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Just Mathematics: Getting Started Teaching Postsecondary Math for Social Justice

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Cover Page Footnote

I am grateful to the hardworking educators who have blazed the path of teaching math for social justice, largely at the K-12 level. I have learned much of what I know from folks like Bob Moses, Jonathan Osler, Eric Gutstein, Rochelle Gutiérrez, Gizem Karaali, Lily Khadjavi, and many others.

Just Mathematics: Getting Started Teaching Postsecondary Math for Social Justice

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Abstract

Following the summer 2020 civil rights movement and increasing attention to the intersections of mathematics with politics and power, many math educators have reported a desire to implement an antiracist pedagogy and to examine the intersections of their subject with issues of equity, inclusion, and social justice. Many resources exist for K-12 math educators interested in incorporating social justice into their curricula, but resources are comparatively scarce for college and university instructors (though this is changing quickly!). We discuss why one may want to teach mathematics for social justice, how to begin to implement issues of social justice into postsecondary math courses, and publicly available social justice materials for postsecondary math courses.

Keywords: social justice, data science, equity, inclusion, diversity, ethics, society.

1. Introduction

The 2020 police murders of Breonna Taylor, George Floyd, Andre Hill, Manuel Ellis, and Rayshard Brooks, among many other Black victims of police violence, sparked a desire on the part of many educators to consider the impact of their subjects and pedagogy on issues of social justice. Moreover, the COVID-19 pandemic demonstrated that we are all impacted by each other's actions — only one infected person is necessary to spark a superspreader event — and the unequal impacts of the pandemic remind us that historical and present marginalization continues to cause inequity today [27].

The origin of the word pandemic — from Greek *pandemos* meaning "all people" — amply illustrates the underlying impact of human society and social injustice even in the context of a virus that seems to "target everyone".

How can mathematics, as a field which favors Western European paradigms of proof and logic and seems to many to be "objective," wrangle with issues of social justice? The author suggests we begin in the way we teach and discuss mathematics. For the large proportion of academic mathematicians who teach undergraduate and/or graduate courses, the way we present and frame mathematics can dramatically impact our students' experiences, whom we welcome into our field, and what our students go on to do with the math they've learned. Many of our students will go on to be employed in data science, work with technology that impacts people's lives, or even become politicians. How do we want them to see and use mathematics in these future careers? What mathematics do we want to investigate in our classrooms? The author suggests that there is no true value-neutral way to teach mathematics; we either welcome students into our classroom and create a just space, or we attempt to ignore issues of justice and end up perpetuating existing injustice (assuming we do not consciously promote injustice). We are used to rigorously examining mathematical paradigms; why not try to analyze the social paradigms in which our math is situated?

Finally, in a moment where democracy is under attack around the globe, social justice at its heart can be seen as a core expression of universal democracy: the idea that everyone should have a say in how they are governed, that all humans deserve the right to live and thrive and seek happiness, that all people deserve access to the material conditions and freedoms necessary for a fulfilling life. The highest aspirations of our education system are to train people to be engaged citizens in a democratic society and to help sustain that society.

As always when implementing new pedagogies, the cardinal rules for all that follows are "keep it simple" (for students, especially) and "take small steps" — no need to completely reinvent a course or immediately design a new course on math for social justice. There are a great many relatively minor tweaks that can be made to existing math and other STEM courses to move them closer to a justice framework, and taking one's time is often the best way to adapt existing techniques and materials to one's own teaching style and context.

2. Evidence-based Pedagogies Promoting Equity and Student Success

Longtime teachers of math for social justice identify two primary ways in which mathematics instructors promote equality of opportunity through their teaching work: promoting the academic success of underrepresented students and encouraging students to critically analyze their sociopolitical environment with an eye toward social change [32, 43]. Of course, the definition of "equality of opportunity" is not universally agreed upon, but in this article it will mean a state in which no systemic or structural bias prevents minoritized groups such as students of color, women, LGBTQ+ students, Disabled students, poorer students, and others from accessing mathematics and the jobs for which it is a gatekeeper.

Though this article will focus mostly on applying mathematics to issues of social justice in the classroom, we first outline some best practices for promoting the success of minoritized students in the math classroom, all of which are useful pedagogies for a class involving mathematics for social justice. We begin by reviewing some cautionary literature on the experiences of students from underrepresented groups in the mathematics classroom.

2.1. The status quo

Preservice instructors often view student diversity as a problem rather than a resource, and many teachers hold a view that "all students should be treated equally in the classroom," even when this leads to disproportionately negative outcomes for students of color [36]. This one-size-fits-all mentality is a common, serious obstacle to true teacher reflection and equity at both the K-12 and higher education levels. Further, the "color-blind" mentality, in which a teacher claims "not to see color," leads to ignoring students' cultural backgrounds in the classroom and decreased student engagement, as well as a refusal to combat historical and actual racist practices. In classroom observations, Rousseau and Tate report several instances of students being "allowed to fail" through their teacher's neglect; such students were disproportionately Black, specifically Black men [36].

In addition, the way mathematics is taught around the world is often grounded in a colonial, Eurocentric narrative in which Europe is a "civilizing" force and the significant contributions of mathematicians from non-European backgrounds are downplayed. Such mathematical leaps as written mathematical

records, a place value number system, the technique of measurement, our base 10 number system, the "Pythagorean Theorem", and algebra all originated outside of Europe and were then used by Europeans to advance their own mathematics [21]. The marginalization of these contributions contributes to the perception of mathematics as independent of economics, politics, and culture; frames mathematical pursuits as confined to a select few (usually white, cisgender, heterosexual men); and enshrines deductive axiomatic logic (i.e. "formal" mathematical proof) as the only method of mathematical discovery.

