Millions, Billions, or Trillions: How to Partition Large Numbers into Friendly Figures

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Millions, Billions, or Trillions:  
How to Partition Large Numbers into Friendly Figures

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**Abstract**

Communicating and making sense of large numbers — millions, billions, and trillions — is a persistent struggle in our society. Using large numbers is a learning requirement for elementary school children, but even adults struggle with it. Hence supporting future teachers in developing their own understanding of the concepts is valuable. To construct, enact, and revise an educational experience for preservice teachers, we apply three frameworks of teaching mathematics for social justice tasks, high cognitive demand tasks, and productive mathematics discussion. The context uses United States educational and defense spending, the national budget, and the gross domestic product. Preservice teachers talk to family members outside of the classroom about large numbers. In class, they read arguments that compare educational and defense funding using the U.S. federal budget (defense receives much more federal funding) to educational and defense spending using the U.S. Gross Domestic Product (educational spending, at local, state, and national levels is much more than defense spending). Through action research, we implemented, revised, and adapted the task several times. We present the completed task and preservice teacher responses.
1. Introduction

Despite feeling comfortable that a billion is 1,000 millions and a trillion is 1,000 billions, even well-educated people can mix up magnitudes, saying “million” or “trillion” when they really mean “billion.” According to Astronomer Jeffrey Bennet, first Director of the Quantitative Reasoning Program at Colorado University “most people see little difference between million, billion, and trillion aside from their first letters” [5]. In fact, researchers found that there is a limit to four items a person can have open in their working memory, or awareness, at once, suggesting that small numbers are easier for human minds [23]. Politicians, journalists, scientists, and others regularly use large numbers to communicate their arguments and decision-making in, for example, health policies and economics [6, 19]. Sensemaking with large numbers is a well-known struggle for the human brain, impacting all of us from schoolchildren to political leaders [15]. Sensemaking in this context is a process where a person engages in noticing a concept or situation, (re)creating meaning, and using the meaning to make inferences or take action [10]. Hence, to be thoughtful participants in their worlds and communities, people must have well-developed numeracy to understand, break down, and judge arguments based on data and numbers. Numeracy, a concept closely related to number sense and quantitative literacy, refers to the capacity, confidence, and disposition to apply numbers and their operations and relationships to better understand and critically examine (i.e., look closely at a situation to judge strengths, benefits, inconsistencies, and constraints) the world around us [18, 7, 16]. All adults benefit from well-developed numeracy, and yet it is common for even well-educated adults to struggle with numeracy. Hence, our focus is on supporting soon-to-be adults, the children who are in our public schools, in developing their number sense as an early form of numeracy (or quantitative literacy) [16]. Maclellan [16] argued that an important part of number sense is the ability to compare magnitudes (i.e., the sizes of quantities). To best support the schoolchildren, we must also support teachers in understanding how to develop these skills.

A number line estimation task described by [15] elicits misconceptions about large numbers. Participants estimate the position of one million on an open number line with zero on the left and one billion on the right (see Figure 1a). Many participants initially place one million halfway (see Figure 1b). One million is one-thousandth of a billion, however, so it should be placed close to 0 (see Figure 1c).
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Because the number line is a visual representation, it is easy to support participants in confronting their own misconceptions by reasoning through the relationships step by step. For example, partition the line from 0 to 1 billion into two equal parts. Many participants would label this point “1 million” so talking through the meaning of “half of 1 billion” by relating it to “1000 millions” can be helpful. That is, there are 1000 millions in 1 billion. Half of 1000 is 500, so the middle of the number line is labeled as 500 million (see Figure 2).

