.

How the logicians struggle with an irresolute fate
in the face of intransigent, unbending \textit{cul-de-sacs}!

Constant questions heaped on Faith, hopes in dust,
\textit{Undecidability} proofs asserting all our mathematics lacks.\ldots\textsuperscript{152}

\textit{“Does Aleph-1 match the Continuum’s elan?”} \textsuperscript{153}

\textit{“Oh! For want of an algorithm to tell if a calculating program halts!”} \textsuperscript{154}

\textsuperscript{151}The three lines from \textit{“Mighty is the chasm. . .”} to \textit{“haunted by herself”} are from William Wordsworth’s poem, \textit{“The Prelude, or Growth of a Poet’s Mind”}, with some changes. The original reads: \textit{“Mighty is the charm / Of those abstractions to a mind beset / With images, and haunted by herself”}

\textsuperscript{152}Over the decades, Logicians have established significant results on the nature of finitary proofs, the limitations of finite axiomatic systems, Undecidability, the quality of mathematical Truth, the nature and limitations of computations, and the like, through the works of Kurt Godel, Alfred Tarski, Paul Cohen, Thoralf Skolem, and many others. They tell us the boundaries of what is possible to establish – and what isn’t – in our current mathematics, while showing possible avenues of going beyond, if we are willing to expand our sense of “self-evident” faith.

\textsuperscript{153}This is the famous \textit{“Continuum Hypothesis”}, which asserts that the cardinal size of the set of real numbers is the first infinity beyond the infinity of natural numbers (which is called “Aleph-0”). Through the joint results proved by Kurt Godel and Paul Cohen, we now know that the Continuum Hypothesis is undecidable in most of our current mathematical frameworks (whether ZFC or other systems). The question of whether it is “really true” or “really false” is no longer meaningful in our mathematics. We would have to expand the definition our number system by adding new axioms, and therefore, ascribe more properties to “numbers” so that in the new system, the Continuum Hypothesis would be either true or false.

\textsuperscript{154}This is Alan Turing’s “Halting Problem”. Effectively, it is the computational form of the first of Godel’s two theorems, proving that there is no algorithm which can take an arbitrary set of instructions for computations and tell us whether that sequence of computations will definitively terminate or not terminate in a finite number of computational steps.
“Undecidable perhaps is the problem of Riemann?” 155

“Do even heavens know the Diophantine problem’s faults?” 156

Doubts, doubts, they litter our endeavors! Oh!
The memories of questions lost on Provability’s lathe!
In the fabric of logic’s demands are those nagging questions snared,
blunted and frayed by the scissoring blades of doubt and faith...

But lose not the religion in contemplation, O Pilgrim!
lose not the cerebral paths to *primum Luminosity*;
for aloft in the heavens a blazing beacon shines,
burning with the beckoning dreams of restless curiosity...

Up there, in the clouded spires of ivory towers - *I know* -
are explorers of the frontiers... of the wild, wild west;
pioneers of *trans-mathematics*, challengers of angry gods,

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155 The Riemann Hypothesis, proposed in 1859, is widely considered the greatest unsolved problem in present Mathematics. Its resolution would have wide-ranging impact on how we think about number theory and its interconnectedness, as well as impact on other open problems in Mathematics. Since the best minds of humanity have not been able to solve it for 175 years now, some have asked whether problems like Riemann Hypothesis, Goldbach Conjecture, etc might be Undecidable using standard techniques. Indeed, the great mathematician, John Conway, had proved that a certain generalization of the infamous Collatz Conjecture is Undecidable...

156 In Hilbert’s list of 23 problems alluded to earlier, the tenth problem, asked whether a procedure existed to determine if a polynomial equation with integer coefficients and finite number of unknowns had integer solutions. The efforts of many logicians and mathematicians – Martin Davis, Julia Robinson, Hilary Putnam, Raphael Robinson, Yuri Matiyasevich, and others – concluded that such a procedure cannot exist, which means that unless explicitly solved, we cannot know if a Diophantine equation actually has a solution set...
brook no transfinite demons, nor dearth of imagination confess…

The unceasing tides in the Logician’s zealous mind-sea,
they wash ashore, dispelling the haze of unknowability,
new paradigms in their quest, new scrapers of the empyrean,
forge, pushing them recondite boundaries of inscrutability.

We, who are the music-makers;157
We, the dreamers of dreams;
We… wandering now by lone sea-breakers,
and sitting by our logic’s desolate streams…
True… that is true, and yet, see we not?!
That We be sparks of divine, both seekers and the sought?!

We are the movers and shakers
of the world forever, it seems;
Seekers of true knowledge,
Seers of pregnant memes;
Playing our pensive tunes of blues,
our ballads and poesies of unknown stars…
since things as they are...
are changed upon a blue guitar…158

157The 4 lines from “we, who are the music-makers…” to “desolate streams”, and the 2 lines from “we are the movers…” to “world forever, it seems” are from Arthur O’Shaughnessy’s poem, “Ode”, with a few changes.
158Steven Wallace, in his poem, “Man With the Blue Guitar”, wrote:
“They said, “You have a blue guitar, / You do not play the things as they are. / The man replied, “Things as they are / Are changed upon the blue guitar”
and so we strive...
and thus do we conspire incessant,
to forge newer axioms; weaving a more verdant foliation,
multiplying our mirrored myths and morrows,
painting fresher themes in star-fired barrows,
unknotting the purple Paradox coloring our Creation...
to dispel this calumny of chaos...
to pierce this waving veil of grey obscurity...
.
.
and if...

No!

When!

When,
in the rising dawn of this ceaseless striving,
our logics converge some day,
and our shackles unlock in our minds,
then...
then...
we shall solve together...
again...
these “Undecidable” questions of metaphysics;
we shall hum deeper tunes of Mathematics;
our symbols shall deduce other schemes;
our cryptic clues shall meet in other guises;
and we shall kiss their radiance to build anew,
for all time,
those topless towers of Ilium once again\(^{159}\)
in other Mathematical skies....
Amen...

\[\text{— finis —}\]

\(^{159}\)The “topless towers of Ilium” is a phrase used by Christopher Marlowe in his poem, *Dr. Faustus*, referring to the extremely tall towers of the city of Troy (Ilium).