How, then, can a mathematics educator actively combat these trends of inequity? In the next section, we emphasize the research showing that engaging with issues of social justice may promote student learning. Additionally, there are pedagogical techniques that have been shown to benefit all students, especially those from backgrounds underrepresented in mathematics.

2.2. Evidence-based pedagogies and "active learning"

One of the most effective teaching techniques involves active learning; however, this concept is not always well-defined in the literature. Perhaps the most famous meta-analysis on the subject, of 225 studies on the impact of active learning on STEM undergraduates, showed that active learning increases average grades by half a letter and that failure rates in a pure-lecture environment are 55% higher than rates observed in an environment that incorporates regular active learning [14]. This observed difference of failure rates (odds ratio: 1.95) is analyzed in the Discussion in the following way:

> If the experiments analyzed here had been conducted as randomized controlled trials of medical interventions, they may have been stopped for benefit — meaning that enrolling patients in the control condition might be discontinued because the treatment being tested was clearly more beneficial.

Not only have active pedagogies been shown to increase the performance of all students, they disproportionately increase the performance of students from backgrounds underrepresented in STEM. Bayesian regression meta-analyses performed by Theobald et al. [41] found that active learning reduced the average gap between students from minoritized groups in STEM (abbreviated MGS and including low-income students as well as Black, Latin@/e, and Native American students) and majoritized students in examination scores by 33% and in passing rates by 45%. Notably, the greatest improvements for

MGS students were observed in courses in which student activities occupied at least two-thirds of class time compared to lower percentages of class time.

It seems obvious from this data that "active learning" should be universally incorporated in STEM classrooms; however, the type of active learning in the analyzed studies varied "widely in intensity and implementation, and included approaches as diverse as occasional group problem-solving, worksheets or tutorials completed during class, use of personal re sponse systems with or without peer instruction, and studio or workshop course designs." Freeman et al. attempted to crowdsource a common definition from over 300 seminar atten dees, arriving at "students' use of higher order thinking to complete activities or participate in discussion in class". Brame [8] synthesizes multiple definitions to arrive at "activities that students do to construct knowledge and understanding" using "higher order thinking" and "metacognition — students' thinking about their own learning." Working from this definition, Brame collects several active learning techniques organized by the amount of class time required. This author recommends that readers "ramp up" the amount of time used and modifications made to their course in order to scaffold their expertise in active learning. Examples of low- or moderate-intensity active learning supple ments to lectures include the use of clickers, group discussion, think-pair-share, one-minute reflections on learning or critical thinking, writing their own test questions, and making concept maps.

For much more information on evidence-based pedagogies, the work of Josh Eyler [13] and Ken Bain [2] is highly recommended.

2.3. Alternative grading

Throughout the vast majority of its history, feedback and assessment in education used a myriad of nonstandardized systems different from the ones used in the United States today. As best as historians of higher education can determine, the four-point GPA system originated at Yale University sometime prior to 1785, when it was recorded in Yale president Ezra Stiles' diary that 58 Yale students received exam grades of "Twenty Optimi, sixteen second Optimi, 12 Inferiores (Boni), ten Pejores" [38]. These four strata likely evolved into the GPA system, though letter grades had not yet developed in their current form. Other colleges and universities, such as William and Mary, used expressive adjectives to describe student performance through 1850 [11].

Though commonly used as a major factor in hiring by employers, college grades have a low correlation to long-term job performance ($r = 0.05$), while cognitive skills are highly related to job performance [1]. This implies that grades are doing a poor job of measuring both future job performance and cognitive skills. Indeed, if our goal is to teach for social justice, it is essential to consider the repeatedly-demonstrated bias of traditional grades against poorer students, students of color, and neurodivergent students, among many others [39].

It is hard to provide a complete list of the problems with traditional grading, but Blum provides a detailed and descriptive list [6]. Grading assumes learning, and students, are uniform; numerical grades are categorical data in disguise, hence averaging them is meaningless; a standard averaging system penalizes students who are going through physical or mental health crises; averages ignore student growth; grades provide transitory extrinsic motivation instead of sparking more durable intrinsic motivation to learn mathematics [34]; students implement a "minimax strategy" to do as little as possible to earn the highest possible grade; students treat college as a game in which the rules of grading are arbitrary and inconsistent (and they're right); and grades prevent students from taking worthwhile risks or risking productive failure.

Given the magnitude and quantity of failures of traditional grading, it is highly recommended to put your social justice commitment into practice in the classroom by adopting a system of "grading for growth." Instead of attempting to describe the many such systems here, we refer the reader to the above cited sources and to mathematicians Robert Talbert and David Clark's excellent open-access Grading for Growth blog [40], which includes articles on subjects ranging from questions to ask when getting started and first steps to building a course structure, earning students' trust, and getting administrator buy-in.

3. Why Teach Applications of Math to Social Justice?

Teaching mathematics with an equity frame doesn't just help fight the many trends of inequity described in Section 2.1; using an equity frame that emphasizes the role of mathematics in democracy and addressing inequality results in increased student learning and achievement [16, 28, 43]. The National Survey of Student Engagement, 2004, found that undergraduate students

are often passionate about social and political issues; bringing such issues into the mathematics classroom can increase student engagement and performance [19]. In a controlled experiment conducted by Winter, students taught precalculus using an approach which incorporated the mathematical analysis of sociocultural issues performed statistically significantly better than a control group taught using a "standard" approach. Students taught using a sociocultural pedagogy performed better on a mathematical assessment (exceeding the control group's scores by 11.9%), and fewer students earned a grade of D or lower in the class (15.4% in the justice pedagogy condition, as opposed to 22.9% in the control group) [43].