Partition the interval between 0 and 500 million into five equal parts. Because 500 divided into 5 parts makes 100 each, the first part will be labeled 100 million. Half of 100 is 50, so half of 100 million is 50 million.
Then partition the interval between 0 and 50 million into five equal parts. Because 50 divided into 5 parts makes 10 each, the first part will be labeled 10 million. Finally, divide the distance between 0 and 10 million into 10 equal parts. Because 10 divided into 10 parts makes 1 each, the first part will be labeled 1 million. So, 1 billion divided by 2, then 5, then 2, then 5, then 10 (i.e., divided into \(10 \cdot 10 \cdot 10 = 1,000\) parts) makes a part of size 1 million (see Equation (1) and Figure 3).

\[
\begin{align*}
1 \text{ billion} & \div 1,000 = 1,000 \text{ million} \div (10 \cdot 10 \cdot 10) \\
& = 1,000 \text{ million} \div (2 \cdot 5 \cdot 2 \cdot 5 \cdot 10) \\
& = 1,000 \text{ million} \div 2 \div (5 \cdot 2 \cdot 5 \cdot 10) \\
& = 500 \text{ million} \div 5 \div (2 \cdot 5 \cdot 10) \\
& = 100 \text{ million} \div 2 \div (5 \cdot 10) \\
& = 50 \text{ million} \div 5 \div (10) \\
& = 10 \text{ million} \div 10 \\
& = 1 \text{ million}
\end{align*}
\]

In Figure 3, we demonstrate the process of partitioning the number line to justify the location of 1 million immediately to the right of zero. We follow the chain of equalities shown in (1) to show that 1 million is one-thousandth of a billion; that is, if we partitioned the number line into 1,000 equal parts, then 1,000,000 would comprise one of those parts.

Figure 3: A million is one-thousandth of a billion.

In a numbers and operations content course for future teachers, students (re)learn basic mathematical concepts while learning to represent, explain, and justify them for their future students. As we discussed potential tasks for a lesson on large numbers, we focused on the needs of our students as undergraduate mathematics learners and as future elementary teachers. In our classroom experience, almost all future K-8 teachers commonly place the
million in the middle, leaving only a few who place the million near zero. So that they may better support their future students, our future teachers need support in understanding relationships between large numbers.

Recent recommendations and policy documents for undergraduate mathematics instruction [17] and for the education of future K-8 teachers [11, 1] urge immediate and decisive changes to engage all students, especially those from traditionally underserved communities, in meaningful and rigorous mathematical experiences. The Mathematics Association of America recommends that all undergraduate mathematics students experience authentic, real-world mathematical tasks in which they propose viable solutions, analyze solutions mathematically, and then argue for (or against) a solution’s mathematical and situational appropriateness [17]. We intended for our students to engage in this process so they could see how they can use mathematics to make sense of and contribute to a real-world argument. The numbers and operations content course for K-8 future teachers, the focus in this article, centers on basic mathematical operations. Hence, we wanted a task relevant to future teachers that would allow them to focus on creating unique solution strategies while applying numbers and operations as support. In this study, therefore, we focus on constructing a messy, rich, and relevant investigation of large numbers. We next describe our process of identifying and adapting a task.

2. Theoretical Background

To assist instructors as they prepare to guide students through the process of proposing and justifying solutions, the Mathematics Association of America (MAA) [17] suggests using Stein and Smith’s five practices for orchestrating productive mathematics discourse [24]. Although Stein and Smith’s [24] framework was developed for elementary teachers, the Mathematics Association of America argues that the practices are just as effective for university students [17]. We drew on Stein and Smith’s [24] framework of developing learning goals and tasks that work best with their five instructional practices. Further, because large numbers emerge frequently in real world discussions about national or state budgets, and humans struggle to develop accurate intuitions about magnitudes, we also drew on a Teaching Mathematics for Social Justice (TM4SJ) framing, which can be a powerful framework for mathematics instruction [4]. In this study, we interpret TM4SJ as supporting students in using mathematics to critically examine authentic situations.
We adapt TM4SJ tasks to be open and ill-defined, based on Tekkumru-Kisa et al.’s [25] framework for high cognitive demand tasks. We critically evaluate our development and enactment of TM4SJ tasks through collaborative action research [12]. Figure 4 shows how the frameworks from [14, 24, 25] are intertwined in this study.

Figure 4: Frameworks overview.

