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Maximum Mass Restraint of Neutron Stars: Quarks, Pion, Kaons, and Hyperons

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This thesis explores the topic of maximum mass stability of neutron stars. The outer structure is detailed and explores nuclear pasta phases, the neutron drip line, and density transitions of matter in the crust and atmosphere layers. Other discussion points include superfluids in the crust and core, vortex roles in neutron stars, and magnetic field effects on the EOS in neutron stars. The inner core is studied in much more detail due to its significant role in EOS. The variety of stars include pion condensate stars, kaon condensate stars, npeu stars, npeu stars with the inclusion of hyperons, quark-hybrid stars, and strange stars. Included with these is a description of nucleon-nucleon, nucleon-nucleon-nucleon interactions, the appearance factors that affect hyperon species, and the formation process of kaons, pions, quarks, and hyperons. The ending EOS are compared with their maximum mass values to determine which ones are likely to limit the mass of neutron stars.

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Introduction and Neutron Star Birth

Supernovae can result in a variety of stellar remnants from black holes, white dwarfs, and Wolf-Rayat stars. Depending on the mass and composition of the star as well as a variety of other factors the ending result can vary. Certain classes of supernovae produce a remnant called a neutron star. A simple explanation one may hear is that a neutron star is a star that has overcome electron degeneracy pressure that would support a white dwarf star and has forced protons and electrons together to create a massive ball of neutrons. This, in turn, is supported by neutron degeneracy pressure. The characteristic diameter is 10,000 meters and the mass is 1.4 solar masses. These are good for rough approximations, but neither the description nor details suffice when discussing neutron stars in depth.

A rational starting point to begin looking at neutron stars is their birth. The most common scenario is for a neutron star to be born in a core-collapse type II supernovae explosion. In this case the pre-supernovae star is several solar masses and as it nears the end of its main-phase life it begins producing heavier and heavier elements. The elements near the start of the elemental table produce a much higher ratio of energy when involved in fusion processes. This is why a hydrogen fusion can last so long and is the most significant portion of main-phases. As each new element is produced the ratio of energy production per fusion reaction decreases and the star must burn elements faster and faster. The critical point of fusion occurs when the star begins iron production, as every element produced after that actually takes energy away from the star instead of producing it. As the star fuses more iron in the core it begins accumulating until it reaches approximately 1.44 solar masses. At this point degeneracy pressures can no longer support the core and the process of star collapse begins.

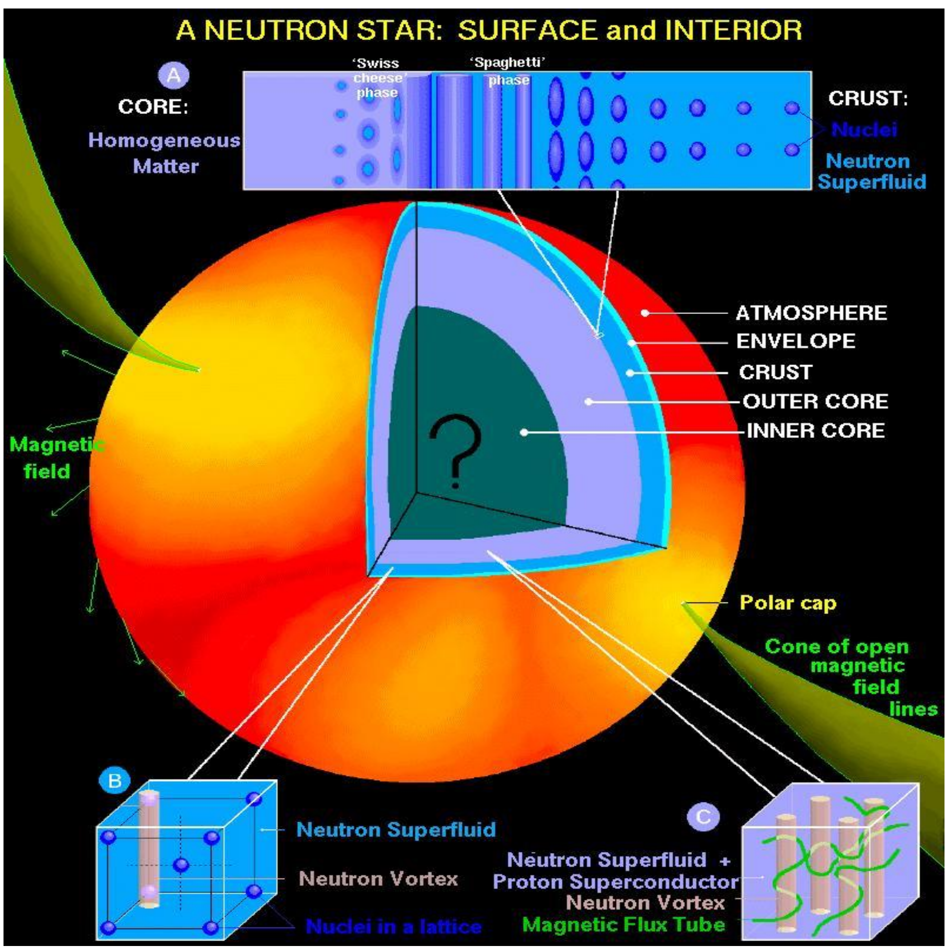
Core collapse is a rather violent process and is caused by the production of gamma rays within the core. Through a process called photodisintegration all the heavier elements that the star has created in its core are destroyed. The core is shredded apart and reduced to lighter elements while it produces neutrinos that can escape through the rest of the superdense material surrounding the core collapse. Eventually, the core collapse is halted by neutron degeneracy and strong forces. The infalling matter then collides and bounces outwards, producing a shockwave of matter. From the outside of the star it is still not apparent that this process is happening, but inside the shockwave rapidly approaches the layers outside of the core before it is halted. The shockwave is then refueled and attempts to push its way through the star via energy absorption processed from neutrinos, a process which is still not fully understood. If it succeeds, the final outcome of the star will be a neutron star, but if it does not, matter will begin to fall back to the core until degeneracy and strong forces are overcome. In this case, the star will be forced to collapse with no other force left to restrain it and a black hole is produced. While the type II supernovae is the most likely way that a neutron star will be produced, it is not the only method.