Since mathematics is a human activity, it is inherently situated in the sociopolitical contexts in which it is done, and even pure mathematics can have unpredictable societal resonance. The work of Cathy O'Neil on bias inherited by seemingly-"neutral" algorithms provides one striking example of the fact that leaving sociopolitics unexamined perpetuates bias [31]. Moreover, mathematical communities are compromised of human beings, and therefore have their own politics and oppressive structures. The American Mathematical Society (AMS)'s appointed Task Force on Understanding and Documenting the Historical Role of the AMS in Racial Discrimination found its own name to be a misnomer; racial discrimination is not consigned to the past but continues in mathematical communities today [37]. The task force found that the effects of blatant racial discrimination in mathematical communities continue today; that the AMS missed multiple opportunities to improve the climate for mathematicians of color; that Black mathematicians continue to face a lack of professional respect; and that Black mathematicians (understandably) disproportionately distrust the AMS.

Therefore, math is already political. A continued lack of attention to issue of social justice at all levels of the mathematical profession is therefore unsustainable. It is as if mathematical communities had been diagnosed with a terminal disease, that of promoting and/or being neutral to injustice; it would be ridiculous to ignore this disease and hope it goes away. Instead, mathematicians must seek treatment — openly discussing and combating injustice through mathematics. Our prognosis depends upon authentic reflection and action, examining questions such as, "What assumptions do our algorithms and databases inherit [31]? What do we think is most urgent or "worthy" of mathematical inquiry: math with applications to petroleum engineering and fracking [18]? Statistical analyses of police stops by race [23]?"

3.1. Qualitative evidence of student success and agency

We now move from quantitative studies to more anecdotal and qualitative evidence. The author of this article has taught a course entitled "Social Justice by the Numbers" at their small liberal arts institution for three years. Several times throughout the semester, students are solicited for their feedback on the course and their own attitudes and beliefs toward mathematics. The following is a sample of two student comments which express sentiments common in these evaluations:

> "The things that I've learned from this class has [sic] helped me to kindle an interest in math that I've never had in my 29 years of life...Whatever my math future may hold I know that this class has been a pivotal part of establishing a foundation.
>
> [...]It was not until I...was able to take a math and social justice class together that I saw that I had the potential to be good at math. Since that first class I have continued to take math classes every semester and this research project [on racial bias in a local police department] is the next step."
>
> "Ever since the project I have gone out of my way more to pick up waste after other people and I know that even my small impact can make a difference in the long run. I also have become noticeably more involved in researching environmental issues since we talked about the topic in class...I pay more careful attention to what is said in the news specifically regarding number statements and taking all statements with a grain of salt...this class has helped me realize that I need to use my voice in certain situations."

Overall, if our goal as educators is to promote student mathematical agency and better citizenship, teaching math for social justice is one way to achieve these goals. Unfortunately, there is a dearth of studies that investigate specific teaching practices which benefit underrepresented students, including teacher-student interaction and its effects, though this may be changing in the last decade. Although trying to specify general practices across a group as diverse as, for example, African-Americans in the US, may lead to essentialization and/or a deficit framing, a variety of historical conditions in the US have meant that many students of color have been told that they

are inferior academically. Therefore, such studies would benefit many groups which are systemically marginalized in US mathematics education, including women and Latin@/e students [22].

Further, although there are several collections of "mathematics for social justice" projects, units, and lessons targeted mostly at K-12 educators, Gutstein [17] claims that no cogent and cohesive curriculum exists and identifies the need for one. Thankfully, Karaali and Khadjavi [24, 25] have recently edited and published two books of resources for postsecondary educators looking to teach math for social justice, the first focused primarily on mathematics classes and the second on statistics and quantitative reasoning. Further research is needed into the pedagogy and impacts of such an approach, and that such research is deeply essential, since equity-focused mathematics is good for students, underrepresented and otherwise, and good for the society in which we live.

For those who would like to read further into the literature on teaching math for social justice, this author reviews twenty journal articles and reports on the topic on their website. (Please see Note on Author's Online Resources.)

4. How do I Get Started Teaching Math for Social Justice?

4.1. Designing a social justice math course (backwards)

It is thus reasonable, in fact urgent, for mathematicians to ask ourselves: how can we educate all of our students in a manner that promotes equity, justice, and a critical analysis of their sociopolitical contexts? How can math be used as a powerful, vibrant tool for investigating and promoting social justice? And, critically, how does one design a course that enables students to use math for social justice?

Perhaps more fundamental to the latter goal is the idea of backwards design — starting with what one wants students to understand by the end of the semester, perhaps overlapping significantly with a list of Essential Questions related to social justice issues (see the next paragraph for more detail), and determining what needs to happen in the course for students to achieve those learning outcomes. Backwards design begins by asking oneself the following questions: "What should participants hear, read, view, explore or otherwise encounter? What knowledge and skills should participants master? What are big ideas and important understandings participants should retain [7]?"

In the second stage, instructors con sider what assignments are necessary to assess these learning goals — term papers, quizzes, homework, labs, and many others. Finally, instructors create learning activities which will equip students with the knowledge and skills necessary to complete these assignments and identify materials that are helpful in achieving the learning outcomes identified in the first stage. The backwards design template available from McTighe & Associates [29] is helpful in organizing the course design process.