2.1. Learning Goals and Task Development
Stein and Smith identify the five practices of orchestrating productive mathematics discussion as: anticipating, monitoring, selecting, sequencing, and connecting [24]. They argue that, before engaging in these practices, instructors must construct a focused conceptual learning goal. They contrast this type of goal with the more common performance goal which describes the actions students will have taken by the end of a lesson. A focused conceptual learning goal is narrowly focused on a particular mathematical insight that students are expected to reach. Developing a clearly focused goal can be challenging, but a well-developed goal supports instructors’ decision-making in planning and enacting a task. Research has shown balancing a focus on a particular mathematical concept and on a particular social justice issue in TM4SJ is also challenging [3]. To navigate the tensions, we accepted that we could not simply write two perfect goals. Part of the method, described below, is our engagement in an iterative process of writing and revising two intertwined goals—one for mathematics and one for social justice—by brainstorming potential goals, working on developing the task and anticipating strategies, refining the goals, and implementing the lesson. After the lesson, we again revised the goals and tasks for the second enactment.
2.2. TM4SJ and Relevance

In developing the goals and the task, we used Gutstein’s first goal to develop sociopolitical consciousness [14]. A key principle of a social justice approach is that students can develop the skill of noticing and being part of solutions for injustices. To do this, they must have a deeper understanding of the relationship between the social and political factors that affect society. Educators may facilitate questions to help students think about and begin to deal with these issues. Freire called this process of assisting students in understanding, formulating, and responding to issues, as well as developing analyses of their world as developing sociopolitical consciousness [13]. To engage our students in this process while supporting their development of an understanding about large numbers, we integrate the process of investigating a real-world situation into an open and ill-defined mathematics task, described next.

2.3. Open and Ill-Defined Tasks

Tekkumru-Kisa, et al. present a framework of cognitive demands in mathematics tasks [25]. For this study, we focus on one level of cognitive demand, “Higher-level demands: Doing mathematics” [25]. Tasks in this category are open and ill-defined, require exploration of rich mathematical ideas or relationships, require metacognitive problem-solving actions, and allow students to draw on their out-of-class personal experiences and knowledge [25].

Open and ill-defined tasks allow for multiple solutions and solution strategies. They require students to make assumptions about the initial situation or the meaning of a solution. Because assumptions vary, such tasks allow for multiple valid solutions and require students to justify validity. Exploration refers to the act of finding out more about an unfamiliar area or situation through experimenting or testing; hence, a task that requires exploration pushes a student to act without knowing the consequences. To familiarize themselves with the situation, students must pay attention to emergent consequences (e.g., characteristics of relationships between mathematical objects and operations) and build on them to revise and try again. To successfully choose an action, fail, adjust, and try again, a student must engage in metacognitive actions, paying attention to their reasoning so they can explain and justify their strategies and attempts to others. Finally, students should be able to draw on their lived experiences and knowledge of the world and how things fit together to make assumptions and decisions about solution strategies.
3. Method

3.1. Research Framework

The goals of this study were to improve and to understand more deeply our teaching practices. The context was the construction and refinement of a messy, rich, and relevant investigation of large numbers as they are used to budget and spend money on Defense and Education in the United States. Currently, we (the researchers) use collaborative action research [12] to agree on initial learning goals and to collaboratively design tasks, instructional supports, and strategies for data collection and analysis. We individually enacted and reflected on the implementation of tasks. We then met to collaboratively revise our goals, tasks, and supports.

3.2. Institution

Using the data from [20], we briefly describe the university. Georgia Southern University is a public university with 22,384 degree-seeking undergraduate students. The course described here was taught on two of the three campuses of the university with differences in locale and race / ethnicity percentages; demographics for each campus are shown below in Table 1.

<table>
<thead>
<tr>
<th>Race / Ethnicity</th>
<th>Campus 1 (Town: Distant)</th>
<th>Campus 2 (City: Midsize)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian</td>
<td>2%</td>
<td>5%</td>
</tr>
<tr>
<td>Multiracial</td>
<td>4%</td>
<td>5%</td>
</tr>
<tr>
<td>Latinx</td>
<td>7%</td>
<td>11%</td>
</tr>
<tr>
<td>Black</td>
<td>26%</td>
<td>26%</td>
</tr>
<tr>
<td>White</td>
<td>61%</td>
<td>53%</td>
</tr>
<tr>
<td>Undergraduate Enrollment</td>
<td>17,431</td>
<td>4,953</td>
</tr>
</tbody>
</table>

Table 1: Race/Ethnicity Distribution and Undergraduate Enrollment by Campus.