Another scenario where a neutron star can be created is in a white dwarf binary system. Such events are rare however, as mass accretion must occur in a certain manner to result in a neutron star. When a white dwarf accretes matter from its companion star it will oftentimes result in a Type Ia supernovae. It reaches a level where it can reignite fusion and this reaction snowballs until the star blows itself apart from the energy produced if it is greater than the star's binding energy. It is rare that a neutron star will be produced in this way so in almost all cases it is a Type II supernovae that produces a neutron star.

The Layers of a Neutron Star

The layers of a neutron star are vital to understanding its mass structure. While white dwarfs can be described by an equation of state (EOS) similar to a degenerate gas and other smaller stars have accurate EOS, this is not the fact for neutron stars. The interaction of general relativity and superdense matter cannot be ignored in neutron stars and there is significant time dilation even at the surface of the star. The outer layers of the neutron star structure can be accurately portrayed to a greater degree, allowing for EOS formulation, and the EOS becomes more theoretical as matter approaches the inner core.

Below is the typical structure that one may find inside of a neutron star.



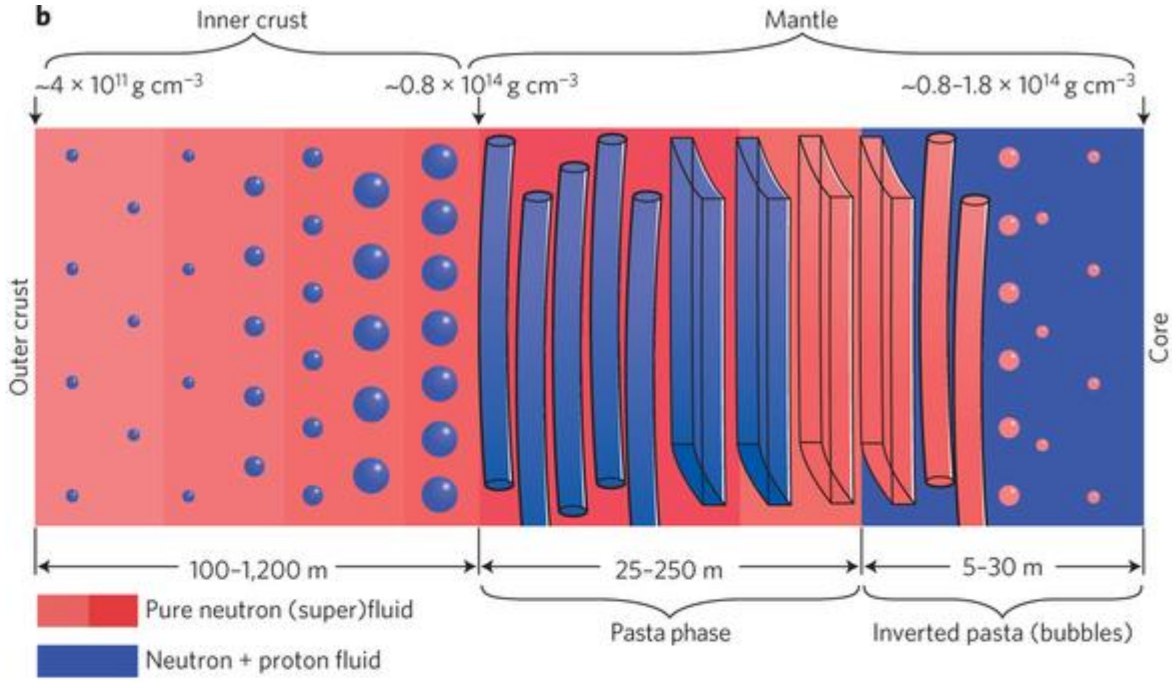
The concepts of neutron superfluids, and high-intensity magnetic fields will be mentioned in a brief explanation and their effects on EOS will be detailed in the next section, but for now it is more important to detail structure. Some models of neutron stars will choose to include the atmosphere/envelope structure of the star while some of them will not. This is due to the fact that if the star is cool enough, the crust structure extends to the surface eliminating the upper layers of the star. The atmosphere itself, if present, can vary from a few millimeters if it is colder, to several centimeters if the star is hot, but it is not an overly complex stucture and is essentially a plasma layer that is above the envelope. The temperature at which the envelope begins to thin from a fewmillimeters is approximately $3 \cdot 10^5$ K and a hot surface would be around $3 \cdot 10^6$ K. The envelope only needs a short mention as it is essentialy just a barrier between the outside of the star and the interior and acts as a method of radiative transfer.

The crust of the star is actually separated into two distinct sections. Overall, the crust can contain anywhere from .2 to 1.4 percent of the a solar mass, with its percent decreasing as the overall stellar mass approaches its maximum mass. The outer crust section is much smaller than the inner crust and is usually a few hundred meters, with the inner crust extending up to 1,000 meters. The outer crust is composed of ions and electrons, with upper sections allowing iron within an electron lattice, and then turns into a degenerate gas that becomes ultrarelativistic. The outer crust takse the form of a Coloumb crystal. In simplified terms it is a 2-d hexagonal lattice that is formed between the repulsion of ions in the Coloumb force. The inner crust is composed of electrons and free neutrons as well as neutron rich-material. At the border of the inner crust and core a phase change coined as nuclear pasta begins.

