Journal of Humanistic Mathematics

Volume 14 | Issue 1 January 2024

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Recommended Citation

Will Tidwell & Amy Bennett, "Undergraduate Mathematics Students Question and Critique Society Through Mathematical Modeling," *Journal of Humanistic Mathematics*, Volume 14 Issue 1 (January 2024), pages 168-195. DOI: 10.5642/jhummath.SEJE4965. Available at: https://scholarship.claremont.edu/jhm/vol14/iss1/9

JHM is an open access bi-annual journal sponsored by the Claremont Center for the Mathematical Sciences and published by the Claremont Colleges Library | ISSN 2159-8118 | http://scholarship.claremont.edu/jhm/

Undergraduate Mathematics Students Question and Critique Society Through Mathematical Modeling

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Synopsis

Mathematics can be used as a tool to question and critique society and, in doing so, give us more information about the world around us and how it operates. This however, is not a common perspective that is conveyed to students during their undergraduate mathematics coursework. This paper contributes to the understanding of how undergraduate mathematics students question and critique society via mathematical modeling tasks. In two courses at two universities, 27 mathematics majors and secondary preservice teachers engaged in the modeling process situated in authentic contexts to learn specific concepts and make mathematical connections across domains and disciplines. Both courses culminated in a final project in which students created and investigated solutions to their own modeling tasks. In this paper, we describe how our courses (1) centered justice as pedagogy, (2) were environments for student agency and exploration, and (3) explicitly demonstrated how mathematics and social justice are intertwined. Drawing on frameworks of mathematical modeling for social justice, we present and analyze student-created tasks to showcase how they utilized mathematics as a tool to question and critique the world around them.

1. Introduction

The GAIMME Report [16] on mathematical modeling claims that the purpose of mathematics curricula is to teach students how to become informed citizens and critique the world outside of the classroom. Modeling as a content standard and mathematical practice is heralded as a crucial connection between mathematical understanding and authentic situations, problems, and phenomena. Students are expected to learn about the mathematical modeling process and teachers are expected to teach this process [e.g., 13]; thus, modeling as a pedagogical tool is a critical area for exploration.

Beyond understanding or “reading” the world through mathematics, students are also called to “write” the world with mathematics, that is, to develop a sense of agency and sociopolitical consciousness toward addressing inequity and unjust situations [17]. In this way, teaching mathematics for social justice can be integrated with a modeling curriculum by posing questions that explore and critique the world. Furthermore, Oldfield [31] called for modeling to be introduced into mathematics curricula so that students are capable of posing and solving real-world problems ethically with mathematics and evaluating their solutions through a lens of social responsibility. To this end, we implemented mathematics tasks that allowed students to examine society through mathematics in our undergraduate classrooms and centered our classrooms with a pedagogy of justice. Students posed and solved authentic modeling tasks, and we analyzed these tasks to see if our students took this opportunity to use modeling as a tool to explore, question, and critique society through mathematics.

2. Literature Review

Our goal in this section is to provide background around mathematical modeling and situate our study in existing literature about modeling as a tool to question and critique society through mathematics. In this section, we will explore studies that illustrate how mathematical modeling can include issues of social justice and have the aim of developing a socio-critical consciousness in students. We will explore frameworks for developing mathematical modeling tasks with students and in teacher education settings, particularly tasks that are authentic and social justice-oriented.

Mathematical Modeling

Mathematical modeling is the "process in which students consider and make sense of an everyday situation that will be analyzed using mathematics for the purpose of understanding, explaining, or predicting something" [4, p. 202]. Modeling is an iterative process with several steps that relate to mathematical competencies, including making sense of a problem and creating a simplified situation, formulating and solving a model, and interpreting and validating conclusions. Mathematical modeling has been integrated into the mathematics curriculum worldwide [38]; yet the GAIMME report claims there should be increased efforts to incorporate mathematical modeling at all levels (K-16) to provide students with authentic experiences to use mathematics [16, 41].

Research has shown that both mathematical modeling and problem-posing support problem-solving abilities and creative thinking [10, 30, 33]. The modeling process challenges students to ask questions and write simplified versions of complex situations that they can address, while problem-posing prompts students to formulate tasks that they can solve through a problem-solving process. Problem-posing is a valuable exercise for both mathematics learners and mathematics teachers [e.g., 9] and is a crucial component of the beginning phases of the modeling process. Additionally, engaging students in the mathematical modeling process allows for the back-and-forth of contextualization and decontextualization between mathematics and the context that it is based in, which is a fundamental practice designated as a standard of mathematical practice in the CCSSM [13]. Thus, it is imperative that all students, but particularly those in teacher education programs, have experience posing and solving modeling problems.