The larger campus (Campus 1) is located in a town with population less than 35,000 and 35 miles from a midsize city with population about 330,000. The smaller campus (Campus 2) is located in that midsize city. Students on the smaller campus are, on average, two years older and often are employed full-time.

1 Enrollment statistics were collected from the National Center for Educational Statistics (2019-2020). Only race/ethnicity categories with at least 2% at one campus included in the table. Nonresident aliens and race/ethnicity unknown categories are not included.
3.3. Course and Students
The Number and Operations course in which the task was implemented is a required mathematics content course for students who plan to enroll in either a special education, elementary, or middle grades teaching certification program. Students enrolled in this course are typically in their first or second year of college and have taken at least one prerequisite mathematics course. The course focuses on addition, subtraction, multiplication, and division of whole numbers, integers, rational numbers, and real numbers.

The task was given in the first two weeks of four sections of the course in Fall 2021 and Spring 2022. For all sections, EMM was the instructor. In Fall 2021, one section each was taught on Campus 1 and Campus 2, respectively. In Spring 2022, two sections were taught on Campus 1.

3.4. Task Development
As we adapted the task to our classroom, we needed to make it fit within the time and structural constraints of our semester planning while anticipating our students’ needs and ability to connect to prior knowledge. At the same time, we needed to develop our knowledge about budgets and gross domestic products as we gathered real-world data and made choices about what to include or exclude. Hence our data and findings focus on our choices and reflections as we planned, taught, and revised the task as shown in Figure 5.

3.5. Data Collection and Analysis
Data collection included researcher journal entries, student work and strategies, and researcher conversation and revision for the next cycle. Researcher journal entries were written 15 minutes before and after implementation of each task. Before the task, we focused on reflecting on what we wanted to remember to include in class, what we wanted students to take away from the task, and anything we noticed or wondered when examining student responses to the pre-class survey (i.e., asking the family member about the tallest building). After the task, we focused more broadly on what went well, what was unexpected or stood out to us, what we would change when re-teaching the lesson, and what we noticed about student responses after class. The qualitative analysis included iterative alternations of inductive and deductive analyzing processes. Initial data categories were developed through an open coding process and continuously modified and refined through the alternating process mentioned above. Discrepancies among researchers were resolved through group discussions.
4. Task Development

We now describe the process of adapting and implementing a large numbers task that incorporates learning goals for both mathematics and social justice based on Peterson’s large numbers mini unit [21]. We briefly describe Peterson’s original purpose and structure. We then describe our original adaptations to recreate it into a 45-minute task appropriate for a numbers and operations content course for future K-8 teachers. Our adaptations included (a) tightening its focus to fit our time constraints; (b) restructuring the task to follow the format of pre-session, launch, explore, post-session, summarize; (c) updating large numbers based on the current year; and (d) revising contexts to be familiar to the students.
4.1. Original Task

Peterson described his task as having a *mathematical focus* on making sense of large numbers (e.g., million, billion) and a *social justice focus* on understanding “the power of math in debates about the future of our communities and world” [21]. To make sense of differences between a million and a billion, students asked family members what they thought these quantities meant. In class, students shared family members’ responses. They then made sense of one million using time (*how many days to count to a million*?), height (*how many stories high is one million hairs, stacked lengthwise*?), and area (*how many pages would one million tiny stars fill*?). After this initial work, the class built on their knowledge in the context of “billions for war, millions for schools” [21]. They made sense of the cost of one year of the Iraq war in terms of the number of schools that could be built (and compared that answer to the number of schools in their state). They interacted with graphs contrasting U.S. military budgets to other U.S. budget items, and to the budgets of other countries. Finally, they calculated and discussed the cost of a stealth bomber in terms of teachers’ salaries / benefits.