Nuclear pasta is a convenient term to describe neutron stuctures as the star densities approach the neutron drip line. The neutron drip line is the density point in the star at about

$4 \cdot 10^{11}\left(\frac{g}{cm^3}\right)$ and is involved in neutron-nuclei interaction . As pressure increases in the star, the potential differnetial outside each nucleus increases. This continues to happen until it reaches the density where the potential differential overcomes the strong force holding the nucleus together and neutrons proceed to start "dripping" out and forming a homogenous neutron soup. Of course at this point, all nuclei will already be neutron rich due to electrons being forced together with protons. As the density increases, the transition phase from inner crust to core goes through four phases of nuclear pasta.

The oddly titled phases in appropriate order are gnochhi, spaghetti, lasagna, and swiss cheese. The last is not a pasta of course, but we'll let it slide. These phase transitions can be seen below as the neutron superfluid transitions into the outer core materials. The gnochhi phase consists of nucleon blobs, which as they are exposed to increasing pressure elongate into the spaghetti phase. This is due to repulsion caused by the protons in the previous gnochhi phase. As the spaghetti shapes get more unstable they meld into nucleon sheets deemed lasagna. The final phase consists of transition to core material and swiss cheese which are bubbles of protons and neutrons. Although the pasta phases, particularly the spahetti and lasagna phases, do not make up an extremely large portion of the star they do still account for an estimated one percent of a solar mass, which is not irrelevant when determining the EOS near the inner crust and outer core.



Reference [28] (pg.2)

The outer and inner core are perhaps the most important components to determining neutron star mass because, unlike the other components of the star, they are pressurized to the point where they are superdense and do not follow standard, well-known EOS models. In general, as density increases EOS formulas become more unreliable. While the crust and other outer layers are still important they contribute less to the uncertainty of mass, as they have well defined EOS. Laboratory testing can only work with the approximate states of core material, as the conditions near the core are at unrealistic levels of reproduction. The outer core consists of free neutrons, protons, electrons, and possibly muons. The inner core is only present in more massive neutron stars, where superdense environments can occur. The inner core could consist of anything from hyperons, to pion condensates, kaon condensates, up , down, or strange quarks. There are a variety of other particle combinations that can be present, but the most critical component is how the EOS of the neutron star is effected based on its composition.

Superfluid Properties and EOS

Within neutron stars, superfluids are mostly attributed to properties involving heat capacity, neutrino emission, and thermal evolution. A simple description of a superfluid would be a fluid of baryons, such as neutrons, that compose a fluid. As the baryons all reach minimum energy states they can no longer lose energy. As a consequence, if a swirl or some other motion is started in a superfluid it will not stop unless acted on by another force since there is essentially zero friction.

There are two, or possibly three, sections of superfluid depending on how one wants to divide each section of the star. In the outer crust there is not sufficient pressure for a superfluid, but in the inner crust a configuration of neutron superfluid can be formed. At a density of $2 \cdot 10^{11}\left(\frac{g}{cm^3}\right)$,compositions and superfluids begin to slightly change. In the inner crust, superfluid is a singlet state consisting of neutrons. In the denser outer core, protons are freed out of nuclei and due to protons low number density they can form a singlet state superfluid. The previous singlet state of neutrons becomes repulsive around the neutron drip line and thus neutrons move onto form a triplet state in the outer core. The role these states play in cooling are quite interesting and further detail about the singlet and triplet states as well as an introduction to Cooper pairs can be found in [26]. Another result of these superfluid states is that vortices will form that can travel inwards or outwards based on certain effects listed below.

Within the outer core, interactions between the neutron and proton fluids form vortices. Although there is significantly more neutron superfluid, the proton superfluid is sufficient to make the vortex into a superconductor. If the vortex happens to be in the right area it can interact with the electron lattices within the crust section of the star. The vortex can then be dragged upwards, and through conservation of angular momentum the star's rotation rate must speed up. Vortices that form in the inner crust can also interact with rotation rate by pinning themselves to lattice structures in the crust. Some of these actions can slow down the rotation rate and some can speed it up depending on where the pinning takes place. The question is if these superfluids have unique EOS and if the interactions between lower layers of the star via these vortex properties are significant enough to take into account when determining the overall EOS. Most papers on the topic agree that it would not affect neutron star mass significantly unless the energy gap is greater than 1MeV. This gap is where the Cooper pairs that make up the superfluids break up. Most estimates have this gap below 1MeV, but if it is significantly over that value then superfluids could exist in greater amounts than thought.

Magnetic Field Properties and EOS

Neutron stars can be born with a very large range of magnetic fields. A typical neutron star has a field of about 10^{12} Gauss, but sometimes a rarer form of neutron star called a magnetar can be formed. These have a field of up to 10^{15} Gauss and inside the core sections it is even higher. These levels are drastic enough to have an effect on structure of the star and are proposed to cause some very unique effects such as star quakes involving warping of the star's iron crust. The main issue is that very few of these magnetars have been studied. They can sometimes be identified by supernovae with two separate peaks in luminosity or by trying to find a star that is rapidly breaking itself against the interstellar medium, but they are not plentiful. Furthermore, it appears that they have short lifetimes before they lose much of their magnetic field.

The easier magnetic fields to identify come from pulsars. While they may be easy to find due to their magnetic poles emitting accurately timed beams of radiation their fields are much weaker. In order to understand the full effects of magnetic field powers above 10^{14} Gauss are desired. At this level it is possible for the magnetic field to affect ion thermodynamics. As a result the Coloumb crystal structure in the crust can be extended much deeper due to the increase in stability granted by the field. Quantum effects can be seen within the star, but only if the particles are in a select few quantum levels that are induced magnetically.