Teacher Education in Mathematical Modeling

Creating rich mathematics lessons that are rooted in authentic situations is challenging ([e.g., 2, 44], especially when the lesson contexts are meant to align with students' lived experiences and knowledge [11]. Many examples in existing literature are from elementary settings, such as elementary teachers generating modeling lessons based on authentic contexts in their schools and communities [44], elementary preservice teachers (PSTs) creating problem-solving lessons that connect to students' funds of knowledge [2], and elementary PSTs creating modeling problems for young children [32]. Within a unit on social justice-oriented mathematical modeling, Jung and Magiera [24] prompted K-8 PSTs to create modeling lessons and analyzed how the lessons attended to macro- or micro-level social justice issues through the posed mathematical problems. Additionally, Hansen and Hana [18] presented five possible difficulties that teachers may face when posing mathematical problems within the modeling process, based on their work with student teachers in eighth-grade classrooms. Across the research on developing mathematical modeling lessons or problem-solving activities, the findings were similar: it is challenging to balance both attention to students' mathematical thinking and connection to their funds of knowledge.

A mathematical modeling lesson may be designed to engage students and draw on their experiences, but it may also act as a vehicle to critique the world. When teachers create mathematical tasks that are rooted in issues of equity or systems and policies in society, they engage in "modelling as critic" [18, p. 42]. Though the modeling process and the mathematics concepts in the task are still important, the goal of "modelling as critic" is to identify and understand authentic situations with a critical eye toward evaluating and improving them.

To reiterate, much of the work on posing mathematical modeling problems is situated in elementary-level contexts. Although research has studied how secondary PSTs and undergraduate mathematics majors solve modeling tasks [e.g., 4, 43] and reflect on this experience as future educators [3], few studies have explored how undergraduate mathematics majors and secondary PSTs develop their own mathematical modeling lessons, and particularly how they attend to issues of social justice (or not) in their lessons.

Social Justice in Mathematical Modeling

There are various perspectives of mathematical modeling, and incorporating authentic or socio-critical aspects into lessons is the objective of only some of these perspectives [25]. For instance, the Flint Water Crisis task that investigates a socially and environmentally unjust situation [1] refers to the socio-critical perspective of mathematical modeling, where the aim is to provide "a critical understanding of the surrounding world" [25, p. 304]. When the context of a mathematical modeling task is both realistic and relevant, the person engaging in the process has the opportunity to investigate social inequities using mathematics, allowing individuals to critique society and the world around them ([1, 34]. The socio-critical perspective of mathematical modeling directly aligns with the *Standard for Preparing Teachers of Mathematics*'s goals to show students that mathematics can be a tool to "examine social and personal issues that arise throughout their lives" [6, p. 107].

There are different ways to attend to social justice issues in modeling lessons. Socio-critical modeling allows for the interplay between the mathematical modeling process and engaging with social inequities to build social consciousness and promote activism [1]. Using mathematics to build social consciousness and promote activism is mathematics for social justice [37]. Stinson and Wager delineates between mathematics for, about, and with social justice, claiming that mathematics about social justice is one where the context is based in a social or environmental inequity but does not achieve the goal of meaningfully discussing the context itself. Stinson and Wager describe that mathematics with social justice is more of a pedagogical act; it is the teaching of mathematics with equitable teaching practices, anti-racist teaching practices, or culturally responsive teaching. Following Gutstein's [17] call to action, several researchers have created curricula and resources to teach mathematics for social justice in undergraduate mathematics classrooms [e.g., 5, 7, 26].

Despite the potential benefits and perceived value, literature supports that curriculum within teacher education programs, textbooks, and online resources do not provide an adequate amount of socio-critical modeling tasks. He *et al.* [20] investigated five mathematics teacher education programs to analyze what modeling perspectives the programs focused on. They found that none of the mathematical modeling tasks addressed the socio-critical modeling perspective in the teacher education programs. Stillman*et al.* [36] and I *et al.* [22] found that very few of the mathematics tasks in popular mathematics textbooks were actual mathematical modeling tasks, and only a small portion of the prompts were socio-critical mathematical modeling. Stohlmann and Yang [40] found when looking at online resources (viz. Teachers Pay Teachers) that only about 10% of tasks labeled as mathematical modeling are in fact mathematical modeling. Given the apparent dearth of socio-critical modeling tasks in mathematics curriculum and confusion about what constitutes a modeling task, it is crucial to better understand how (future) teachers interpret and develop modeling tasks for questioning and critiquing society.

Developing Mathematical Modeling Tasks

When developing modeling tasks, Moore and Diefes-Dux [28] describe the process as being iterative, similar to the modeling process itself. To better inform our construction of modeling, we looked at two design structures for mathematical modeling problems. Galbraith [15] provides five principles for mathematical modeling problem design (PD):

1. There is some genuine link(s) with the real world of the students.
2. There is opportunity to identify and specify mathematically tractable questions from a general problem statement.
3. Formulation of a solution process is feasible, involving the use of mathematics accessible to students in making of necessary assumptions, and the assembly of necessary data.
4. Solution of the mathematics for the basic problem is possible, together with interpretation.
5. An evaluation procedure is available that enables checking for the mathematical accuracy, and for the appropriateness of the solution with respect to the contextual setting. (p. 55)

When comparing Galbraith's [15] design principles to I *et al.* [21]'s criteria for mathematical modeling problems, there are significant overlaps between these two frameworks. I et al's criteria for mathematical modeling problems consist of *Realistic Context, Problem, and Solution* (which align with Galbraith's PD 1, 2, and 5), *Multiple Representations* (PD 4), *Generalizable/Transferable Knowledge* (PD 5), *Shareable Approach* (no alignment with PDs), *Focus on Mathematics* (PD 2), and *Unpredictable Methods* (PD 3). Both frameworks discuss the importance of a feasible mathematical solution process that allows students to have multiple methods for solving — allowing for students to create their own method for solving the problem — and for students to be able to represent or arrive at a solution by use of different or multiple representations.