4.2. Task Adaptation

We first developed focused conceptual mathematics and social justice learning goals. We started by using Peterson’s goals: Making sense of large numbers (mathematics goals) and understanding the power of math in debates about the future of our communities and world (social justice goals). As we designed our adaptation, we returned to discuss the learning goals multiple times. Our first revision focused on what we wanted the students to do during the task. The mathematical goal for the first iteration was to understand how grouping quantities helps visualize differences in magnitudes of large numbers. The social justice goal was to understand how to use math to support positions about spending on the military and education.

As a meaningful context for our students who will be teachers, we focused on a common rhetoric in which federal budgets on military and education are compared. The clear difference between defense and education that can be discerned through this rhetoric is often used as evidence of the need to increase education funding. This argument ignores, however, that education spending comes from local and state budgets in addition to federal budgets.
Comparing the data on spending as a percent of the U.S. gross domestic product (GDP), education spending is a few tenths of a percentage higher than defense. Percentages vary, but comparisons are relatively stable. For example, the most recent data from academic year 2018-2019 shows that, at the federal level, education is 7% of discretionary spending while defense is 51% [6] (see Figure 6a). But, across all levels, the total U.S. public elementary and secondary education spending was about $764.7 billion [26]. When compared as a percent of GDP, Defense is 3.2% while Education (at all levels) is 3.6% (see Figure 6b).

![Figure 6: Defense and Education Spending in FY2019 (in billions of dollars).](image)

4.2.1. Before Class
The before class homework includes students watching two videos. The first one shows a visual comparison of the top 20 countries’ military budgets, which isolates the United States (U.S.) military budget as the largest by several times.\(^2\) The second compares education spending and teacher expectations in the U.S. with those of different countries [27]. Students then ask several questions of a family member or friend at least ten years older than themselves to interact with a perspective different from their own.

\(^2\) We used a video titled “Countries with the highest military expenditure,” posted on YouTube on January 14, 2021 by We Love Stats, retrieved from [https://youtu.be/S9f1eCTqJkQ](https://youtu.be/S9f1eCTqJkQ) on August 9, 2023. This video is unfortunately no longer available. Readers might instead use the video “Top 10 Countries by Military Spending (1870-2020)” posted on YouTube on June 18, 2021 by RankingCharts, available at [https://youtu.be/UrUp5Rm_Ncw](https://youtu.be/UrUp5Rm_Ncw), last accessed on July 23, 2024.
Students finish by converting one million seconds into days and one million dollar bills (stacked flat) into feet. Figure 7 shows questions students asked and images from videos they watched.

**Ask someone 10 years older:**
1. What does one million mean?
2. What’s the difference between a million, a billion, and a trillion?
3. Who is the oldest person you know? How old are they? (It can be a guess.)
4. What’s the tallest building you’ve ever seen? How tall is it in feet or levels?

**Figure 7:** Before Class Task: Interviewing a Friend and Watching Videos.

### 4.2.2. In-class launch

In the *launch* portion of the class, we share examples of family members’ explanations of one million and comparisons of a million, billion, and trillion. Then we use student examples of largest buildings and oldest people to convert age to seconds and heights to numbers of dollar bills stacked flat. Ages reported ranged from mid-50s to 104. Students used a simplified calculation to convert years to seconds (ignoring leap years) to calculate the number of seconds that their person had lived. For example, one response for age was “the oldest person I know is my great-aunt and she is 89.” The student found their great-aunt was about 2.8 billion seconds old. Responses about tallest buildings ranged from Georgia’s capitol building (about 272 feet) and the Bank of America Plaza in Atlanta (55 levels; 1,023 feet) to the Willis Tower in Chicago (100 levels; 1700 feet) and the Empire State Building in New York City (102 levels; 1454 feet). Students used the conversion factor of a height of 1,000 dollar bills to 4.3 inches. Then, for example, Georgia’s capitol building is 272 feet \(\times 12 \text{ in/ft} \times 1,000 \text{ dollar bills} \div 4.3 \text{ in} = 759,070 \text{ dollar bills laid flat and the Bank of America Plaza in Atlanta is about 2.9 million dollar bills laid flat.}

### 4.2.3. In-class exploration

In the exploration portion of the class, we ask students to read two letters to the editor that advocate for different perspectives on military funding and educational funding. The students are asked to use a representation
to compare U.S. budgetary items for education and military spending, take a stance on the letters to the editor, and use mathematical evidence and representations to justify their stance. We created the letters to the editor based on [21] and common arguments for funding education (shown in Figure 8).