Overall, the upper layer EOS are altered the most. Usually the magnetic field will result in the maximum mass increasing as plasma, Coulomb crystal, and the envelope of the neutron star are influenced at high levels. A solid conclusion is not possible however, until more high level magnetic fields can be observed. In most cases particular temperature, pressure, and quantum state requirements must be met for significant changes in the EOS to take place. Even if just the outer layers are drastically changed they do not account for much of the star's mass and won't alter the maximum mass of the star by a large amount. It may be likely that magnetic field plays a larger role in young neutron star EOS or that it is not that significant at all to the overall EOS. The majority of studies done that concluded a significantly changed EOS used magnetic field levels that are on the border of unreasonable. In reference to magnetic fields and superfluids there are no widely used EOS. Many prefer to obtain an accurate model of nuclear matter states before adding extra complexities.

History of Neutron Star Mass and EOS

The first formulation of EOS of neutron- star-like matter was devised in 1933 by Sterne shortly after the Chandrasekhar limit, the mass where white dwarfs must collapse, was formulated. Sterne used a model which consisted of electrons, neutrons and protons. Due to its simplified nature it is not extremely valuable to modern day mass and EOS calculating, but it is significant in the fact that it was the first detailed account to approach dense matter in neutron stars. No maximum mass was produced, as Sterne simply studied the effects of neutron appearance with density change.

In 1939, Oppenheimer and Vollhoff used hydrostatic equilibrium and general relativity equations to make EOS models. They still made some large assumptions such as free Fermi Gas in thermal equilibrium at T=0, but for the time period it was a rather advanced model. Using this, they obtained a maximum mass of .71 solar masses. This presented some problems, the foremost of which was that it was significantly below the Chandrasekhar limit. In part, the low value for the EOS was due to the fact that it was still believed that P was held to the inequality of one-third of epsilon as density approached infinity. The truth is that for dense matter P actually approaches e rather than 1/3 of its value. This resulted in inaccurate calculations of pressure and flawed EOS. Over the next few decades very few new valuable EOS breakthroughs occurred, except for a few that lowered the Maximum mass by about .01 solar masses by studying npe(neutron, proton, electron) core material. Other studies attempted to evaluate equations of densities $10^{18}\left(\frac{g}{cm^3}\right)$ to several powers higher, but eventually held no relevance since it was determined that nucleons could no longer can support themselves at those densities.

Finally, in 1959, Cameron used newly introduced nucleon-nucleon interaction to help evaluate superdense levels of neutron stars to obtain a mass of 2 solar masses. Cameron also discussed muons and hyperons as possible occurrences, but did not incorporate them in his equations. Hyperons are considered a class of exotic bosons and are composed of one or more strange quarks with no charm, top, or bottom quark and are typically very instable outside of nuclear matter. He noted that the Σ^- hyperon would be the most likely one to appear first and that heavier hyperons would probably not appear until higher densities. Victor Ambertsumyan and Suakyan in 1960 presented an EOS involving a free Fermi gas of baryons, electrons, mesons, and muons. The assumption of thermodynamic equilibrium was still used. They also brought hyperons into heavy consideration, as due to the Pauli Exclusion Principle, hyperons could exist in a stable state at core pressures. Hyperons' appearance pressures, of where they could form, were determined and the Σ− line was put at $1.1 \cdot 10^{15}\left(\frac{g}{cm^3}\right)$. As will be detailed later, this value can vary greatly based on various super dense effects. Around the same time in, 1961, Yakov Zeldovich presented an EOS that proved P must be less than epsilon at superdense levels. This was proven with a special relativity model that broke the previous P and epsilon one with a new EOS.

Most of these equations were not brought to the front of the scientific field when they were created. The discovery of pulsars in the late 1960's pushed them into the main scientific field. Many regarded neutron stars as hypothetical and a waste of resources, but with their confirmation scientists rushed to solve superdense EOS problems. In the late 60's and early 70's many EOS models of pure neutron material stars were proposed that attempted to integrate nucleon-nucleon(NN) and nucleon-nucleon-nucleon(NNN) interactions. These EOS were too simple and the ones that relied on NNN interaction were using various proposed NNN models that varied to significant degrees.

The first hyperon integration models of the 70's veered all over the place. Initial results pointed toward a maximum mass of 1.4 solar masses, but hyperon-hyperon(HH) and nucleon-hyperon(NH) formulations varied widely at the time resulting in a large range of estimates. Some simulations had hyperons occurring at unrealistically low pressures and composing almost the whole star, while others had unreasonably high pressure appearances for hyperons, resulting in them never being able to appear in a neutron star. By the 90's, computational methods and technology advanced to acceptable points to run more adequate simulations. At this point HH and NH interaction models had become more reliable as well. Many of the presently used equations were formulated in the 90's.

The Known and Unknown Equation of States

As mentioned earlier the EOS within the core structure can introduce many problems due to its uncertainty in a variety of factors, mainly involving the matter it is made out of and high density interactions between particles such as hyperons and other nucleons. In order to properly determine maximum mass, stability between gravitational pressure and the pressure pushing outwards must be found. The upper layers, such as the crust, can use relatively well defined equations that don't vary to any great degree. The core can still alter the crust by varying its thickness and structure, but the EOS for the crust mainly varies because of unknown factors in the core. Most of the outer equations are dependent on an unknown inner EOS. Below are the EOS results for the crust provided by Haensel, Potekin, and Yakovlev in *Neutron Stars: Equation of State and Structure [1].* Although the inner crust EOS has more variables that can cause it to fluctuate, it is not at a sufficient density become unreliable until it begins the later nuclear pasta phases that transition to the core.