I *et al.*'s [21] framework does include one criterion that does not align with Galbraith's [15] design principles: a Shareable Approach. This criterion aligns well with a mathematical modeling competency, reporting out, which is one of seven modeling competencies within the modeling process [4, 42]. I *et al.* [21] provide a criterion for differentiating mathematical modeling problems that address a social justice issue from other tasks: "the context involves unjust situations of the real world and encourages learners to be an agent of change by identifying mathematical conflicts and resolving conflicts" (page 891). This criterion is the key difference for a *Social Justice Context*.

Notably, these frameworks discuss the importance of a real-world context that authentically connects to the lives of the students. Historically in mathematical modeling, a realistic context is a necessary criterion for aligning a mathematics task with the modeling process [15, 21, 39]. Sevinc and Lesh [35] posits that realness and what is realistic may be perceived differently and that realism exists on a spectrum. They found that two different perspectives existed for the relative nature of realness:

1. Children live in a world where they speak with their toys, so a context that is not realistic for adults may be realistic for them.
2. Realistic problems should include real people, real objects, and real actions because the world we live in includes those (page 680).

We suggest that the same may be true for undergraduate students and what they view as realistic. Mathematical modeling has an important role in reflecting the mathematics of the world to our students; however, we believe that discussing mathematical modeling exclusively in this way limits its potential impact. Mathematical modeling can be a "window" to show real-world issues that students may not be familiar with to gain a broader, better understanding of the world. Additionally, limiting modeling situations to exclusively real-world opportunities limits fun, engaging modeling opportunities such as modeling a zombie outbreak, investigating how fast Sasquatch can run, or analyzing what the tides would look like for a planet with two moons. We posit that the context does not need to be real-world AND relevant to those engaging in this process, but that this context needs to be real-world OR relevant to those engaging in the modeling process. To this end, this paper addresses the following two research questions:

1. What issues do undergraduate mathematics students select when given the opportunity to investigate a topic of their choice through modeling?
2. How do undergraduate mathematics students create mathematical modeling lessons about social justice issues?

3. Methods

Setting and Participants

This study took place at two universities. The first university is a large, public, research-oriented, predominantly white institution (PWI) in the United States. At this university, the participants included 10 secondary PSTs in one section of a capstone mathematics course focused on mathematical modeling for secondary teacher preparation. These students had no introduction to mathematical modeling prior to this course.

The second university is a medium-sized, teaching-focused, PWI located in a rural region of the United States. This university had 17 students participate in the study during a half-semester course on mathematical modeling. All students enrolled in the course were mathematics majors, with a small proportion of them double majoring in secondary mathematics education. The students in this class largely had no experience with mathematical modeling, with the exception of two or three students who experienced modeling in their Ordinary Differential Equations course. The students participating from this university had varying mathematical backgrounds, as the prerequisite for the course was a first course in calculus. Most students in the class had not had exposure to differential equations or linear algebra; however, some students had already taken partial differential equations and mathematical statistics.

The instructors of these two courses are also the authors of this paper. Both instructors piloted modeling curricula and engaged students with tasks from the MODULE(S2) Project. The instructors met monthly to discuss the progress of their courses, upcoming tasks, and triumphs and struggles related to previous tasks implemented.

The students in both courses were introduced to the modeling process and solved modeling tasks in small groups throughout the semester. Weekly tasks focused on realistic topics that have potential applications of specific mathematical concepts. Additionally, the tasks varied in terms of their potential math concepts (e.g., analyzing geometric properties, graphing data and finding trends, generalizing patterns), task topics (e.g., addressing a lack of resources, improving on or resolving a social injustice, predicting future data), and task structures (e.g., map-based, choosing the best option from a given list, using a news or research article with authentic data event).

Both courses included a final project which prompted students to create their own modeling tasks. At the first university, the instructor asked students to base these modeling tasks on the weekly tasks that they had experienced together. To support students in this process, this instructor created scaffolds and guidelines to help students focus on important aspects of modeling and problem-posing. This instructor directed students to analyze three components of the tasks they solved during the semester (i.e., the math concepts, task topics, and task structures) and make connections to their generated modeling tasks. In this way, the instructor assessed students' understanding of the modeling process and specific mathematics concepts through the modeling tasks that they created.

The instructor at the second university asked students to pose a question that interested them and find an appropriate solution using mathematical modeling. This instructor asked the students in the class to be creative and find problems that truly interested them, even if they were not traditionally classified as "modeling problems" due to their non-realistic context [cf. 21]. At the beginning of the final project, this instructor consulted with students to make sure that problems were viable and appropriate for the class. The students were given parameters about problems that were not allowed, which included problems where modeling solutions had been published or discussed widely on online forums (e.g., r/theydidthemath or r/theydidthemonstermath on Reddit). This instructor assessed students' understanding of and proficiency in engaging in the modeling process through their projects and presentations.