4.2. Practices to promote socially just mathematical experiences

Once learning outcomes are established, high school educator and Radical Math founder Jonathan Osler [32] provides helpful advice for structuring the student experience in the course. Osler's tips include: (1) work with a textbook with a strong mathematical framework and scaffolding; (2) discuss student interests before deciding on equity/justice issues to cover; (3) set up units around "Essential Questions"; (4) introduce the equity/justice issue first; (5) then begin the mathematics, devoting as much time as necessary to the concepts and scaffolding both math and equity/justice along the way; and finally (6) end with a great project. Some specific unit ideas: for exponents, studying compound interest and population growth; for probability, exploring the possibility that a traffic stop should be (and is) a person of color [32, 17].

Following a similar structure, Winter [43] provides a model of equity- and social justice-oriented pedagogy in STEM courses which introduces sociocultural phenomena through readings, videos, and activities, then assigns a problem and a structured worksheet to help students apply mathematics to the problem. Finally, students interpret their mathematical solution in the context of the original issue, often leading to discussion about the broader implications of their solution. This model has been applied to precalculus classes; teaching functions, graphing, and statistics through examining political corruption and economic development worldwide; climate change in Zimbabwe and its impact on agriculture; and water security and Native peoples' rights in Botswana, to name a few.

Winter derives several principles for justice-focused education from Marilyn Frankenstein [15], including using real situations and real information; emphasizing situations that students might have learned about through news or other media; and using controversial materials to help students increase

their curiosity, understand other perspectives more accurately, solve problems more effectively, and generate creative ideas [20].

In the author's experience, staying responsive to student interests has been key. It is recommended to follow campus activist and/or student union groups and use campus or local issues as a warm-up, or analyze the data behind a local issue for a full-class activity. Have students vote — perhaps through a ranked-choice poll followed by a discussion of voting methods — on the social justice issues they'd like to discuss toward the beginning of each semester. The website RankIt, among others, facilitates the process of ranked-choice polling (see Author's Online Resources for an example of such a poll).

Too easily, a social justice math course can become a litany of mathematical and statistical arguments that the world is deeply unjust. Moreover, students from minoritized groups may be retraumatized by discussions of issues that already impact their daily lives. To help avoid these pitfalls, it is important to bring a solution-based component to a social justice math course. One example is to assign a semester project in which students promote social justice issues in their own communities by using math or statistics in some way. This type of project is often left open-ended aside from the stipulations in the previous sentence; see Author's Online Resources for an example.

Students should present and share their work, both on class worksheets and their semester project. Many proponents of social justice pedagogy engage in service learning in their communities, for example by sending students to analyze data for a local nonprofit or tutor at a public school or community center. However, service learning has the potential to mire students in shock and dismay about issues of injustice, or to promote exoticization of minoritized community members, if not carefully managed. It is essential to provide background that situates community minoritization in proper context within social structures and systems of oppression, promote just solutions, and attend to student cognition and development throughout the experience. If done well, service learning can lead to significant changes in student attitude about social justice, equity, and civic responsibility [35]. When students reflect on their service experiences through regular journaling, engage with a longer-term institutional or community program, and discuss relevant social justice issues in the classroom while experiencing them in the community, students can have emancipatory mathematical experiences that promote future community engagement [42].

Again, one of the biggest potential pitfalls involves thinking that one has to reinvent the wheel, while starting to teach math for social justice using others' materials can be more like airing up a tire — the machinery is already there; it just needs an infusion of instructor and institutional context and personality. This author started small by implementing a half-day lesson on measuring inequality using Gini coefficients in a Calculus I course. Only then did they move on to spending multiple class sessions discussing racial bias in police use of force in a statistics course, performing a literature review on teaching math for social justice, and eventually designing a first-year seminar course entirely focused on social justice mathematics.

It is essential as well to begin with a strong mathematical framework, one that the instructor keeps in mind as a guiderail through various Essential Questions and course topics. A great text such as the ones described in the next section can provide strong scaffolding for the instructor. Many mathematical concepts, especially those taught in lower-level courses, lend themselves to many different social justice issues; the resources in the next section can point the instructor to some options. As Osler [32] says, "Always fit the issues to the math," not the other way around.

5. Public Resources for Teaching Math for Social Justice

This section provides links and information about the most thorough, and ideally open access, resources found by the author. All open-access resources can be found in the Author's Online Resources. Some of the most suitable textbooks to scaffold a full course in social justice math are listed below; these resources are also excellent as sources of individual modules to implement in an existing course.

5.1. Module collections & textbooks

Resources are beginning to be developed for full postsecondary courses, largely in quantitative reasoning, for social justice. The most complete as of the submission of this article are the lesson plan collections edited by Karaali and Khadjavi [24, 25], which consist of independent modules capable of insertion into various courses. A (perhaps outdated) table listing the modules in both collections by both mathematical content and social justice topic(s) can be found in the Author's Online Resources and is shared with the kind permission of the editors. The author has successfully implemented modules on

function composition and sea level rise, linear modeling and climate change, reported health status of those living near fracking wells in Pennyslvania, residential proximity to highways by race, the impacts of student loan debt, and the demographics of minimum wage earners, among others. Many of the modules in these texts are appropriate for general-education or quantitative reasoning courses, but these collections also include material suitable for courses ranging from statistics for STEM majors to linear algebra and multivariable calculus, discrete mathematics and graph theory, and combinatorics.

Even more recently, several authors and organizations are developing their own texts designed for implementation, either as individual modules or in sequence, in undergraduate courses without prerequisites. One very promising open-source text in development, Math for the People [5], can be freely accessed online and features modules with a wide range of authors, all collected and edited by Mark Branson and Whitney George. Modules are targeted at a first-year-level quantitative reasoning course and include social justice topics such as environmental racism, gerrymandering, payday loans, police racism, and more. This collection is still growing.

