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Elena Anne Corie Marchisotto
California State University Northridge

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Cover Page Footnote

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Anneli Lax: They Think, Therefore We Are

Elena Anne Corie Marchisotto

Department of Mathematics, California State University Northridge, USA
emarchisotto@csun.edu

Synopsis

Anneli Lax (1922–1999) was my mentor and thesis advisor at New York University in the late 1980s. I was fortunate to be included among her circle of friends and collaborators in the ensuing years. Dr. Lax made important contributions to mathematics and mathematics education. This article describes her pioneering work in promoting good exposition of elementary mathematics in support of mathematics and its pedagogy. The design and implementation of her views illustrate her indefatigable spirit and impressive work ethic for causes she championed.

1. Background

Anneli Lax (1922–1999) was my mentor and thesis advisor at New York University (NYU) in the late 1980s. I was fortunate to be included among her circle of friends and collaborators in the ensuing years. Our relationship was one of heart and mind. The words of the song "For Good" [26] capture how I have been changed by my experience of her: "So much of me is what I learned from you. You'll be with me like a handprint on my heart."

I wrote before about Anneli's "quiet determination, her openness and acceptance of the weaknesses of others, and her joy in simple pleasures" [19]. Here I want to focus on her indefatigable spirit and impressive work ethic for causes she championed, by describing her pioneering work in promoting good exposition of elementary mathematics in support of mathematics and its pedagogy. I thought it apt to use as my title a variation of the words of

René Descartes that Anneli adopted as an underlying theme for exploring the dual aspects of language in the teaching and learning of mathematics. What I share, in concert with excerpts from published works, emerges largely from my recollections of our times together, as well as from her written communications with me. In addition to our conversations, I include excerpts from letters she wrote to others and notes on talks she was planning to give or had given, which she had shared with me.

2. Anneli Cahn Lax: A Brief Introduction

Anneli Cahn Lax was born on February 23, 1922, in German Kattowitz.[1] Her family moved to Berlin in 1929, then Paris and Palestine, before relocating to New York City in 1935. Anneli received a bachelor's degree from Adelphi College in 1942, and earned her doctorate in mathematics at NYU in 1955. She wrote her dissertation, "Cauchy's Problem for a Partial Differential Equation with Real Multiple Characteristics," under the supervision of Richard Courant. She met Peter Lax in Courant's graduate class on complex variables. Reuben Hersh, Peter's thesis student, a life-long friend of the couple, described Anneli then as Peter's "brilliant and beautiful fellow student and future bride" [7, page 605]. See the Photo Gallery in Appendix A.

In 1961, Anneli became a member of the faculty of the Department of Mathematics at NYU. Prior to that time she held various positions at NYU: Assistant (1943-44) in the Guggenheim Institute of Aeronautics (which was part of NYU), Research Assistant and Instructor (1944-1949), Assistant Research Scientist at Courant Institute (1950-57), Instructor (1957-1961). Anneli noted that in the 1940s she had been teaching soldiers older than she was who were pursuing their educations through the GI bill [2, page 32].

Professor Lax's important contributions to the theory of hyperbolic equations are well known to those who study the complex behavior of many kinds of waves. See, for example, [11, 12, 13]. A more heterogeneous audience knows

[1] A Prussian town until 1921, German Kattowitz is today the capital city Katowice of the southern Polish province of Silesia. In the year of Anneli's birth, following the defeat of Germany in World War I, German Kattowitz had become part of Poland. The city then fell to the Germans in 1939, and was "liberated" by the Soviets in 1945 again as a Polish city, albeit renamed *Stalinogród* between 1953 and 1956.

of her work in reinterpreting strategies for the teaching of mathematics, endeavors for which she earned the George Pólya Award (1977), the Golden Dozen Teaching Award (1988), and the Mathematical Association of America's top honor: the Distinguished Service Award to Mathematics (1995). Mount Holyoke College awarded Professor Lax an honorary degree in 1997.

Anneli remained at NYU until her retirement in 1992, but retirement did not mean she slowed down. Indeed, in an email on October 13, 1997, she told me, "When mathematicians retire, they do exactly the same thing that they did before retirement, except no formal teaching and more freedom in scheduling their time." Anneli did precisely that up until the day she died on September 24, 1999. I was in California, but was fortunate to be able to speak to her by phone and say goodbye. Peter told me the last thing she was working on were my notes.

3. The Interplay of Mathematics with Language

Professor Lax pursued several projects that promote the use of language — reading, writing, listening and speaking — as a fundamental part of mathematics. She designed and implemented different strategies to confront the challenge of broadening access to interesting mathematics in ways that

- do not distort the subject, i.e., do not sacrifice technical accuracy; and
- do not impede its learning – in particular, by failing to consider, in words she often used, "a diverse, balanced diet of teaching methods".

Her ideas continue to resonate and offer salient advice in today's world.