Vignette of Classroom Culture and Norms for Interaction

The following vignette describes the classroom culture in which students engaged in modeling and demonstrates how our two courses (1) centered justice as pedagogy, (2) were environments for student agency and exploration, and (3) explicitly demonstrated how mathematics and social justice are intertwined.

Centered Justice as Pedagogy

We view mathematical modeling as a collaborative activity and thus established norms with our class to create a collaborative environment. Students often worked on a task in small groups during class and presented their solutions with the whole class. Due to the highly interactive and participatory structure of our class, we were transparent about our efforts toward equitable teaching practices so that all students felt supported and comfortable speaking with peers. We anchored our equity-centered pedagogy with research on teaching mathematics in collaborative settings, such as elements of Complex Instruction [12], a focus on revisions and a growth mindset for learning mathematics through "Rough Draft Math" [23], and the NCTM's publication on asset-based ways of reflecting, noticing, and engaging in mathematics education [29]. Our ultimate goal was to empower students to critique their world through mathematics, which often meant addressing implicit biases and working to develop a consciousness of injustices. To accomplish this goal, we first ensured that students felt comfortable in our learning environments.

Demonstrated How Mathematics and Social Justice are Intertwined
Throughout the course, students engaged in mathematics for social justice lessons, such as the Food Desert task [14], the Flint Water Crisis task [1], and the Shrinking Area of the Sioux Reservation task [5]. In all of these tasks, students justified their position on a social justice issue through their mathematical models. For instance, in the Food Desert task, students argued for the best location to place a new grocery store based on measuring distances and formulating equations of circles.

Students participated in open conversations during class and through asynchronous assignments about how social justice issues arose within tasks and how the PSTs might implement this task (or a similar one) in their own classroom. These conversations were often personal and students shared parts of their identities to convey their opinion about the task context. Several remarked in reflection assignments that were more comfortable having discussions about unjust situations when they could support their position with mathematics.

Supported Student Agency and Exploration
Students worked together to develop a model and find a solution for each task. After solution presentations, the instructors provided student work examples, where the undergraduate students would analyze other strategies and solutions. This was particularly salient for the PSTs, who could use the example solutions to discuss the diverse ways that their future students will approach modeling tasks. These conversations often became debates where PSTs (respectfully) argued and advocated for their favorite solution strategy. Initially, the undergraduate students would look toward the instructor

for an indication of who was correct and which student work sample earned the highest marks. However, once it became clear that multiple solutions can achieve high marks when evaluated through a modeling rubric, the students became more confident and agentive in their mathematical arguments. Students came to realize that modeling tasks are different from traditional textbook word problems, where there is typically one correct solution and one solution path that is most efficient. In this supportive environment, students felt comfortable posing creative, complex, and off-the-wall mathematical questions to explore.

Data Collection and Analysis

In this paper, we focus on the final project submissions from the 27 undergraduate mathematics students. These projects are student-created modeling lessons, where they solved their created modeling task and reflected on and presented their work. We used an adjusted version of Galbraith's [15] modeling design principles to verify that the final projects met the criteria for modeling lessons. All 21 final projects were classified as mathematical modeling tasks, that is, mathematics problems that align with the modeling process.

We then utilized I *et al.*'s [21] framework to assess if the mathematical modeling tasks contained elements of social justice mathematical modeling problems. This framework claims that social justice problems are a subset of modeling problems and provides one criterion that distinguishes *social justice mathematical modeling problems*: "The context involves unjust situations of the real world and encourages learners to be an agent of change by identifying mathematical conflicts and resolving the conflicts (page 891)". We initially analyzed the projects individually through artifact analysis [19] to document key characteristics of the created tasks. Then, we met to compare our analyses and sorted tasks into tables, based on the I *et al.* [21] framework. Later, we drew on a framework by Jung and Magiera [24] for posing social justice-oriented modeling problems to describe the social justice components at macro- and micro-levels that are connected to students' lives. We provide two examples in the discussion section. We found that both frameworks [21, 24] provided insights for how students critique the world by posing mathematical modeling problems.

It is noteworthy that our analytic frameworks do not explicitly situate their work as mathematics *with*, *about*, or *for* social justice. I *et al.* [21] initially defined their criteria as describing mathematical modeling *with* social justice; however, their reference to students being "agents of change" aligns more closely with mathematics *for* social justice. Indeed, they clarify that "social justice problems are not defined as the tasks that just involve social justice contexts" (page 890). In other words, their criterion is stronger than mathematics *with* or *about* social justice. Here, we differ from the I *et al.* [21] framework in our analysis, as we did not directly prompt students to create social justice-oriented modeling tasks. We gave students the agency to explore any topic, which included encouraging them to "explore unjust situations of the real world" but did not assess their work based on their ability to be "an agent of change" and resolve the explored conflict. We classified tasks as attending to social justice issues if we interpreted the context as "involving an unjust situation of the real world" or if students explicitly made connections from their task to social justice issues. Two examples of this classification are presented in the next section.