Table A.1. The EOS of the outer crust derived by Haensel & Pichon (1994). The last line with a nucleus observed in laboratory and present in the ground state of dense matter, as well as the line corresponding to the neutron drip point, are printed in boldface.

ρ (g cm^{-3})	P (dyn cm^{-2})	n_b (cm^{-3})	ρ (g cm^{-3})	P (dyn cm^{-2})	n_b (cm^{-3})
3.303E7	3.833E24	1.991E31	2.091E10	1.938E28	1.257E34
6.592E7	1.006E25	3.973E31	2.533E10	2.503E28	1.522E34
1.315E8	2.604E25	7.926E31	3.315E10	3.404E28	1.991E34
2.625E8	6.676E25	1.581E32	4.174E10	4.628E28	2.507E34
3.305E8	8.738E25	1.991E32	5.039E10	5.949E28	3.025E34
5.239E8	1.629E26	3.156E32	6.619E10	8.089E28	3.973E34
8.303E8	3.029E26	5.001E32	8.337E10	1.100E29	5.002E34
1.0455E9	4.129E26	6.296E32	**9.631E10**	**1.450E29**	**5.777E34**
1.212E9	5.036E26	7.299E32	1.091E11	1.495E29	6.545E34
1.606E9	6.860E26	9.667E32	1.4155E11	2.033E29	8.485E34
2.545E9	1.272E27	1.532E33	1.701E11	2.597E29	1.0195E35
4.166E9	2.356E27	2.507E33	2.096E11	3.290E29	1.256E35
6.606E9	4.362E27	3.974E33	2.730E11	4.473E29	1.635E35
8.031E9	5.662E27	4.830E33	3.325E11	5.816E29	1.990E35
1.011E10	7.702E27	6.081E33	4.188E11	7.538E29	2.506E35
1.319E10	1.048E28	7.930E33	4.299E11	7.805E29	2.572E35
1.661E10	1.425E28	9.982E33	**4.321E11**	**7.857E29**	**2.585E35**

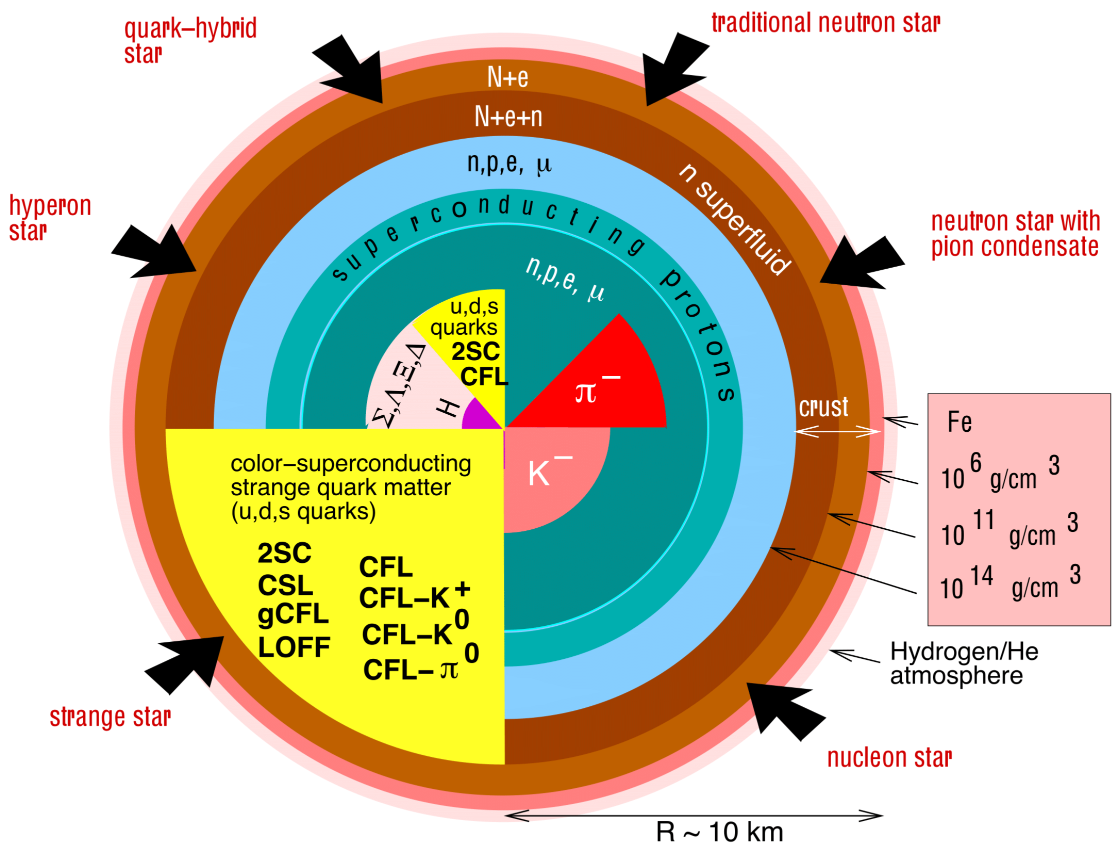
Reference [1] (pg. 518)

Table A.2. The SLy EOS of the ground state of the inner crust, together with the adjacent segments of the SLy EOS of the outer crust and the core (calculated by Douchin & Haensel, 2001). The first and last lines corresponding to the inner crust are printed in boldface.