<table>
<thead>
<tr>
<th>Letters to the Editor (162 words each)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sofia</strong>: Our students attend overcrowded, worn out classrooms. Our teachers spend an average of $745 out of their own pockets to provide basic supplies for our children. That’s $3 billion that our teachers donate each year! Our children are our future! But we aren’t spending the money necessary to provide safe and quality education. The 2021 Federal Budget was $4.8 trillion, and Defense got 16%, leaving less than 2% for Education. We already spend three times as much as China on Defense, and they have four times our population. We could decrease defense to 15% of the whole budget and increase education to 3%, and still spend many times what China spends. We could replace or renovate the schools that need it and still meet needs for adequate classroom supplies. Why do we always find money for war, but not for education? As Biden withdraws troops from Afghanistan, we can afford to spend less on Defense. Our tax dollars should benefit taxpayers.</td>
</tr>
<tr>
<td><strong>Ayaka</strong>: Our troops fly broken airplanes and forfeit training for vital maintenance. Budget cuts have created a crisis for our troops, threatening the safety of our heroes in uniform. We ask too much of those who serve! For six years, we’ve barely been getting by. The world is becoming more dangerous, and our resources are cut again and again. US Gross Domestic Product (GDP) is $19.5 trillion. We spend only 3.5% of the GDP on Defense. Yes, Defense gets more Federal money than Education. But Education is supported at all levels: Federal, State, and Local. All levels combined, the US spends about 3.9% of our GDP on K-12 Education. If we shifted only one quarter of a percent of the GDP to Defense, we could replace 250 of our broken aircraft with brand-new F-35s. Repairing and rebuilding our military is key to defending our country. It’s also the way we can care for the men and women who protect us.</td>
</tr>
</tbody>
</table>

Figure 8: Arguments for Education (Sofia) versus Defense (Ayaka).

In these arguments, we included two ways of thinking about Defense spending compared to Education spending (as shown in Figure 7); specifically, federal spending on Education is $581 billion less than its spending on Defense while U.S. spending on Education at all levels is $88.7 billion more than spending on Defense. Differences connected to ideas in the pre-class work where U.S. spending on Defense and Education was compared to other countries’ spending. The two arguments are equivalent in the actual change in funding.
That is, Sofia argued for a difference of 1% of the budget while Ayaka argued for a difference of 0.25% (a quarter of 1%) of the GDP. Because the budget was about $4.8 trillion, 1% (or dividing by 100) is $48 billion. The GDP was about $19.5 trillion, so 1% is $195 billion and a quarter of that is $48.8 billion.

4.2.4. In-class summarization

During the next class session, student groups present their visual and written arguments. As a whole class, we discuss what they noticed about magnitude and government spending. Even though the students have to make a lot of assumptions to make the task do-able, they are also developing an internal sense of what it means to spend $48 billion on 49.5 million students or on four million teachers. The calculations, drawings, and written arguments are intended to support students in reaching the conceptual mathematical and social justice goals: understand how grouping quantities helps visualize differences in magnitudes of large numbers and understand how to use math to support positions about spending on military and education.

5. Findings: Reflecting After Task Implementation

Here we present our findings as we developed, taught, revised, and re-taught the lesson. We interpret findings broadly, describing our process of revising learning goals, presenting students’ (and their families’) responses to questions, and presenting examples of students’ arguments in response to the task.

5.1. Development of Focused Conceptual Learning Goals

As we discussed the meaning of performance goals compared to conceptual goals, we realized we had written performance goals because they describe students visualizing and comparing. After the first teaching, we met to discuss what we really wanted our students to discover and know after the lesson. We then revised the goals to be more conceptual by focusing on what we hoped they would notice by visualizing and comparing (see Figure 9c). That is, for the mathematics goal, we hoped they would understand how to make large quantities (like a million) more manageable by creating groups to visualize and compare them. For example, Peterson [21] had students creating groups by dividing the large quantity into a number of airplanes or schools. Breaking a large quantity into smaller, more familiar groups can help
students see differences in magnitudes. For example, if an F-35 airplane costs $160 million to build, then students can visualize the difference between a billion and a million as the difference between building 6 airplanes and 6,000. For the social justice goal, we hoped they would notice how mathematics helps them understand a situation, take a stance, and construct a convincing argument to support the stance.