4. Discussion of Findings

Altogether, the students in our classes created 21 modeling tasks. As we described, our classrooms were environments for student agency and exploration which we made transparent through our justice centered pedagogy. In the first section, we present Table 1, which describes 12 student-created modeling tasks that we determined did not directly address a social justice issue. In the next section, we present Table 2, which describes nine student-generated tasks that addressed a social justice issue. To determine which tasks belonged in each table, we used the I *et al.* [21] criteria for differentiating modeling tasks versus social justice mathematical modeling task. In the final section, we focus on two selected tasks from Table 2 and discuss them through the lens of our analytic frameworks.

Mathematical Modeling Tasks without Social Justice Applications

The modeling tasks in Table 1 did not identify a mathematical conflict or unjust situation of the real world, nor did they attend to macro- or micro-issues of social justice. Most, 8 of 12 (Tasks 1-6, 8, 11), of the tasks in Table 1 were based on realistic contexts or events, and some, 4 of 12 (Tasks 7, 9, 10, 12), were hypothetical or imaginative settings or situations. However, all 12 of these tasks were aligned with the modeling process and allowed students to engage in rich mathematics concepts while answering a question about "their experienced world". Students explored topics that interested them and posed mathematical problems that they genuinely wanted to solve.

For instance, the "How many licks does it take?" task is based on a classic question from a popular candy: How many licks does it take to get to the Tootsie Roll center of a Tootsie Pop? In this project, students created a visualization of an ideal Tootsie Pop (two concentric spheres) by dissecting and measuring multiple Tootsie Pops and found the minimum volume that need to licked to reach the center of the Tootsie Pop (see Figure 1).

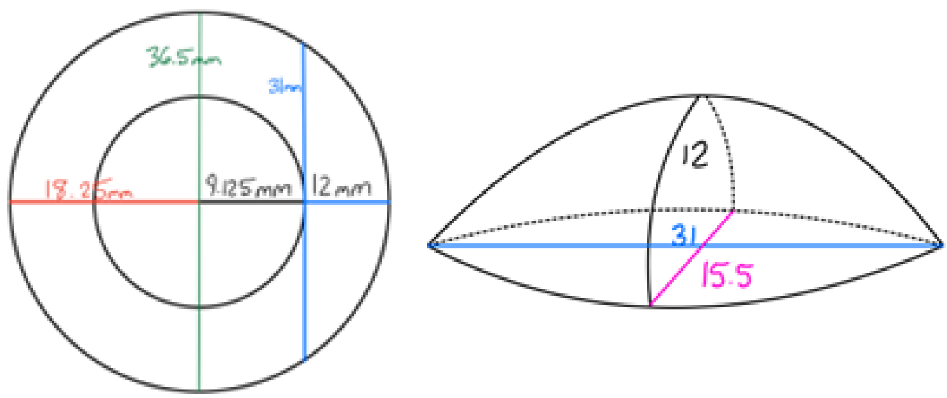

Figure 1: The 2-dimensional visualization of the Tootsie Pop (left); the proposed minimum solid needed to be licked to get to the center of the Tootsie Pop (right).

The students then created a chemical solution similar to that of saliva and experimented with how long it would take to dissolve all of the hard candy portion of the Tootsie Pop. They used the result to create a proportion for how much volume is eroded in one second, which they equated to one lick. After some unit conversions based on the information they collected, the students were able to find how many licks it would take to erode the minimum amount of volume needed to reach the center of a Tootsie Pop. After experimentation, the students found that their estimation was higher than the average amount of licks taken and determined that it was a limitation due to their assumption regarding the amount of volume eroded by one lick. They assumed that "tongue pressure" was negligible in their model and believed that in experimentation that was not the case; therefore, this created the inconsistency between the model and the real phenomena. Thus, this was a rich mathematical exploration of an authentic topic that interested students, but it did not address a social inequity or unjust situation.

Table 1 shows the various math concepts that students used to solve their posed modeling tasks. One of the most common content areas was related to identifying variables and creating equations. Students in both courses experienced modeling tasks where a key step of developing a model and finding a solution was to first identify the relevant variables and research unknown information. Once the relevant factors were identified, these became parameters in a generalized equation, which could be replaced with specific values from the assumptions made about the situation. Based on the twelve generated modeling tasks in Table 1, students followed this modeling task structure as a way to explore their posed problem.

Modeling Task Title	**Driving Question**	**Math Concepts in Model**
1. California Dreamin'	Is it better to drive, fly, or take the train from Nebraska to California?	Rates, Variables and Exploring Equations
2. Most Efficient Route	What is the most efficient route to take when running errands?	Graph Theory
3. Baseball Pitcher Lineup	Given a list of possible pitchers, who should the Yankees choose as their pitching lineup?	Statistics
4. Planning a Boxer's Diet	Design a diet for a boxer to accommodate his goal weight and workout schedule.	Rates, Variables and Equations
5. How far would a car travel?	How far would a car travel after a driver perceives a need to stop?	Variables and Exploring Equations
6. Starting a New Tradition	How much money does an organization need to collect each year to purchase and deliver gifts to every child between 4-11?	Variables and Exploring Equations
7. Tardigrades in Space	How long would a nickel plate with a colony of tardigrades survive in space?	Variables and Exploring Equations
8. How many licks does it take?	How many licks does it take to get to the Tootsie Roll center of a tootsie pop?	3-Dimensional Geometry, Descriptive Statistics
9. How much to put a Dollar General on the moon?	How much would it cost to get a "symbolic" (non-functioning) Dollar General on the moon?	Variables and Exploring Equations
10. Falcon Punch vs. Human	At what force would a peregrine falcon strike be fatal to a human?	Variables and Exploring Equations
11. Baking potatoes with potatoes	How many potato batteries would it take to bake a potato in a standard oven?	Variables and Exploring Equations
12. Too Many Tuna Fish?	How many Atlantic Bluefin Tuna Fish would it take to raise the sea level so that the Appalachian Mountains would become an archipelago?	The volume of ellipsoids and portions of the Earth's troposphere.