n_b (cm^{-3})	ρ ($g\ cm^{-3}$)	P ($dyn\ cm^{-2}$)	n_b (cm^{-3})	ρ ($g\ cm^{-3}$)	P ($dyn\ cm^{-2}$)
1.7590E35	2.9398E11	5.0926E29	7.6609E35	1.2831E12	1.3370E30
1.8297E35	3.0582E11	5.3344E29	1.2616E36	2.1141E12	2.1547E30
1.9024E35	3.1800E11	5.5843E29	1.8947E36	3.1766E12	3.4272E30
1.9772E35	3.3052E11	5.8426E29	2.6726E36	4.4827E12	5.2679E30
2.0540E35	3.4338E11	6.1094E29	3.6062E36	6.0511E12	7.7976E30
2.0791E35	3.4759E11	6.1968E29	4.7097E36	7.9058E12	1.1147E31
2.0823E35	3.4810E11	6.2078E29	7.4963E36	1.2593E13	2.0894E31
2.0905E35	**3.4951E11**	**6.2150E29**	1.1197E37	1.8824E13	3.5841E31
2.1604E35	3.6121E11	6.3573E29	1.5999E37	2.6920E13	5.7611E31
2.2306E35	3.7296E11	6.4675E29	2.2073E37	3.7170E13	8.8117E31
2.3114E35	3.8650E11	6.5813E29	2.9477E37	4.9677E13	1.2947E32
2.4014E35	4.0158E11	6.6998E29	4.2684E37	7.2017E13	2.1620E32
2.4997E35	4.1805E11	6.8228E29	6.2200E37	1.0509E14	3.8475E32
2.6426E35	4.4199E11	6.9945E29	7.3174E37	1.2372E14	5.0462E32
3.0533E35	5.1080E11	7.4685E29	**7.5959E37**	**1.2845E14**	**5.3711E32**
3.5331E35	5.9119E11	8.0149E29	7.7100E37	1.3038E14	5.3739E32
4.0764E35	6.8224E11	8.6444E29	9.7100E37	1.6441E14	9.2059E32
4.6800E35	7.8339E11	9.3667E29	1.1710E38	1.9854E14	1.5028E33
5.3414E35	8.9426E11	1.0191E30	1.3710E38	2.3281E14	2.3136E33
6.0594E35	1.0146E12	1.1128E30	1.5710E38	2.6722E14	3.4072E33

Reference [1] (pg. 519)

Since these upper boundaries on the stellar body are well-defined, attention to EOS generally shifts to ultra-relativistic and super dense particle combinations. Several of these are proposed for the core and many of them are considered exotic. Depending on stellar interactions, the particles either stiffen or soften the EOS. In the case of softening there is less repulsion and a resulting smaller maximum mass. Stiffening is the exact opposite of softening. For example, the presence of a large amount of hyperons would soften the EOS and result in a lower maximum mass, and if there were less hyperons it would stiffen the equation resulting in a higher maximum mass. The exotic materials will be listed in order of pions, kaons, and quarks. Below is a picture from *Neutron Stars and Pulsars* (Becker, 2009) showing possible core states.



Reference [29] (pg. 4)

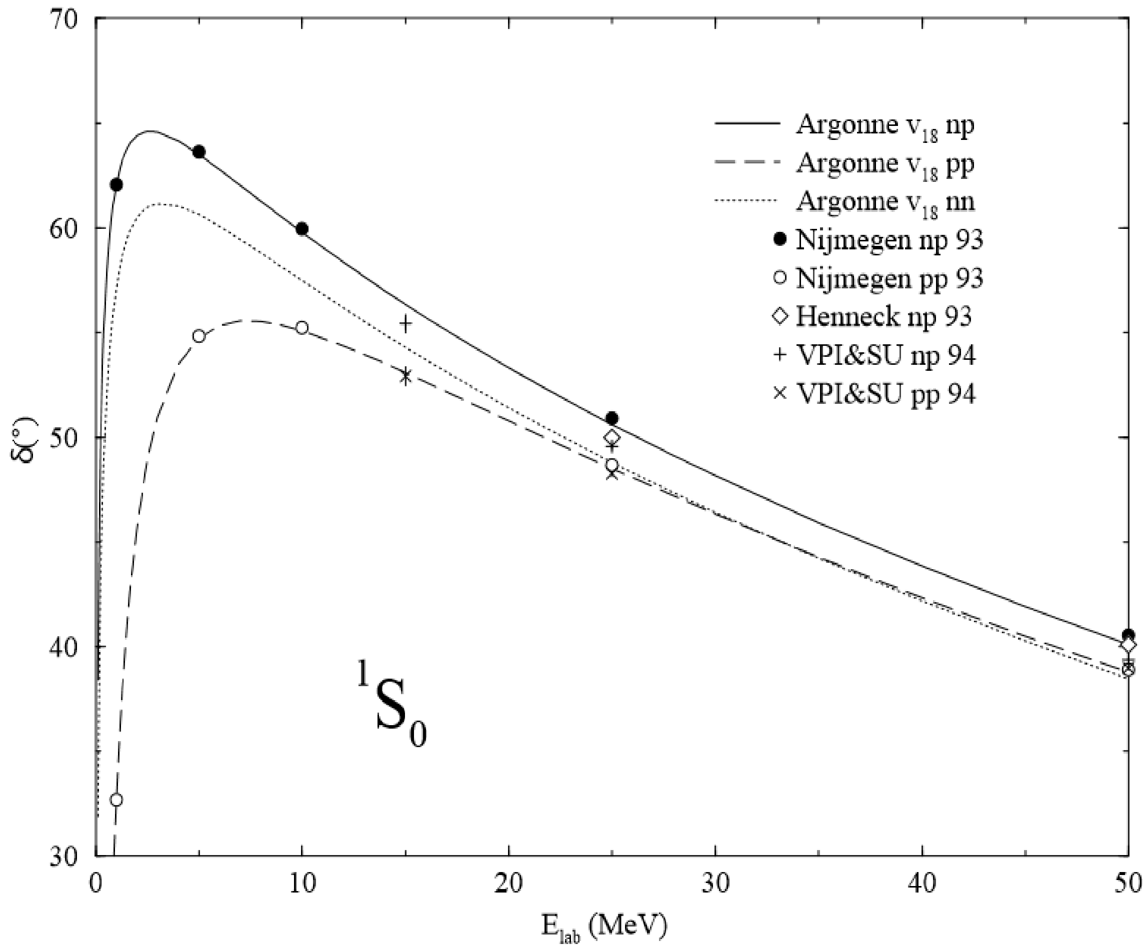
Nucleon-Nucleon Interactions (NN) and (NNN)