3.1. The New Mathematics Library (1958-1996): Looking at the World Mathematically

Anneli was the founding editor and driving force behind the New Mathematics Library (NML), the publication of a series of monographs written by various authors, designed to engage teachers, students (high school and beyond), and average readers (non-mathematicians and mathematicians alike) to explore elementary mathematics from new perspectives. See [14, 2]. She was responsible for "virtually every aspect of the NML . . . , including acquisition, copyediting, mathematical editing, layout, cover design and typesetting" [21, page 28].

Professor Lax saw the "main purpose" of the series as getting "good mathematical expositions to interested people in a language they could understand without having to immerse themselves for years in big mathematical tomes [2, page 30]. The first part of its educational mission was directed toward helping students develop, "among other skills, one of their natural talents: looking at the world mathematically" [16, page 32]. The second part of that mission was directed toward American mathematicians, "who had, for the most part, never written anything except papers for professional journals, i.e. for other mathematicians (preferably working in the same field)" [16, page 31]. Anneli was very successful in recruiting distinguished authors to address the intended audience for the series. Don Albers called this a "remarkable achievement", noting that Anneli had "something of a Midas touch when it came to attracting mathematics manuscripts" [1, page 11].

Anneli described the NML as a "collaborative projective" and a "mathematics cooperative learning experience." She spoke often, publicly and privately, about the help and encouragement of many who contributed in a variety of ways to the series, including her husband Peter, "when the spirit moved him." [16, page 31]. Ivan Niven recalled the reciprocal part of that scenario in which the authors themselves, himself among them, learned much from her careful editing. Indeed, Anneli once described a "memorable moment", when she added what she perceived was a needed chapter to a manuscript written by two authors. Each of the authors assumed the other had written it, and the augmented manuscript enjoyed the kind of success that the NML volumes received under Anneli's tutelage. See [2, page 30], [21, page 29], and [9, page 34].

The far-reach of the NML is evident in a story Victor Pambuccian, professor of mathematics at Arizona State University, recently shared:

> In tenth grade, I became a member of the MAA while I was living in Bucharest. I learned about the NML from the *American Mathematical Monthly* and *Mathematics Magazine.* As I had no currency to buy anything produced in the US, I ordered, through my uncle who lived there, I. Yaglom's *Geometric Transformations I-III*, and N. Kazarinoff's *Geometric Inequalities.*

Professor Pambuccian, who earned his doctorate in 1993 at the University of Michigan, with a thesis focusing on the axiomatics of Euclidean geometry, and who has published more than a hundred papers in mathematics journals, all sterling examples of good mathematical exposition, added: "The rest is history."

Anneli was at the helm of the NML for thirty-eight years, bringing thirty-nine volumes to publication. In an email on April 19, 1996, she told me, "My role in the NML is getting less active. I am trying to fade out gradually, but cannot find anybody to take my place." To be sure, it would have been difficult to fill her shoes. The series was renamed the "Anneli Lax New Mathematics Library" in her honor in October 1999 to recognize it "as an enduring monument" reflecting her contributions to "quality mathematical exposition" [1, page 11]. For her stewardship of the NML, as well as for her work on several committees of the Mathematical Association of America (MAA), Anneli earned the Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished service in Mathematics. See [21, 16].

Reflecting on her goals for mathematical exposition, Anneli urged her fellow mathematicians,

> Let us practice what we preach, read and write carefully, avoid trendy slogans, and go beyond mathematical correctness, syntactic correctness, and political correctness in serving our discipline in our individual ways [16, page 32].

Her efforts on behalf of mathematics education echoed this advice.

3.2. Ford Foundation Grants (1985-1989): Language-Linked Approaches to Mathematics Instruction

Among Professor Lax's concerns about public education were these:

- The failure to address the idea of "mental fitness" as an objective for learning mathematics.
- The increasing trend to robotize students instead of educating them as thinking human beings.
- The tendency for schools to treat the cultural and linguistic diversity of student populations as an enormous obstacle.
- That, in spite of lip service to respect for the individual," the schools keep trying to impose "crippling uniformity" in the teaching of mathematics.

Anneli believed that there was insufficient emphasis in teaching and learning on the role of language for clarifying one's own thoughts, for getting one's ideas across, and for becoming a good listener. In 1986, she and her NYU collaborator, Erika Duncan (with whom she had been teaching a double-credit course that offered intensive work in mathematical thinking and expository writing for freshmen), were awarded a grant from the Ford Foundation to work with high school teachers in some of what Anneli described as "at risk" city schools. In her letter to me dated September 8, 1986, Anneli explained,

> The reason we are so interested in reaching younger students (via their teachers) is that by the end of high school, most students' attitudes towards — and misconceptions about — mathematics and science are pretty firmly established and difficult to dislodge.