Table 1: Twelve student-generated modeling tasks that did not attend to social justice issues

Mathematical Modeling Tasks with Direct Social Justice Applications

We interpreted nine of the 21 modeling tasks as attending to social justice issues. Here, we briefly describe two examples from Table 2 that demonstrate our analysis of tasks that attend to social justice.

Modeling Task Title	**Driving Question**	**Math Concepts in Model**	**Social Justice Application**
13. School District Expansion	Where should a local school district build a new high school?	Distance, area, equations of circles, density	Analyze population density and driving distances in a city to distribute students fairly between schools and alleviate demand on resources at those schools
14. Buy or Rent?	Is it financially smarter for a recent graduate to rent or buy a house over a 5, 10, and 20 year period in our city?	Finding line of best fit for a graph of data points	Explore the best financial option for a recent college graduate analyzing the housing market, including loans and debt
15. Energy Production	In what year will the combined production of renewable resources and nuclear energy surpass that of fossil fuels?	Rates, Variables and Exploring Equations	Explore energy production between renewable and nuclear energy resources and fossil fuels with implications for impact on the environment
16. Location for a New Trader Joe's	Where should a new Trader Joe's store be built in our city?	Distance, equations of circles	Analyze the areas of a city that are "food deserts" in terms of being near a health or specialty food grocery store
17. Cattle Stock Tank	What is the optimal size of cylindrical stock tank for a farmer to provide water for his cattle?	Volume and surface area of a cylinder, optimization	Focus on agricultural settings and helping farmers to make efficient use of water
18. Caterpillar v. Caterpillar	Is there a correlation between Caterpillar equipment sales and the number of monarch butterfly sightings?	Data Correlation	Examine deforestation and its impact on the wildlife population
19. Great Pacific Garbage Patch	How large is the Great Pacific Garbage Patch?	Analytical Geometry, Area of Closed Figures	Analyze the pollution of the Great Pacific Garbage Patch and discuss how to quantify its size
20. School Funding vs. Student Success	Does school funding affect student educational success?	Data Correlation	Investigate if rural vs. urban school funding in their state correlates with the percentage of college and career ready students
21. Planting Shady Trees	How many trees would it take to shade the entire contiguous United States not including waterways and what would be its effects?	Tessellations of hexagons	Investigate the effects of planting more trees on the offsetting of carbon emissions and greenhouse gasses

Table 2: Nine student-generated modeling tasks that attended to social justice issues

The "Location for a New Trader Joe's" task was classified as modeling about social justice (see Table 2). Although determining a new location for a specialty grocery store may not seem like an unjust situation, the PST who created this task drew on her knowledge of food deserts and experience with the Food Desert task [14] to analyze regions of her city where health foods stores are located. This PST was motivated by an online article she read that claimed Trader Joe's stores are almost exclusively in neighborhoods with above-average median household incomes and with small percentages of households on SNAP [8]. Her development of the task showcased mathematical modeling *about* social justice, and her reflection on the importance of the task demonstrated that she modeled this situation *with* social justice. Thus, even though her solution did not describe how to be an agent of change for a macro social justice issue like food insecurity, we classified this task as social justice oriented.

The "Energy Production" task analyzed the production percentages of different types of energy sources and tried to determine when the production of renewable resources and nuclear energy will surpass that of fossil fuels. The PST who created this task expressed that he was motivated by the varying electricity costs in households and how different types of energy sources may affect those costs. In this sense, the task attended to the macro social justice issue of decreasing reliance on fossil fuels and addressing climate change and the micro issue of determining lower and equitable electricity costs across households. However, throughout the course, this student expressed a hesitancy toward emphasizing social justice in mathematics lessons and wrote "I'm not getting in this debate on climate change" when reflecting on his created task. We interpreted the Energy Production task as having clear implications for environmental social justice, despite the student not intending to extend their explorations to a call to action for environmental justice. So, although this student would perhaps choose to not classify their task as attending to social justice, we classified it so.

A Comprehensive Description of Two Examples of Social Justice-Oriented Modeling Tasks

Here, we focus on two of the modeling tasks that addressed a social justice issue in Table 2. For each task, we describe the driving question and motivation for this exploration, the developed mathematical model, and the ways this task addressed a social justice issue. For each of the tasks, we examined whether the social justice issue addressed was micro-level or macro-level. The attention to different levels of issues were distributed roughly equally across the tasks, with six tasks addressing micro-level (Tasks 13-17, 20), seven addressing macro-level (Tasks 13-15, 18-21), and four of the tasks addressing both micro- and macro-level social justice issues (Tasks 13-15, 20).