In order to understand where NN and NNN models stand today it is important to review a brief how strong and nuclear forces have been interpreted in the past. The main control factors in EOS for neutron stars are these models so their accuracy is essential. Initial models were first created in the 50's and 60's and involved meson interaction. It was initially thought that the meson could explain NN interaction and that it was the main conveyor of forces between particles. While it did accurately fit experimental data in the past and modern era, it only does so for low energy situations due to the meson not actually being the particle responsible for these forces. Many theorists tried to integrate the pion into their equations, but failed to produce accurate models. In addition to these meson-nucleon interaction models, multi-interaction models were attempted as well. Due to the higher energies and complexities, these models became inaccurate and it was clear that a breakthrough was needed to progress interaction models. Interestingly enough the Bonn and Paris potentials included pion and multi-boson exchange and were the select few to survive to more modern day models. Oftentimes portions of them will be used in the lower energy sections of neutron star EOS.

The introduction of quantum chromodynamics (QCD) and chiral symmetry revolutionized the way NN and NNN interaction was handled. QCD was introduced as a multi-body interaction of the quark building blocks of nucleons and interactions with gluons. Early models simply tried to integrate the concept of QCD and did not use it fully due to limitations of understanding. With the addition of chiral symmetry, the well-known models of today began to evolve. The fell model integration of chiral symmetry is often called chiral effective field theory (chiral EFT). Attributed to Goldstone, in this theory for massless particles chiral symmetry must be observed, which is to say each, in this case quarks, must be in one state or another. Quarks are small enough in mass that the concept of chiral symmetry can be applied. However, due to particles of similar mass not show this symmetry it is considered “broken” in the case of quarks. Normally, for each broken symmetry generator a physical particle of 0 mass and spin must be contained, but quarks are not really massless so a very light particle must result from the perturbations of this breaking. It turns out, ironically, that the resulting particle responsible for particle interaction is a pion due to its super light mass. The reason why the initial models of pions did not work was due to the non-existence of QCD and chiral symmetry. Essentially all modern models are based off of these concepts. Because this paper is not solely focused on these forces, I will forego a lengthy conversation of down, up, strange, charm, bottom, and top quarks, as well as topics such as quark color and flavor. The conversion theory is rather complex, but a brief summary of QCD and chiral symmetry is contained in [30].

The two main models used in NN and NN are the Argonne series and Urbana series. Both are based off of chiral EFT and QCD. The earlier versions of Urbana and Argonne both try to fit neutron-proton (np) data. The most current version of Argonne (A v18) updates its previous np data. Some models were based off of scattering and energy data, but many used a baseline of np or proton-proton (pp) data. That is to say, they would fit their data to only one line and this would result in skewed results for the other one. Since Argonne v14 was based on np, its pp estimates were not accurate. In order to fix this A v18 integrates data from SAID, which is a program that uses accelerators to scatter various particles. Of particular interest in NN scattering is deuterium due to its two nucleons. Using this data, phase shift energies can be calculated. In the graph below, it can be seen that widespread integration is attempted by the A v18 model. Not only is the A v18 used to obtain pp and np interactions, but it also includes neutron-neutron (nn) interactions.

Fig. 2b



Reference [22] (pg. 43.-Fig. 2b)

The A v18 also has a set of over forty adjustable variable and can be extended to NNN interactions. Many of its processes are somewhat similar to the Urbana model, but Urbana uses Helium 3 interactions to reproduce its binding energies. This allows Urbana a direct NNN interaction with which it can construct its model. However, there are still present problems such as Neutron-Deuterium (nd) scattering and pion-nucleon interactions beyond just chiral symmetry breaking. Current experiments try to obtain more accurate scattering data with more complex particles that have short stability times. However no massive breakthroughs have been made such as the chiral symmetry and QCD case.

Hyperon-Nucleon(HN) and Hyperon-Hyperon (HH) Interactions

Hyperon-Nucleon(HN) and Hyperon-Hyperon(HH) are the two models of hyperon interaction. For now, the field of physics is relegated to these two particle interactions. Although it would be optimal to know Nucleon-Nucleon-Hyperon(NNH), Nucleon-Hyperon-Hyperon(NHH), and Hyperon-Hyperon-Hyperon(HHH) that is a pipedream for now considering that HN and HH are still just experimental and are very poorly known. To try to extrapolate further when the base is unknown is almost useless. For now there is at least some understanding of HN and HH. Unlike NN and NNN models, there is not a universal set of interactions that are used. Many will just splice in different interaction models depending on what works best. Current progress is mainly based on understanding of QCD, chiral symmetry, and the ability to produce hyperon scattering data. Part of the problem lies in the instability of hyperons making it difficult to study them for scattering data due to the fact that they can only last about 10^{-14} seconds in their most unstable forms.