Anneli observed that mathematics was often promoted to students for its tremendous utility and almost universal applicability ... and for its playful, puzzle-like, fun aspects. She noted, however, that "an important third aspect" was being neglected ... namely, "the feeling of mental power, or at least mental fitness on the part of those who use mathematical reasoning when appropriate." She wrote,

> Just as having physical control and coordination over one's body adds to one's sense of wellbeing and confidence, so does having mental control over one's thought processes. I believe most normal human beings would work quite hard to achieve both. Both help people gain control over their lives.

Professor Lax believed that schools were depriving students of learning how to think for themselves. She proposed that mathematics be taught in a way that encouraged students to use their individual experiences as members of a culturally and ethnically heterogeneous population. Since "doing mathematics consists in large part of constructing connections between concepts, topic, experiences and applications," Anneli suggested that "students examine one another's mathematical strategies and ideas" [16, page 32]. When students are given the opportunity to talk and write about what they are learning, they are able to connect new material with past experiences and future goals. In this way, what they learn becomes part of them.

Providing such opportunities involves teaching students to express themselves so that their classmates and teachers can understand. This requires paying attention to language as well as developing the art of listening. Anneli expressed her thoughts about this in a letter she wrote on April 11, 1988, to George S. Monk, then professor of mathematics at the University of Washington, who had published an article about calculus reform [20]. She wrote,

> I am extremely interested in how people learn mathematics and in what they are thinking when they are not thinking what we think they ought to be thinking. But it is not easy to find this out. In my experience, one learns much more by listening to students as they explain how they reached their conclusions than by asking questions.

Indeed, for Anneli, one of the first steps in getting students to become responsible for their own learning is for teachers to *listen* to them (her emphasis). The plan for the Ford Foundation Project was to incorporate what the teachers "heard" from their students into strategies for connecting topics to contents of their syllabi.

With their grant, Anneli and Erika designed weekly workshops with teachers focused on researching their experiences. They then explored how to make productive use of the teacher's stories, and create links between disciplines, primarily between mathematics and language. These workshops were models of collaborative problem solving, exportable to the classroom. They were intended to lead students away from an egocentric mode ("the once heralded, now often abused, notion of relevance -- 'me-ism', i.e., of immediate use to me"), into "some delight in the interrelatedness of ideas, in the possibilities of understanding new things and internalizing them" (Ford Foundation Proposal Abstract, 15 May 1987).

Professor Lax suggested that these strategies would be especially more promising in inner city schools than methods based on presumptions of uniform standards and preparation. That being said, she believed a "diverse balanced diet of such strategies" should enjoy more widespread implementation. For example, on November 24, 1989, Anneli wrote to me about a regional conference she attended on Making Mathematics Work for Minorities. She said, "I was glad to hear several speakers voice the opinion that strategies effective for minorities are likely to be effective for everyone else as well."

In the second year of the grant, Anneli and Erika expanded their project at the high schools, both horizontally (across different student populations and across disciplines) and vertically (across the grades and into elementary and junior high schools). In a letter dated January 5, 1987, Anneli shared with me "a few instances where teachers have successfully involved students in meaningful responsible learning, through innovative use of writing, reading, respectful listening and thoughtful guiding of student explorations." She noted, however, the challenges in implementing such good ideas, having observed "the unreasonable pressures on teachers, the contradictory messages received by the teaching and administrative staffs, and the social problems compounding the difficulties."

The Ford Foundation project recognized that most students mistake their linguistic deficiencies in reading problems for lack of mathematical talent. Linking language and mathematics instruction, Anneli and Erika sought to address the need for familiar common vocabulary, for understanding and using idiomatic English and mathematical conventions, and for relating the structure of language to the meaning it conveys (Ford Foundation Proposal, 15 May 1987, page 2). Seeking to include parents in the conversation, the Lax Foundation gave the Howard University Mathematics Department start-up funds to create forty page booklet for parents of children in kindergarten and first grade. Entitled *A Parent's Guide – Helping You to Understand Math Taught in School*, these booklets were used in workshops in the District of Columbia at Public Schools and Churches. Louise Raphael, now emerita professor at Howard University, reported, "Anneli was a constant inspiration and guide — tireless advisor from start to finish."

As with virtually all Anneli's projects, there were, and continue to be, ripple effects. At Sarah Hale Junior High School in Brooklyn, Anneli had worked closely with a forward-thinking teacher Peter Egleston, who taught English and Communication Arts. He created a variety of imaginative writing assignments for his students that captured and actualized Anneli's insights. One involved a project having his students exchange a series of letters with a fictional "Aunt Rosanna" who was trying to establish a spaghetti sauce company and needed help solving a series of practical mathematics problems. Louise Raphael recently forwarded Egleston's project materials to a former Junior High School mathematics teacher who is currently teaching the Math for Elementary School Teachers course at Howard University.

3.3. They Think, Therefore We Are: Dual Aspects of Language in the Context of Learning Mathematics

In an essay based on her experiences working on the Ford Foundation project, Professor Lax spoke about the roles of language that "intrigued her most" and gave examples of how they are related to mathematical thinking. She noted how harmful it is to the learning process to separate out the twin functions of language, i.e. "its dual input-output aspects—reading and writing, listening and speaking, formulating (synthesizing) and analyzing, decoding and encoding."