Example 1: *School District Expansion*

Problem posing and motivation.. Two secondary PSTs worked together to develop this task. They were inspired by their experiences as students in a large school district and their experience with map-based modeling tasks from our course. In particular, the PSTs stated that they derived their task structure from the Food Desert task [14], in which a new grocery store must be placed in a region with limited access to fresh food.

The School District Expansion task prompt stated: "[Our hometown school district] is looking to build a new high school to keep up with increasing student enrollment figures. The district is currently deciding on where to place the new high school at. Develop a model that will display the location of the new school along with the updated boundaries for each high school."

This prompt was not contrived but represented an authentic situation in their hometown. As former students of this district and future teachers, the PSTs were invested in understanding how a new location for a high school was determined. Thus, they posed a mathematical problem that allowed them to explore relevant topic and connect it to the mathematics in the course.

Connection to the mathematical modeling process.. Aligning with the Jung and Magiera [24] framework, this pair of PSTs chose a realistic context that was meaningful to them, developed a mathematical model to find a solution to the posed question, and generalized this solution into a shareable process for similar situations. They created three mathematical models using different strategies and mathematics concepts; the first relied on calculating areas of each school's boundary and dividing up the largest region (this model was deemed unsatisfactory). The second solution also focused on the geometry of the district and used circles centered at each existing school in the district, similar to their solution for a new grocery store in the Food Desert task [14]. In the third model, the PSTs calculated each school enrollment and graphed the population of the school region over time. They placed the new high school in the most densely populated area, which decreased the area of the two most populated schools (See Figure 2). In sum, the models for the location of a new school were interpreted, validated, and revised based on which seemed most realistic and addressed the goal of their exploration (i.e., alleviating the stress on resources and space).

Figure 2: The original school boundaries within the district (left) and the proposed boundaries for the new high school in the most densely populated area (right).

Although the PSTs drew on their personal experiences and knowledge of the school district, their models were generalized so that others could use this process in similar situations. Since they were genuinely interested in the final decision made by the school district, they continued exploring this topic after the final project assignment and found that the school district voted to build two new high schools in almost the exact locations that they selected through their second and third models. In this way, they experienced the validation step of the modeling process in an authentic way that emphasized the importance of using mathematical models to address real problems. By engaging in the validation step of the modeling process, they were able to critique their developed mathematical models, the policies of their school district, and the eventual new school locations determined by the district.

Attention to social justice.. These two PSTs explicitly stated how the School District Expansion task attended to issues of social justice. They referenced systemic, macro issues such as overcrowded schools having inadequate funding and resources for students, and explained how their mathematical solution can address this issue. One PST said: "The social implications of my solution is to help the district and Title I schools alleviate the stress from having an increasing student population and not enough space or resources to accommodate for the amount of students." Although this is a broad, systemic issue, these students connected it to a local setting and addressed the micro issue of placing a new high school in their hometown district.

The PSTs referred to other macro issues, like school segregation based on race and socioeconomic status. This connection was meaningful to one student since she experienced these issues on a personal (micro) level at her high school. She said,

> The social [justice] issues from where I attended schools are similar to the issues from this problem situation because I went to high school in a very populated building with a dire need of expansion. Another social [justice] issue would be coming from [my city], which is a very segregated city, there is a huge discrepancy between SES [socioeconomic status] and race. Mathematics can be used to solve the question of how the discrepancy affects housing prices, crime rates, or other social [justice] issues.

Thus, this task attended to both macro- and micro-level social justice issues.

EXAMPLE 2: *Great Pacific Garbage Patch*

Problem posing and motivation.. Two mathematics majors chose to investigate the Great Pacific Garbage Patch (GPGP); they primarily wanted to gain "some perspective about the size and aspects of the garbage". Despite there being some estimates regarding the size of the GPGP, they cited that the combination of the slow breakdown of microplastics, the compacting of plastics to make it more dense, and the rapid growth warranted their own estimate. The GPGP is one of the largest collections of garbage that floats in the oceans. In a study to determine the ocean plastic concentration inside and surrounding the GPGP, Lebreton*et al.* [27] surveyed the region covered by 25°N to 41°N and 129°W to 156°W — which is the northern part of the Pacific Ocean roughly in between China and the United States — giving a general region in which the mass resides. It primarily consists of plastic waste and fishing nets and is amassed due to gyres in the ocean currents. While neither of these students had seen the GPGP, the students felt drawn to it as a way to help environmentalists know more about this issue and to provide actionable solutions to this problem.