David Lippman's Math in Society [26] is open-source, available online, and though not explicitly framed around issues of social justice, includes topics such as voting theory, cryptography, and counting systems used by various cultures throughout history. Each chapter could be implemented as a standalone module or in sequence in a quantitative reasoning course targeted at liberal arts majors.

5.2. Other open-access social justice math resources

The Author's Online Resources folder contains many other resources ranging from individual lesson plans to two collections of worksheets used by Dave Kung and Robin Wilson, respectively, as the backbone for their semester-long no-prerequisites social justice math courses. The materials are organized by type: social justice-related datasets, materials for getting started, others' public social justice math resource lists, readings on social justice issues, and even the author's full Canvas course from their first-year Social Justice by the Numbers course. These materials are free to use with attribution and have been shared by permission of the original authors. A table summarizing the resources in the folder organized by mathematical as well as social justice topic can be found at Author's Online Resources.

The resource folder is always looking for more materials suitable for teaching math for social justice at the postsecondary level. The author hopes the reader will consider openly sharing materials they create for their courses through the Google Form linked at Author's Online Resources.

Finally, the author would be remiss not to mention that K-12 educators have been doing the work of developing and implementing social justice math resources for decades longer than postsecondary educators. One of the earliest such resources, the venerable Radical Math website [33] was recently redesigned and relaunched by curator Jonathan Osler; it is searchable by social justice topic or mathematical content and contains a great deal of material suitable for middle school, high school, and preservice college-level courses. It was partially through reading and utilizing Osler's work that this author came to realize the liberatory, inspiring work of teaching math for social justice.

6. Developing Your First Social Justice Math Lesson

At some point after implementing various existing resources in the classroom, the reader may wish to create their own social justice math lesson. Following the design process above, and assuming that the learning objectives for the course have already been established, this may be done through the following steps from Osler's excellent guide [32], illustrated with examples developed by the author.

Even if the reader is not at the point of wishing to design their own social justice math lesson, it may be helpful to follow these steps when adapting existing lessons to the reader's context.

The pedagogical best practices described in Section 2 should permeate each step, including the use of scaffolding to guide students through both the mathematical concepts and the social justice issue and the importance of promoting student agency through solution-based projects.

We recommend following steps similar to the above when designing one's own lesson. We provide an exercise for the curious instructor below, which we hope helps to kick-start lesson development.

Exercise 1. (Begin to design your own social justice math lesson).

1. Identify a single class into which you would like to introduce a social justice math lesson or unit.
2. After writing the learning objectives for that course, choose one objective and identify a resource in one of the tables at Author's Online Resources or, for a (pre)service/QR course, `https://radicalmath.org` that would fit your objective.
3. Set the unit in the context of a few broad, open-ended questions that do not have one specific answer (often called Essential Questions). For example: "Does race play a factor in who is getting mortgage loans in our city?" These questions should have both a mathematical and social justice component to them.
4. Share your Essential Question and resources from steps 2-3 with a few other instructors, if possible, and offer each other feedback and thoughts.

Step	**Example**
Identify essential mathematical concepts for the lesson.	Summarize the relationship between two categorical variables using two-way tables. Lead up to discussion of chi-squared tests.
Talk to students to decide which issues to focus on related to these concepts.	After a ranked-choice poll, students in 2021 voted to discuss racial justice.
Create broad, open-ended Essential Questions for the lesson.	How can statistical exploration contribute to understanding/fighting for racial justice?
Start the lesson by introducing the social justice issue.	Students free-write for 5 — 10 minutes on the following (adapted from [3]), then share their thoughts with their groups and then the whole class: (1) What do you know about the relation ship between policing and race, ethnicity, or class? Do you know of statistics that back this up? (2) What do you see (and/or create) on line or in the news related to policing? (3) What do you wonder about policing? How could you find data to investigate this question?

Begin introducing the math.	Students follow a worksheet to create observed and expected relative frequency tables as they examine police arrest, stop, use-of-force, or similar data by race.
The social justice issue doesn't have to be the focus of every lesson.	Students spend time practicing exploratory data analysis in situations with a categorical explanatory and numerical response variable, among others. Students discuss how to measure differences between categories in observed and expected two-way tables, leading up to the introduction of the χ^2 test statistic.

Table 1: A selection of Osler's [32] recommended steps for designing a social justice math lesson, together with examples of each step used by the author to design a lesson on police bias.

7. Addressing Difficulties in Implementation

In this section, we describe some common issues encountered while teaching math for social justice, together with proposed solutions learned from teaching a full course on social justice mathematics.

7.1. Set class norms for discussion and get student buy-in

First, when discussing social injustices, minoritized students who are impacted by the issues discussed are at risk of retraumatization. Moreover, majoritized students may feel as though their identities are being attacked, and some may feel they're at risk of losing a privileged status. To those who have lived their lives in privilege, having that privilege challenged may feel like oppression.

Though we have found no single way of eliminating these issues, setting classroom norms for discussion of sensitive issues and allowing students to opt-out of discussions in which they are at risk of retraumatization without penalty may help. As an example, an instructor may have students discuss the following in order to get buy-in for the norms that follow, as well as to create their own classroom norms for the semester. The first exercise below is adapted from Dana Ernst's "Setting the Stage" activity [12]. It is recommended to utilize a similar activity on the first day of class to promote student buy-in for both active pedagogies and safe/brave space discussion norms.