<table>
<thead>
<tr>
<th>(a) Peterson (2013)</th>
<th>(b) 1st iteration</th>
<th>(c) 2nd iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics goal</td>
<td>making sense of large numbers (e.g., million, billion)</td>
<td>use commonly experienced concepts to visualize differences in magnitudes of large numbers</td>
</tr>
<tr>
<td>Social justice goal</td>
<td>understanding the power of math in debates about the future of our communities and world</td>
<td>use lived experiences and mathematical analysis to compare government spending for military and education</td>
</tr>
</tbody>
</table>

Figure 9: Development of focused conceptual learning goals.

5.2. Family Members’ Explanations of Large Numbers

For the in-class launch portion, family members’ explanations of one million and comparisons of a million, billion, and trillion varied and some showed misconceptions. An example of a misconception was: “A million is a lot of money, a billion is double a million, and a trillion is triple a million.” Another: “My dad answered ‘A lot’ and then said that a billion is a million millions and a trillion is a billion billions.” This finding supports the results by Landy et al. [15] about sensemaking with large numbers being a well-known struggle even for adults.

Many responses described mathematical differences. One said “My dad, being the smart aleck that he is, says, ‘The difference is about three zeros. Add three zeros to a million and you get a billion and do that to a billion and you get a trillion.’” Another responded, “Each one is a thousand times more than the one before.” Other responses showed individual ways of making sense of the values. For example, one person said “A million is exceptional, a billion is rare and unique, and a trillion is almost unbelievable.”

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3 Indeed in older British English, a billion did equal a million millions, but this is listed as archaic in dictionaries today.
5.3. Students’ Arguments on Large Number Task

In teaching, revising, and reteaching these tasks, we have made the task more open so the students make decisions about an issue that is important to them. For example, students might focus on the costs of new desks, improved meals, or more reliable transportation. These issues are important to our students because they remember high school classes where they or classmates had to sit on the floor, classmates who needed breakfasts, lunches, and weekend meals, or rides to school that took an hour or more because the schools could not pay enough drivers or rent enough buses. In groups, students choose an issue, do quick research, and work outside of class to create a visual and written argument. See Table 2 for a list of issues chosen by groups.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>New desks</th>
<th>(49.5 million students) × ($157.85 per student)</th>
<th>$7.8 billion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 2</td>
<td>Better meals</td>
<td>(49.5 million students) × ($720 per student)</td>
<td>$35.6 billion</td>
</tr>
<tr>
<td>Group 3</td>
<td>Improved salaries</td>
<td>(4 million teachers) × ($12,000 per teacher)</td>
<td>$48 billion</td>
</tr>
</tbody>
</table>

Table 2: Large Numbers Task: Student Arguments.

For example, during the in-class summarization, one group found that a new student desk costs $157.85. In the United States there are about 49.5 million students. Even though not every student needs a new desk, for simplicity the group calculated the costs of buying a new desk for each student, which would be about $7.8 billion. Another group estimated, based on median costs for breakfast and lunch, that schools currently spend about $5 per day for each student across both meals. The group decided (through discussion) that improving the meals would cost at least $2 per student per meal and $720 per student per school year (180 days). The group calculated how much it would cost if the government paid for the meal improvements rather than the students’ families. They estimated it cost about $35.6 billion across 49.5 million students and 36 weeks of school.

Other groups focused on making an argument for increasing teacher salaries. One group estimated an average teacher might spend around 2,312 hours each year teaching, prepping, grading, communicating with administration, colleagues, and parents, working during the summer to prepare for the school year, and engaging in professional development. The estimate was based on information they looked up online and information they had from parents or friends who were teachers. They estimated a teacher works about 60 hours per week during the school year and about 150 hours during the summer.
They compared the estimate to an average teacher salary of $37,000. They found that (based on their assumptions) teachers make about $16 per hour. They argued that teachers should make at least $20 per hour, and worked back the other way to argue that the average teacher should make at least $47,000. They found that, with about 4 million teachers in the United States, if the average teacher salary was raised by $10,000 per teacher, then it totaled $40 billion. That left about $2,000 per teacher for classroom supplies so they do not pay out of their own pocket.