Connection to the mathematical modeling process.. To determine the size of the GPGP, the students utilized data about confirmed locations of this body. They plotted the data on a map and tried to find a circular region that encompassed these points (see Figure 3). They assumed that the GPGP would be elliptic in nature due to the natural gyre that aids it in formation. Their circular region was sized so that they had a large density of location markers inside the region; they were concerned that including markers that were outliers would hyperinflate the estimation. Their first estimate of the GPGP closely related to “1 Texas” — a unit of measure where they utilized a roughly known quantity, the area of the state of Texas, to better grasp its size. However, when comparing this model to other accepted models in the community, they felt that this severely undersized the GPGP. In a second model, the students decided to include some of the more outlying data points with an ellipse. They kept the elliptic shape due to the gyre, but decided to go to an ellipse instead of a circle due to the density of location markers. With this model, they did get a closer estimation — they expressed it as “more than 2 Texases” — to that provided in the literature [viz. 27]. In this way, they were able to validate their model.

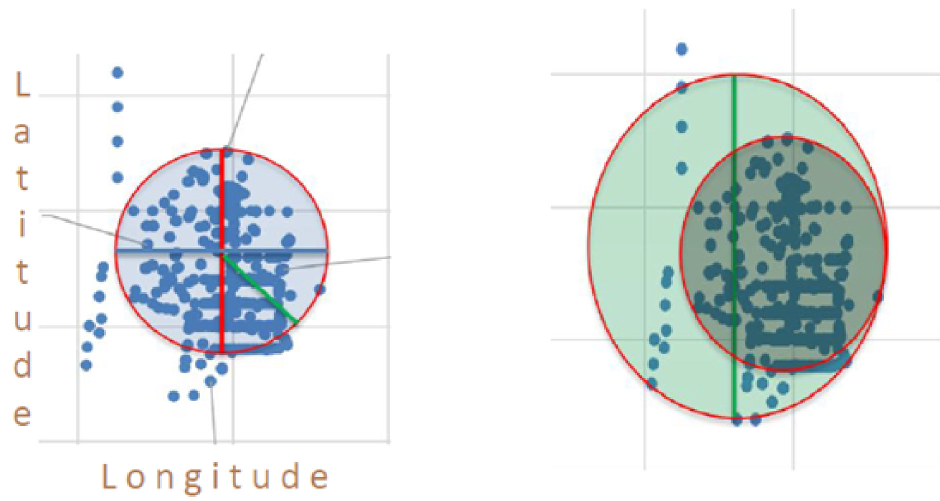

Figure 3: The first iteration of the estimation of the Great Pacific Garbage Patch (left) and the final iteration compared to the initial (right).

Attention to social justice.. The students in this task made connections to macro-issues. This problem brings attention to broader social justice issues: ocean pollution and plastic waste. Although students initially set out to provide actionable solutions to reduce the GPGP, they were overwhelmed by the size of the GPGP that they found through their model and were unsure how to address this problem. These students did not make connections to micro-issues with this task like plastic waste in their community or household.

Conclusions

We found that students posed mathematical problems about both the "real-world" and their perceived reality, which includes science (fiction) theories and pop culture references. Within our agentive classrooms, we encouraged authentic exploration and found that this led to rich mathematics. We believe that some tasks that are not realistic (e.g., Dollar General on the Moon task) can provide a quality mathematical modeling experience. These tasks still allow students the opportunity to engage in the modeling process and use mathematics to better understand a complex situation.

On the other hand, exploring non-realistic contexts does not allow for critiquing the world in which we currently live, nor does it address unjust situations and inequity. Perhaps exploring hypothetical and non-realistic contexts is an entry point for future explorations that attend to social justice issues. We emphasize the importance of providing students with diverse examples of modeling tasks that address a wide range of topics and attend to multiple levels of issues of social justice.

The findings of this study suggest that attending to micro-level issues was more accessible for these undergraduate students when posing mathematical problems; nevertheless, several students attended to both macro- and micro-level issues in their lessons. Our results showed that 4 out of 9 created modeling tasks addressed both macro- and micro-levels of social justice and that 3 out of 9 tasks focused solely on macro-level issues, which was more than the number of tasks that were solely focusing on micro-level issues. This conflicts with Jung and Magiera's [24] results, and we posit that this difference is due to the populations sampled and the tasks we solved during class. Together, our results suggest that different groups of PSTs may attend to different levels of social justice issues.

During our analysis, we realized there is a difference between mathematical tasks that had a primary goal to address social justice issues and tasks where connections to social justice issues was a secondary goal or even unintentional. In other words, there is a tension between how a task was intended and how it is perceived or interpreted. We found examples of tasks that were initially intended to address a social justice issue but were not able to make that connection (e.g., the Too Many Tuna Fish? Task initially intended to address overfishing and the effects of global warming but did not make any environmental conclusions) and tasks that did not set an objective to address a social justice issue but their conclusion and implications suggested otherwise (e.g., the Energy Production Task explored the environmental impact of renewable energy sources even though that was not the PST's stated initial intention). As instructors of the course and researchers analyzing the tasks and student reflections, we ultimately decided if the tasks were social justice oriented. However, we acknowledge that this tension between intended and actualized task goals could (and should) be more clearly discussed with students, particularly with future teachers of mathematics.

In conclusion, we encourage instructors of undergraduate mathematics courses to incorporate opportunities for students to critique the world through mathematical modeling activities. We found that even when a mathematics course did not include a formal unit on mathematics for social justice, students developed a sense of agency that led to exploring and critiquing unjust situations in society via mathematical modeling.

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