Exercise 2. (Setting the stage). Get in groups of size 3 — 4. Group members should introduce themselves: name, pronouns, hobbies outside school, goals for the semester/year, and the reason they're taking the course. For each of the questions that follow, I will ask you to:

1. Think about a possible answer on your own.
2. Discuss your answers with the rest of your group.
3. Share a summary of each group's discussion.

The questions are as follows:

1. What are the goals of a college or university education? How does social justice fit in? How does math?
2. How does a person learn something new?
3. What is the value of making mistakes in the learning process?
4. How do we create a safe environment where risk taking is encouraged and productive failure is valued?
5. What does respect look like for you? How might ideas of respect vary with cultural context? How might you firmly challenge the views of someone else in a respectful manner?
6. What are the differences betweeen a personal attack and a challenge to an idea that makes an individual feel uncomfortable? What are some situations that might blur the lines between the two? How can we acknowledge when certain beliefs (e.g. trans people not existing) are inherently personal attacks?

Students will often arrive at some of the following norms on their own through discussion. Often, students will also suggest helpful class norms to add to the list below while answering the Setting the Stage questions above. Here are some examples of classroom norms that have been useful to the author:

1. *Controversy and vulnerability.* Different views are expected and honored with a group commitment to understand the sources of disagreement and to work cooper atively toward common solutions without putting a disproportionate responsibility on marginalized groups to educate us.

   (a) No one's identities are under question: e.g. no arguing that queer people are wrong for existing.

   (b) Please come talk to your instructor anytime you feel uncomfortable enough that it's affecting your work in class.

2. *Own your intentions and your impact.* Acknowledge that the impact of your actions is not always congruent with your intentions and that positive or neutral intentions do not trump negative impact. Participate truthfully and accept when your actions or words harm others.

3. *Challenge by choice.* Individuals may determine for themselves if and to what degree they will participate in a given discussion or activity. It is possible not to participate in a few activities and still receive an A in the course. Be aware of what factors influence your decision about whether to challenge yourself on a given issue.

4. *Respect.* each other and yourselves. Investigate how your cultural context affects what you think of as respect for others and acknowledge what respect looks like in other contexts.

5. *No attacks.* Personal attacks are a form of extreme disrespect. Disagreeing with ideas is welcome; attacking individuals for their ideas is not.

Interestingly, the author has found that they receive less pushback from students averse to social justice when bias is openly admitted to students. Mentioning something along the lines of, "In this course/lesson, I admit that I am convinced by the data that an environmental justice approach is necessary to understand the human impacts of fracking in Western Pennsylvania, so we will consider questions of racial oppression in this lesson," may help students realize that "both-sidesism" may not be appropriate for a particular course or lesson framework.

7.2. Pushback form math/stats departments

One common issue heard from people interested in teaching math for social justice is, "I don't know if my department would support this. How could I frame a social justice math lesson so as not to get in trouble with my chair, Dean, Provost, course coordinator, advisor, etc.?" Though this issue takes myriad forms based on institutional context, starting by emphasizing the

ways in which the mathematical status quo promotes obvious injustices (for example, Black folks being marked as higher recidivism risks by algorithms designed by statisticians because of their race alone [31]) may be a way to frame the discussion.

The fact that mathematics as a field is disproportionately likely to force out women, gender-expansive individuals, queer folks, and people of color, according to the AMS itself [37] and the Columbia University Teacher's College [BFM20] among many others, could be used to justify changes in the way mathematics is taught, perhaps along the lines recommended by Just Equations [10]:

> Additionally, math content should be relevant to a student's social contexts as well as their desired area of study ... researchers have found that students, particularly Black and Latinx students, are more likely to engage in math and form a math identity when they can see the links between what they are learning and their communities.

Finally, especially for those hoping to design a social justice math course, consider interdisciplinary offerings. The author's social justice math course is listed under the Social and Behavioral Sciences core, fulfilling a college breadth requirement as well as meeting the required Quantitative Emphasis. In 2022, it was offered as a first-year Learning Community coordinated with the college's Justice Studies department; students are required to take both Social Justice by the Numbers and an introductory Justice Studies course entitled Power and Social Change.

While the reader's institutional context may be very different, perhaps opportunities exist to work with the sociology department to co-teach or cross-list a math course with a course that provides students disciplinary background on social theory. Mathematicians are not required to be experts on social issues in order to discuss these issues; they must only be willing to educate themselves and seek collaboration with those who are experts if possible.

7.3. How could social justice fit into upper-level math classes?

The module collections edited by Karaali-Khadjavi, especially the first [24], contains modules appropriate for courses in differential equations (modeling the spread of the 2010 Gulf oil spill), discrete mathematics/combinatorics (e.g. voting with partially-ordered preferences), graph theory (e.g. modeling

the acceptance of queer relationships), and geometry (compactness measures for gerrymandering).

In many fields of theoretical mathematics, an application can be found which lends itself to a social justice lesson. For example, in a number theory or abstract algebra course, a brief introduction to cryptography and RSA encryption can serve as a springboard for a discussion of NSA surveillance in which students wrangle with questions such as, "Do you agree with the statement that, 'Mathematics can be used for good, but it can also be used for evil?' What does this mean? Was the NSA's reading of private emails necessary for national security? How much privacy is appropriate to sacrifice for security?" A course in algebraic geometry could discuss elliptic curve cryptography using similar questions. A course in topology could take a detour into persistent homology and data analysis with an emphasis on modeling political structures [30] or gerrymandering [9]. In almost any upper-level pure math course, some application exists with resonance to social justice. As mathematicians, we are trained to sit with uncertainty and brainstorm multiple approaches until we find one that works; this process could just as easily apply to designing social justice math lessons.