6. Discussion
6.1. Reflections
Using the five principles of action research described by Feldman et al. [12] to structure our study, we committed to sustainability, practicality, increased social justice, using small changes, and continuing reflection. As we collaborated on adapting this task and then teaching, revising, and reteaching it, we wanted it to be meaningful and relevant to the students. Because each student has a unique background, the task cannot be “one size fits all” as it was in our first iteration. In that iteration, students compared the costs of F-35s to schools, which might have been meaningful to some students but we saw it was not meaningful to them all. After revising the task, we had students bring their own meaning to the task by choosing their own ways of breaking the large quantity into groups. The students were much more excited and engaged. The fear of openness, however, has to be acknowledged. Many of our students have encountered so-called open tasks where they made a choice that was rejected. Because of bad experiences, some students struggled to find the courage to make a choice. Our recommendation is that several similar tasks can be used throughout the semester, with a focus on valuing the choices students make. We as instructors also need to support our students by explicitly acknowledging that some faculty will not value their choices and that it is not fair when that happens.

Because the task was open to choice, it meant that questions were often unexpected and required us as faculty to have processing time. EMM noticed that some students immediately thought that the processing time meant they had done something wrong. She reassured students that mathematics can take time to process, so sometimes she needed time to do that. By acknowledging that she needed processing time, she noticed other students would use similar language when they did not immediately understand something.
This growth was surprising and exciting, even though EMM had initially been scared about what would happen if she was unable to answer a question immediately.

One necessary change to the task was to update the context. When Peterson [21] taught his task, the Iraq war and occupation was relevant to the country, and the renovations to their school were relevant to his students. Because our students are future teachers, we decided to focus on teachers and teaching in the United States. Keeping the focus on a comparison of funding for the Department of Defense and Department of Education, we found updated information about the percent of the federal budget going to each department. We found, however, that other arguments compare funding by considering the percentage of the Gross Domestic Product (GDP). This perspective surprised us because educational spending at all levels — local, state, and federal — adds up to a higher percentage of the GDP than defense spending (see Figure 6). We decided to use those two perspectives to provide two balanced arguments, one for increasing educational spending and one for increasing defense spending.

One aspect of the task that we did not change was prompting our undergraduate students, who themselves will be future teachers, to ask their family members to describe one million, one billion, and one trillion. Each time we taught this lesson, we saw a similar range of responses as those reported by Peterson [21]. We list a few that we saw in our class:

- precise (e.g., “A million is ten one hundred thousands, a billion is a thousand million, a trillion is one thousand billion.”)

- comical (e.g., “million = rich, billion = more rich, trillion = super rich.”)

- practical (e.g., “One million is something I could make in a lifetime. A billion and a trillion are unreachable and just insane to really think about how much money that really is.”)

This similarity persists despite differences in regions of the country, levels (undergraduate rather than fifth graders), and time (ten years later). Some family members even answered by comparing the numbers to units of time which connected to the in-class activity, “A million seconds is 11 days. A billion is about 32 years. One trillion seconds is about 32000 years.”
6.2. Future Directions

We found that the openness of some parts of the task meant we needed to be more directive in other ways. When we teach the task again, we will ask students to answer questions at the end of the first day that push them to make choices. For example, asking them specifically what they decided their stance would be and what they would focus on to construct groups (i.e., teacher salaries, student meals). Then, we would ask them what information they needed for their argument and ask them to look up the information before the next class. Despite our best efforts to contain the task to one class, it seems that it would work better as a small unit project that students could work on and get feedback on before presenting to the rest of the class.

References


[27] Vox, “Teaching in the US vs. the rest of the world,” video on YouTube, January 11, 2020; available at https://youtu.be/wFqQm1541aA, last accessed on July 22, 2024.