Anneli advised that both instructors and students need feedback "to strike the right balance" between what instructors say and what their students hear. This requires the desire and the time to develop a common language. She proposed writing "mathematical vignettes" to represent conceptual and experiential aspects of learning and problem solving. She saw them as attempts "to unify theory with practice, educational sermons with the nitty gritty details of procedures." Anneli hoped that these vignettes would enable instructors to see "how easy it is to have a group of students play with a problem in a way that is at once meaningful, and would lead to many kinds of mathematical thinking." The idea is to use language and notation as a speculative tool for exploration. This routine consecutive integer problem illustrates the process:

> *Problem*: The first of three consecutive integers squared exceeds the sum of all three by 7. What are the three integers?
>
> Solution Steps:
>
> 1. *Questions regarding the statement of the problem.* Does everyone know what the problem is asking? What are consecutive integers? Are these whole numbers? Must they be positive? How do the cognates such as "sequence", "consecutive", "order" apply? Must we assume, for example, that there is an ordering in terms of increasing size? If so, why not write, "The smallest of ...." ? Is it possible to rephrase the problem in ways that give greater insight into what is being asked?
>
> *Exploration:* Task the students to paraphrase the question in writing, then read and debate until they feel they have clearly expressed the question.

2. *Questions after a solution is found by trial and error.* "Are there other solutions?" Can algebra help us find a systematic way of finding them? Can we translate the problem into an equation, i.e., to a statement that two numbers constructed from the conditions of the problem are the same? If so, what are the various conventions for naming things for such an equation? If we name the smallest number n, what does the definition of consecutive tell us? Do we have to name the smallest number? How would the notation vary if we named the biggest, or the one in the middle? Could we traverse three different paths to the same systematic solution on the basis of these three notational variants?

Exploration: Partition the students into three groups, each tasked with translating the conditions of the problems into symbols according to a given variant. Invite them to discuss the translation, and produce written versions of it. Ask them to present their findings to the whole class, giving detailed explanations of their solution attempts. Have them summarize their discussions with a comparison of the three notational schemes, including any additional results and questions.

3. *Questions arising from compare and contrast.* The students will observe that all three ways give the same two correct answers to the question posed and in all three cases the symbolic representation of the condition stated in the problem leads to a quadratic equation. Can they know there are no other triples besides the ones they found? What is different about the three schemes and how do these differences impact the derived equations?

Exploration: Ask the students to explain, in writing, which notational scheme they preferred and why.

4. *Questions about "what if"?* What if the problem had asked for three consecutive natural numbers fulfilling the other conditions? Suppose we had 5 consecutive integers, would their sum be just 5 times the middle integers?" Suppose the three integers were evenly space, but not consecutive, say 2 apart, is their sum still 3 times the middle integer? What if there was a condition that the numbers are odd? What if they are even?

> *Exploration:* Assign these "what ifs" as short writing assignments.

Although this vignette emanates from a rather unexciting problem, Anneli proposed it as an opportunity for students to acquire mathematical and metamathematical skills that they could then apply to more interesting problems. She explained, "If the students can become involved in solving this not very interesting problem, if their oral and written contributions to its discussion are taken seriously, how much more involved might they become in tackling less artificial problems?" Anneli also believed that the use of vignettes had the potential to mitigate frustrations students often experience in problem-solving, providing an empowering environment that gives greater opportunities for them to experience success.

3.4. Fashioning "A Community of Learners" in Which Prospective and In-Service Teachers Take Part

In a talk entitled "Developing Mathematical Thinking" for the New Jersey Section of the MAA on April 29, 1995, Professor Lax asked, "How can we demonstrate that doing mathematics at any level with its triumphs and frustrations is, on the whole, a gratifying activity with many unexpected intrinsic rewards?" She shared stories to "tempt math persons" into collaborative activities to help in the education of prospective and in-service teachers.

One story recalls her asking her NYU colleague Martin Davis about ways to engages mathematics-anxious freshmen in some fruitful elementary investigations. Davis advised, "Have them stare at the multiplication table for a long time and see what they come up with." Anneli reported that Martin's advice "had miraculous consequences," with engaging "people of all ages and levels of sophistication" to discover patterns and formulate conjectures.

Another is revealed by a conversation between Anneli and Lucy Moser-Jauslin,[2] regarding the frustrations she and many teachers experience when students are unable to comprehend basic ideas. Lucy reported that Anneli,

> very patiently explained to me how one has to find the ways to help students find the solutions on their own terms. She was a

[2] Professor Moser-Jauslin is an algebraic geometer, the daughter of the German-American mathematician Jürgen Kurt Moser and granddaughter of Courant.

big believer in allowing people to develop original methods for solving problems. Math classes shouldn't make things worse.