7.4. Where can I find homework or quiz questions to assign?

When an instructor is not working from a particular textbook and instead using a pre-designed social justice math module, it can sometimes be difficult to come up with problems or questions for assessment of students. Though this is changing wtih the advent of more college-level social justice math textbooks, one may have to design their own questions based on a reading. Mathematicians are not often trained to do this. However, the following process has been fruitful for the author:

1. Search for a reading (ideally from a .edu or known reputable site, book chapter, Web article, etc.) that gets at the social justice or mathematical issue.
2. Read it, paying attention to questions related to social justice issues and jotting them down. For example,
   (a) who benefits from this presentation of the situation? Who is harmed?

   (b) what data or mathematical analysis would help get more insight into the issue?

   (c) what role does historical oppression play in the present situation?

   (d) what role do existing racism, sexism, queer-/transphobia, xenophobia, ableism, and other oppressive social structures play in the issue?

3. Consider what questions the reading brings up and how you may scaffold students to consider, and perhaps even answer, those questions.

4. Depending on the desired assignment length, trim down to the few most compelling or central questions for instructional purposes and assign those for students to con- sider, and discuss, toward the beginning of the next class period.

This emphasis on discussion also promotes active learning.

The author wishes you the best of luck in your teaching and hopes that the strategies and ideas described above are helpful in promoting greater justice and greater appreciation of the power of mathematics in your students.

Author's Online Resources

The author has created a curated collection of a wide range of online resources for instructors aiming to teach math for social justice, available at https://www.kenanince.org/OER4SJ.[1] Further resources are hosted on a Google Drive folder titled "Teaching Postsecondary Math for Social Justice Resources Folder (ed. Kenan Ince)" and can be accessed at this time via https://drive.google.com/drive/folders/1crKiL54la0s5FbOgICIsWj-Dd-6RhS5p, last accessed on July 31, 2023. Some of the references in the paper are to specific files in one or both of these folders.

[1] As of July 31, 2023, this link is no longer active. The contents can still be mostly accessed via the WayBackMachine: https://web.archive.org/web/20230602185411/https://www.kenanince.org/OER4SJ.

References

[1] J. S. Armstrong, "Natural Learning in Higher Education," In N. M. Seel, ed., *Encyclopedia of the Sciences of Learning*, Springer US, 2012, pages 2426–2433.

[2] Ken Bain, *What the Best College Teachers Do*, Harvard University Press, Cambridge Massachusets, 2004.

[3] Robert Q. Berry, Basil M. Conway, Brian R. Lawler, and John W. Staley, *High School Mathematics Lessons to Explore, Understand, and Respond to Social Injustice*, Corwin, Thousand Oaks California, 2020.

[4] Jessica Brathwaite, Maggie P. Fay, and Adnan Moussa, "Improving Developmental and College-Level Mathematics: Prominent Reforms and the Need to Address Equity," 2020.

[5] Mark Branson and Whitney George, eds., *Math for the People*, 2022.

[6] Susan D. Blum, "Ungrading," *Inside Higher Ed* (November 2017); available at https://www.insidehighered.com/advice/2017/11/14/significantlearning-benefits-getting-rid-grades-essay.

[7] Ryan S. Bowen, "Understanding by Design," Vanderbilt University (2017); available at https://cft.vanderbilt.edu/guides-sub-pages/understanding-by-design//

[8] Cynthia J. Brame, "Active Learning," in *Science Teaching Essentials*, Elsevier, 2019, pages 61-72.

[9] Moon Duchin, Tom Needham, and Thomas Weighill, "The (Homological) Persistence of Gerrymandering." *Foundations of Data Science*, Volume **4** Issue 4 (2021), pages 581–622.

[10] Dina Buck, Pamela Burdman, and Mina Dadgar, "Solving for Equity: Design and Implementation of New Postsecondary Math Pathways," Tech. report, Just Equations, May 2021.

[11] Mark W. Durm, "An A is not an A is not an A: A History of Grading," *The Educational Forum*, Volume **57** Issue 3 (1993), pages 294-297.

[12] Dana C. Ernst, Setting the Stage, http://dcernst.github.io/setting-the-stage/, January 2015.

[13] Joshua R. Eyler, *How Humans Learn: The Science and Stories behind Effective College Teaching*, West Virginia University Press, Morgantown, 2018.

[14] Scott Freeman, Sarah L. Eddy, Miles McDonough, Michelle K. Smith, Nnadozie Okoroafor, Hannah Jordt, and Mary Pat Wenderoth, "Active learning increases student performance in science, engineering, and mathematics," *Proceedings of the National Academy of Sciences*, Volume **111** Issue 23 (2014), pages 8410-8415.

[15] Marilyn Frankenstein, "Incorporating Race, Gender, and Class Issues into a Critical Mathematical Literacy Curriculum," *The Journal of Negro Education*, Volume **59** Issue 3 (1990), pages 336-347.

[16] Eric Gutstein and Bob Peterson, *Rethinking Mathematics: Teaching Social Justice by the Numbers*, Rethinking Schools, 2005.

[17] Eric Gutstein, "Reinventing Freire: Mathematics education for social transformation," Proceedings of the Fifth International Mathematics Education and Society Conference, Albufeira Portugal, 2008.

[18] Katie A. Hendrickson, "Fracking: Drilling into Math and Social Justice," *Mathematics Teaching in the Middle School*, Volume **20** Issue 6 (2015), pages 366-371.

[19] IUPUI, 2004: National Survey of Student Engagement (NSSE), https://irds.iupui.edu/students/student-surveys/nsse/2004.html, 2004.

[20] David W. Johnson and Roger T. Johnson, "Conflict in the Classroom: Controversy And Learning," *Review of Educational Research*, Volume **49** Issue 1 (1979), pages 51-69.

[21] George Ghevarughese Joseph, "Foundations of Eurocentrism in mathematics," *Race & Class*, Volume **28** Issue 3 (1987), pages 13-28.

[22] Kara Jackson and Jonee Wilson, "Supporting African American Students' Learning of Mathematics: A Problem of Practice," *Urban Education*, Volume **47** Issue 2 (2012), pages 354-398.

[23] Lily S. Khadjavi, "Driving While Black in the City of Angels," *CHANCE*, Volume **19** Issue 2 (2006), 43-46.

[24] Gizem Karaali and Lily S. Khadjavi, eds., *Mathematics for Social Justice: Resources for the College Classroom*, American Mathematical Society, Providence Rhode Island, 2019.

[25] Gizem Karaali and Lily S. Khadjavi, eds., *Mathematics for Social Justice: Focusing on Quantitative Reasoning and Statistics*, American Mathematical Society, Providence Rhode Island, 2021.

[26] David Lippman, *Math in Society*, 2.6 ed., 2022.

[27] Katherine Mackey, Chelsea K. Ayers, Karli K. Kondo, Somnath Saha, Shailesh M. Advani, Sarah Young, Hunter Spencer, Max Rusek, Johanna Anderson, Stephanie Veazie, Mia Smith, and Devan Kansagara, "Racial and Ethnic Disparities in COVID-19 — Related Infections, Hospi talizations, and Deaths," *Annals of Internal Medicine*, Volume **174** Issue 3 (2021), pages 362-373.

[28] Robert Moses and Charles E. Cobb, *Radical Equations: Civil Rights from Mississippi to the Algebra Project*, Beacon Press, 2002.

[29] McTighe & Associates, Resources, 1998.

[30] Andrea Mock and Ismar Volic, "Political Structures and the Topology of Simplicial Complexes," *Mathematical Social Sciences*, Volume **114** November 2021, pages 39-57.

[31] Cathy O'Neil, *Weapons of Math Destruction: How Big Data Increases Inequality and Threatens Democracy*, Crown, New York, 2016.

[32] Jonathan Osler, A Guide for Integrating Issues of Social and Economic Justice into Mathematics Curriculum.

[33] Radical Math, https://www.radicalmath.org, 2022.

[34] Richard M. Ryan and Edward L. Deci, "Intrinsic and Extrinsic Motivations: Classic Definitions and New Directions," *Contemporary Educational Psychology*, Volume **25** Issue 1 (2000), pages 54-67.

[35] Kerry Ann Rockquemore, Harwell Schaffer, "Toward a Theory of Engagement: A Cognitive Mapping of Service-Learning Experiences," *Michigan Journal of Community Service Learning*, Volume **7** Issue 1 (2000), pages 14-24.

[36] Celia Rousseau and William F. Tate, "No Time like the Present: Reflecting on Equity in School Mathematics," *Theory Into Practice*, Volume **42** Issue 3 (2003), pages 210-216.

[37] Adriana Salerno, Francis Su, Dylan Thurston, and The Task Force on Understanding and Documenting the Historical Role of the AMS in Racial Discrimination, "Towards a Fully Inclusive Mathematics Profession," report, American Mathematical Society. Available at https://www.ams.org/about-us/Towards-a-Fully-Inclusive-Mathematics-Profession.pdf, last accessed on July 31, 2023.

[38] Ezra Stiles, *The Literary Diary of Ezra Stiles*, ed. Under the Authority of the Corporation of Yale University, Arkose Press, 2015.

[39] Jesse Stommel, "Ungrading: An Introduction" (June 2021); available at https://www.jessestommel.com/ungrading-anintroduction/.

[40] Robert Talbert and David Clark, "Archive - Grading for Growth" (2022); available at https://gradingforgrowth.com/archive?sort=new.

[41] Elli J. Theobald, Mariah J. Hill, Elisa Tran, Sweta Agrawal, E. Nicole Arroyo, Shawn Behling, Nyasha Chambwe, Dianne Laboy Cintrón, Jacob D. Cooper, Gideon Dunster, Jared A. Grum mer, Kelly Hennessey, Jennifer Hsiao, Nicole Iranon, Leonard Jones, Hannah Jordt, Marlowe Keller, Melissa E. Lacey, Caitlin E. Littlefield, Alexander Lowe, Shannon Newman, Vera Okolo, Savannah Olroyd, Brandon R. Peecook, Sarah B. Pickett, David L. Slager, Itzue W. Caviedes Solis, Kathryn E. Stanchak, Vasudha Sundaravardan, Camila Valdebenito, Claire R. Williams, Kaitlin Zinsli, and Scott Freeman, "Active learning narrows achievement gaps for underrepresented students in undergraduate science, technology, engineering, and math," *Proceedings of the National Academy of Sciences*, Volume **117** Issue 12 (2020), pages 6476-6483.

[42] Alana Unfried and Judith Canner, "Doing Social Justice: Turning Talk into Action in a Mathematics Service Learning Course," *PRIMUS*, Volume **29** Issues 3-4 (2019), pages 210-227.

[43] Dale Winter, "Infusing mathematics with culture: Teaching technical subjects for social justice," *New Directions for Teaching and Learning*, Volume **2007** Issue 111, pages 97-106.