Anneli then revealed an experience she had with her class for remedial students at NYU. Before teaching any systematic methods, she asked students to solve a standard high school algebra word problem (e.g., Joe is twice as old as Mary was, when . . . .). Allowing the students to choose their own strategies generally resulted in their getting the right answer. Afterwards, the students were taught the standard methods of solving this type of problem using algebraic equations. Some months later, a problem similar to the original one was posed. The students could no longer solve the problem.

The story Anneli shared with Lucy reminds me of one the prominent mathematics educator Robert Davis (see [17]) once told me regarding an experiment he had conducted in an elementary school. Students learned how to subtract, and then were interviewed several times afterwards over a period of some months to discuss what they learned. One very bright young student devised an incorrect method of subtracting. The teacher "corrected" her, and afterwards the young girl always subtracted "correctly." When Professor Davis interviewed the student, he asked about the correct method for subtracting. The young girl informed the professor that her own method was the correct one, but that she was only using the other method because the teacher wanted her to do that. The take-away was that the young girl "owned" her own method and not that of the teacher.

Anneli believed strongly that what students acquire from the process of trying to create their own strategies helps them understand deeper concepts. The role of the teacher is to facilitate the process, guiding them to correct strategies by conversing with them rather than asking them to follow the standard ways. Anneli gave the example of this problem: "Find two fractions, both in lowest terms, with different denominators, such that their difference is 2/13." She said that it confounded "even mathematics persons". It takes a while to realize that there are infinitely many solutions. Anneli proposed several ways stimulate an exploration process in conversations with and among the students:

- examine subtraction and addition as processes that undo each other.
- look for the "missing addend."

- think of the first addend playing the role of an initial condition.
- recognize that given the amount by which something changes does not tell us its final value unless we know its initial value.

Anneli observed that these explorations touch upon something calculus teachers often forget, even as they "take off credit" when a student forgets the constant of integration.

4. The Faces of Mathematics and Its Pedagogy

Dr. Lax's work with the NML and the Ford Foundation Projects were important vehicles for her to exploit the interplay of mathematics with language in support of good mathematical exposition. These experiences infused her quests to expose the many faces of mathematics that invite understanding and appreciation of it.

Among the faces of mathematics that Professor Lax sought to promote were those espoused by the Humanistic Mathematics Network and the Task Force on Ethnomathematics.

4.1. Humanistic Mathematics: Un-fragmenting Mathematics and Its Teaching

Alvin White, professor of mathematics at Harvey Mudd College, founded the Humanistic Mathematics Network (HMN) in the mid-1980s—a movement that advocated teaching humanistic mathematics and teaching mathematics humanistically. The interests and goals of the network embrace humanistic ideas in the education process and in the mathematical content of what is being taught. The underlying idea is that since mathematics is a beautiful creation of the human spirit, it is incumbent upon those who teach to come together to explore strategies that expose its creative nature and beauty to students in the classroom in a way that empowers them as learners. See [28, 29, 30, 24, 18, 3, 8].

Professor White established the *Humanistic Mathematics Network Newsletter* (HMNN), in the summer of 1987, to foster the ideals of the Network. He served as editor until it, or rather its immediate descendant *Humanis-*

tic Mathematics Network Journal (HMNJ), ceased publication in 2004.[3] By then the readership of the HMNJ included nearly 3000 subscribers worldwide, Anneli a supporter and contributor among them.

At Alvin's request, Professor Lax participated in a panel discussion, *Mathematics as a Humanistic Discipline,* held in a special session of the annual American Mathematical Society (AMS)/MAA meeting on January 23, 1987. She focused her remarks on ways to "put together fragments" emanating from mathematics perceived as being isolated from the rest of life [15].

Discussions about the fragmentation of mathematics can be traced to Plato's writings; see [10]. Plato described the "unity of mathematics" in terms of the mutually benefiting links between its branches. He saw the study of mathematics best undertaken not only by recognizing the kinship between its branches, but also by acknowledging its relation to the rest of life. Arithmetic, for example, is common to all thought, arts and sciences.

Anneli believed that an un-fragmented view of mathematics improves its study, as "humans retain what they learn best if they can put new material into a human context, e.g., connecting it to past experience, future aspiration, previously acquired mental structures." Mathematical reasoning is a natural part of general reasoning. A failure to understand this causes mathematical reasoning to "get pushed out of other school subjects" (like social science, literature, even the natural sciences[4]). This is just one instance of the "harmful fragmentation of human activities." Another is "the early elimination of reasoning from the humanities, separation of reading from writing, and listening from speaking."

[3] See [27] for a review of the contents of the six issues that make up the HMN Newsletter (Issues 1–6, 1987–1991) and [5] for a review of the contents of the HMN Journal (Issues 7–27, 1992–2004). Mark Huber (Claremont McKenna College) and Gizem Karaali (Pomona College) have perpetuated its spirit with their editing of this *Journal of Humanistic Mathematics* (JHM). The HMNN / HMNJ archive is fully scanned and available at `http://scholarship.claremont.edu/hmnj/`, last accessed on January 26, 2023.

[4] I recall our conversation regarding the rejection of an offer to rewrite the California standards for teaching science, made by several Nobel Laureates in science in conjunction with university science professors who had expertise in pedagogical issues; see [6]. In an email dated November 26, 1997, Anneli asked, "Who made this crazy, cost-ineffective decision?" She wondered why the decision to pay education professors was deemed preferable to accepting services volunteered by a team of both scientists and science educators.

Students should learn to appreciate the kinship between the branches of mathematics. Instead, they experience a mathematics that is fragmented into arithmetic, algebra, geometry, etc., and these sub-disciplines "get cut up into sections or modules or skills to be mastered, tested and forgotten." Dividing mathematics instruction into therapy for the "math-anxious" and instruction in mathematical techniques exacerbates the problem. Anneli advised, "Human learning involves both intellectual and emotional attitudes and students must get acquainted with both."

Fragmentation is not restricted to those being taught. Those who teach are also victims of fragmentation, being experts in this or that. Anneli advocated that efforts at de-fragmentation be designed to address both students and teachers.

Professor Lax spoke at another session on humanistic mathematics at the joint AMS/MAA meetings in Phoenix in January 1989, on the topic of discussing and debating conjectures.[5] She detailed several specific problems related to the failure of educators to know their audiences, in particular with respect to

- what their mathematical and linguistic experience was; and
- what kind of things they were ready to learn and expect to hear about.

Anneli cited Sheila Tobias for her "Peer Perspectives" work, and [22] among others that addressed the issues of innumeracy. She paid homage to great expositors of mathematics (e.g., Hugo Steinhaus, Wacław Sierpiński, Hans Rademacher, Otto Toeplitz, Courant and Herbert Robbins, Lynn Steen, Barry Cipra and others), who bridged the gap between mathematics and the humanities, disseminating good mathematics to novices. She discussed [23], which emanated from letters to the Hungarian writer Marcell Benedek, who felt poorer in literary expressivity because he was unable to understand mathematics, and Rózsa Péter's successful attempts to demonstrate the common features of mathematics and literature.

In her talk, Anneli used seven "posters" to draw attention to what she considered "urgent educational tasks."

[5] See Abstracts of All Papers Presented at the Humanistic Mathematics Sessions in Phoenix 1989. *Humanistic Mathematics Network Journal*: Iss. 4, Article 14. Available at: https://scholarship.claremont.edu/hmnj/vol1/iss4/14

1. *Is Teaching an Ego Trip?* Let a teacher's ego be associated with enabling enjoyment of the growth of students into independent responsible learners and thinkers, rather than being the omniscient authority on rules and techniques to be followed blindly. Make room for both, being a charismatic lecturer and a facilitator who models and supports the learner's trials and triumph.

2. *Don't Let Johnny Put Beans Up His Nose and Other Pre-correcting Codes.* Suspend preconceptions about what common mistakes students will make. Such preconceptions and larger classes of misconceptions are closely related to our expectations of their effects on students, about which there is a growing literature.

3. *The Unreasonable Effectiveness of Little Words.* By listening to students or looking at their mathematical writing, we see ambiguous, sometimes confusing, uses of words such as "by", "into", "all", "some", "between", "at most", etc. I signal "with" and "for" because of my plea to my colleagues to do mathematics with rather than for people.

4. *Compare and Contrast: Innumerate Literati and Artificial Intelligentsia vs. Literacy and Numeracy in Public Education.* Difficulties that "poor" mathematics student and many innumerate intellectuals have in common include, among other things, misconceptions about the nature of mathematical thinking. We need to pay greater attention to building a common language of general discourse. Listen to students and having them listen to themselves and one another, guide them in the use of linguistic and mathematical conventions, point out the demands mathematics makes on correct language.

5. *Cautionary Tales about Buzz Words or The Education of Journalists and Politicians.* Words such as "basics", "critical thinking" and "problem solving" mean different things to different people and have the effect of polarizing people into adversarial camps. The advice is to find out what speakers mean before pigeon holding them into ideological camps[6]. Promote language that minimizes misinterpretation. In educational reform, pool strength, talents and interests, strive for balance,

[6] Anneli gave the example of "student-centered instruction" vs. "learning basic skills", the former that suggests to some the bleeding heart, hand-holding teacher, and the latter, the sergeant-like drill instructor.

and avoid adversarial stances that make the pendulum swing farther away from equilibrium.

6. *Be Competitive — Dog Eat Dog vs. Be Cooperative — We Sink or Swim Together.* Of the several contradictory messages highlighted was that concerning parents of "over-achievers" who push school staff toward high test scores via unhealthy tracking procedures and uncooperative study habits. The staff is under pressure to avoid peer hierarchies and inequities.

7. *For New Yorkers, Their Mayor, General Readers of the* NY Times*: Don't Ax Questions! Don't have an "Ask" to Grind!* The "in-joke": Clippings from the *New York Times* (When Speaking to Mayor Koch, Ax No Questions, January 29, 1988; Attention Students: Never Mind Bush's G's, January 9, 1989) prompted a reminder that questions should be encouraged and listed to, not used to grind a class to a halt.

At the end of her talk, Professor Lax congratulated the Network for its promotion of "humanistic instruction—some toward 'humanistic', others just toward 'human' and 'humane aims". She noted how fortunate many of the speakers were "in being permitted by their institutions (mainly colleges) to experiment in their mathematics classes and to build a collegial networking community, in contrast to pre-college teachers who rarely get that opportunity."

Anneli saw the Humanistic Mathematics Network as an important vehicle for efforts to engage others in addressing the impediments to K-12 mathematics instruction. She wrote to me on January 30, 1989, to discuss her reading of a newly published report to the nation on the future of mathematics education (the National Research Council's "Everybody Counts"), and her attendance at a recent conference on educational reform at Bard College. She said that both compelled her "to call the attention of the Humanistic Network the luxury of being able to experiment in classes with what they believe in, and the need for having this happen in K-12." Anneli once sought to encourage more and more humanistic mathematicians to "acquaint themselves with conditions in local schools and start modeling, with teachers, the student-involving instruction they had described," noting that many were applicable to K-12 topics.

Anneli always endeavored to find her optimal role to support the Network. In a letter dated November 24, 1989, she wrote:

> So far I have contributed mighty little to the writing project Alvin is trying to launch. Perhaps my role should be to react to or help edit stuff others are writing. I should ask you for advice. After all, you have read the stuff I have written, you have heard me hold forth on this and that; how would I fit into Alvin's plans?

Anneli indeed fit into Alvin's plans, serving as a model for the goals of the Humanistic Mathematics Network and ever vigilant in encouraging ways to achieve them. She contributed mightily to the promotion of humanistic mathematics, as usual in her quiet ways, behind the scenes with support and out front, spreading its message. Anneli frequently wrote to the HMNN / HMNJ contributors regarding their publications. Even after she had been diagnosed with pancreatic cancer in October, 1997, she remained engaged with the promotion of the Network. In an email on January 22, 1998, she wrote: "I am about half through with my treatments and am still feeling pretty well. If you see Alvin White, congratulate him for me on the most recent *Humanistic Newsletter*; it carried several excellent articles."

4.2. Ethnomathematics: Studying the Relationship between Mathematics and Culture

When the Humanistic Network was portrayed as "a white, Western-dominated outfit," Anneli pointed out, in a joint letter dated February 24, 1989, to Alvin and me, that

- More than half the papers of the HMNN stress the human and humane parts of doing mathematics, many with special attention to engaging and retaining minorities.
- The more traditional "humanistic" part, while closely tied to prevalent "history of ideas" scholarship, which in most universities is tied mainly to Western culture, in no way excludes other cultures. In fact, the Humanistic Network is interested in working with the Ethnomathematics group led by U. D'Ambrosio.

In 1985, Ubiratan D'Ambrosio defined Ethnomathematics as the maths practised among cultural groups such as national-tribal societies, labour groups, children of a certain age bracket, professional classes and so on. See [4, 25]. Anneli was a strong supporter of the efforts put forth in Ethnomathematics by the Task Force on Minorities; in particular, those focused on the nationwide reform of the teaching of calculus in the 1980s.

In 1990, Anneli and Peter Lax conducted a Workshop to Improve the Teaching of Calculus at California State University Northridge (CSUN) as part of a joint faculty workshop series for the California Community College-California State University systems. See the Photo Gallery in Appendix A. In her presentation, Anneli promoted vector approaches to teaching calculus and gave several examples for student projects. She spoke about freedom from prescribed lists and schedules of topics. She discussed alternatives to time-tests. The participants of the workshop described her discussion as provocative, informative, creative and stimulating. They saw her development of the formulas for the derivative of sine and cosine on the unit circle as elegant, and her exploration of the relationship between intrinsic properties and analytic representations as insightful. Anneli concluded her talk by posing the noted calculus problem about reflection through a curved mirror, which many of the participants subsequently worked on.

As was her habit, Anneli was not content to participate and move on. In a letter dated March 21, 1990, she asked me if there was any feedback from the participating faculty, in particular with respect to incorporating writing into their courses. When I shared the feedback, she thanked me for "flattering" comments, and told me that she filed them in her "fan mail" folder, which she used "periodically as antidepressant". Anneli also offered to correspond with any workshop participants who were interested in the curved mirror problem or any of the other projects she had suggested.

Professor Lax was tireless in her efforts to engage those teaching, as well as those planning to teach, in endeavors to improve mathematics instruction. In 1996, she asked Jim Leitzel to send me information about the MAA program New Experiences in Teaching (NExT). Anneli advised that since people in their first year or two of teaching in colleges are eligible to become NExT FELLOWS, this program would be of interest to my graduate students, and that more "seasoned" people as she and I could join the electronic conversations. She added, "For example, there has been one on the teaching of calculus possibly combined with pre-calculus for those who need it, there have been questions about selecting texts for various elementary courses, about what to put into a course for elementary school teachers, and many other things connected with educational reform."

As always, Anneli's suggestion opened pathways for listening, learning, and expanding horizons, both for me and for my students.

5. Conclusion

In writing this remembrance, I had the great pleasure of reviewing many of Anneli's writings, but also the bittersweet task of rereading her letters and emails. Throughout it all, I was constantly reminded of her skills in giving and in receiving. I am hopeful this essay conveys the depth and breadth of her efforts on behalf of good mathematics exposition in support of those teaching and learning, and her focus on all of us hearing one another.

I am also hopeful that this essay reveals how Anneli was the embodiment of her ideals. Anneli was an extraordinary listener. With thought-provoking words, she consistently raised my awareness of issues and problems and drew me into exploring creative strategies to solve them.

I am not alone in reaping such rewards from knowing Anneli. With words that echo my experiences as well, Louise Raphael recently reflected on Anneli's skills in helping her be a better expositor of mathematics, "What I especially admire in [her] letters is Anneli's kind and incisive editing — which she shares as a friend. I treasure that she included me in her circle." Lucy Moser-Jauslin speaks for me, and I am certain for many like us who pursued mathematics at a time where there were few women in the field:

> I knew how to handle being a mathematics student, but, when looking at the older generation of scientists, it was hard to imagine how it could be done as a woman. Can one be a serious researcher and have a family? How can science and life mesh? I was lucky to have two role models: Anneli Lax and Cathleen Morawetz. They were both extremely talented, and dedicated to mathematics. At the same time, they were extremely wonderful people. Being around great mentors like them was extremely meaningful to me.

Anneli's and Peter's son, Dr. James (Jim) Lax recently shared recollections of how his mother empowered him when he did not understand a problem or got the wrong answer — taking the time to see where he got "derailed". He recalled,

> She didn't see the derailment as wrong or stupid but as a valid, intelligent, creative, and sometimes humorous alternate way of

> thinking. She taught me that often there isn't a right or wrong way of thinking, but there are certainly different ways, which are the essence of creativity and innovations.

Jim noted that Anneli's ability to teach in this way required "time, patience, emotional strength, and empathy." It facilitated optimal communication and mutual understanding that was especially significant for him because he was dyslexic. He revealed that he did not know of his condition because sixty years ago no one did. He was called "an underachiever." Clearly not, as Dr. Lax recently retired from a decades-long practice as one of the preeminent gastroenterologists in New York City. He spoke about the import of what his mother advocated and modeled, with respect to his profession, and indeed in all areas of life:

> This kind of understanding and communication is important in many professions in addition to math education, and perhaps in all areas of life. I believe it to be essential in my profession: medicine. If politicians could be taught these skills, they might consider that their opponents may at least have a point and compromises could be made. Our country and the world would be in far better shape.

I am among the many who are in better shape because I knew Anneli. I am appreciative of the opportunity to raise my voice in appreciation of this extraordinary woman, in a journal devoted to the ideals she cherished.

Acknowledgments: I am grateful to Dr. James Lax and Professor Louise Raphael for sharing their memories and their stories. I am appreciative of their salient comments and suggestions for this paper. I thank Gizem Karaal for encouraging me to write this paper, and for her usual insightful editing of it. I thank Jim for granting permission to share his family photos. Above all, I owe the deepest gratitude to Anneli and Peter Lax who opened their hearts to me, and from whom I have learned so much.

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A. Photo Gallery

A.1. Courtesy of James Lax: Anneli - daughter, sister, wife, mother

Anneli Cahn, Age 9.

Anneli, Age 19, with parents & brother, Dieter in Mannsville, New York.

Peter and his future bride Anneli.

Peter with son John

Lax Family Sledding in Central Park.

Anneli with Jim on her lap and John.

Anneli (second row, fourth from right) and Peter (second row, first from right) at the International Congress of Mathematics, Moscow, Russia, 1966.

Anneli, recreating in the Adirondacks. Sailing on Loon Lake.

Reading at Prince Camp.

Anneli and her son Jim

A.2. Courtesy of Elena and Joe Marchisotto: 1997 Visit to Prince Camp, Loon Lake.

Anneli and Joe.

Peter and Elena.

Making dinner at Prince Camp.

Enjoying the repast.

Sharing Good Times

A.3. Courtesy of Elena Marchisotto: 1990 Calculus Workshop CSUN

Alvin White, Anneli, Peter, and Elena

Anneli Presenting

Elena and Peter in Discussion