Equation of State for the Core of a Traditional Neutron Star

Before the exotic states are presented it is important to question that maybe nothing exotic happens at superdense levels. There are many models present at this level that are dependent on nucleon and hyperon interactions. These models can produce maximum masses anywhere from 1.4 solar masses to 2.5 solar masses depending on the interaction of particles and whether or not one determines hyperons to be present at certain levels. Although the handy sheet provided below compares a variety of EOS trial results in *Neutron Stars: Equations* *of State and Structure [1]*, they are not expounded upon as to how each one differs in its determination of EOS. Every EOS except BPAL12 will be covered due to BPAL having a somewhat inferior model in terms of NN interactions and its resulting maximum mass.

EOS	M_{max}	R	r_g/R	n_c	ρ_c	$E_{bind}^{(Fe)}$
	$[M_\odot]$	[km]		$[fm^{-3}]$	$[10^{15}\ g\ cm^{-3}]$	$[10^{53}\ erg]$
BPAL12	1.46	9.04	0.478	1.76	3.94	3.19
BGN1H1	1.64	9.42	0.516	1.59	3.71	3.82
BBB1	1.79	9.67	0.547	1.37	3.09	5.26
FPS	1.80	9.27	0.572	1.46	3.40	5.37
BGN2H1	1.82	9.54	0.564	1.46	3.51	4.83
BBB2	1.92	9.50	0.596	1.35	3.20	6.17
SLy	2.05	9.99	0.605	1.21	2.86	6.79
BGN1	2.18	10.9	0.591	1.05	2.46	7.28
APR	2.21	10.0	0.651	1.15	2.73	9.13
BGN2	2.48	11.7	0.626	0.86	2.02	9.40

Reference [1] (pg.290)

Main EOS Table

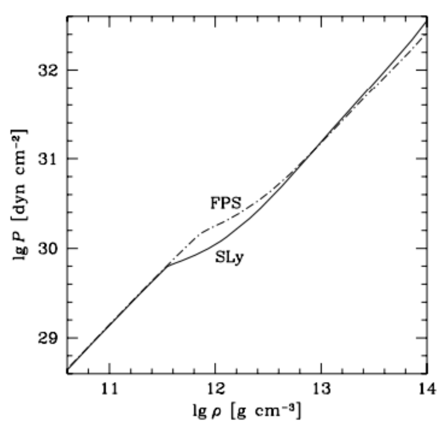
EOS	model	reference
BPAL12	$npe\mu$ energy density functional	Bombaci (1995)
BGN1H1	$np\Lambda\Xi e\mu$ energy density functional	Balberg & Gal (1997)
FPS	$npe\mu$ energy density functional	Pandharipande & Ravenhall (1989)
BGN2H1	$np\Lambda\Xi e\mu$ energy density functional	Balberg & Gal (1997)
BGN1	$npe\mu$ energy density functional	Balberg & Gal (1997)
BBB2	$npe\mu$ Brueckner theory, Paris NN plus Urbana UVII NNN potentials	Baldo *et al.* (1997)
BBB1	$npe\mu$ Brueckner theory, Argonne A14 NN plus Urbana UVII NNN potentials	Baldo *et al.* (1997)
SLy	$npe\mu$ energy density functional	Douchin & Haensel (2001)
APR	$npe\mu$ variational theory, Argonne A18 NN plus Urbana UIX NNN potentials	Akmal *et al.* (1998)
APRb*	$npe\mu$ variational theory, Argonne A18 NN with boost correction plus adjusted Urbana UIX* NNN potentials	Akmal *et al.* (1998)
BGN2	$npe\mu$ effective nucleon energy functional	Balberg & Gal (1997)

Reference [1] (pg. 263)

The Sly and FPS EOS

The Sly and FPS EOS are very effective formulas and their outer EOS for crust is used by many other EOS formulations. The next section discussing Balberg and Gal reveals that they used the Sly EOS for outer core layers as well. It is relatively pointless to recalculate outer layers when there are already decent calculations, so oftentimes EOS formulations will differ wildly within the more superdense layers and agree to a large extent on outer

Reference [1] (pg.153)



layers. The main difference between these two within outer layer is the neutron drip line, but beyond that their transfer points from drip line to the inner core are very similar as seen to the right. The EOS between the two align, except for on the outer ends and at the drip line. Regardless of which EOS is used for outer layers the final result will not have large deviation from the other model.

Both FPS and Sly rely on matching equations for different transitions in the star. In other words, there needs to be a smooth transition without discontinuities from crust to core, drip point to pasta phases, and so forth. However, forcing the model to work can sometimes create uncertainties and it is sometimes better to go with the latest experimental data in super dense areas regardless of if they match up perfectly. In addition to these properties, the Sly has a rather unique value in that it has a adiabatic index as a weak function of density indicated by Haensel as equal to $\frac{Nb}{p} \cdot \frac{\mathrm{d}P}{\mathrm{d}Nb}$. Haensel describes this function as a "smooth dependence on neutron and proton number densities" This property is also present in BGN1 and BG2.

In further exploration of these two EOS it can be noted that nothing is exceptional about core composition. It is a regular npeu (neutron, proton, electron, muon) combination and no hyperons or exotic particles are considered. The Sly is slightly stiff and the FPS is slightly soft, but overall they lie in the middle of maximum mass results. Neither make any unrealistic assumptions due to their need of smooth transitions.

APR and APRb

These two EOS formulations were completed by Akmal in 1998. The full paper that included descriptions of the formulations referenced in this paper are from 2008 however. Akmal uses several corrections that build upon each other in his models. Previous models used older NN interactions to explain nucleon-nucleon interactions, but these models saturated at certain densities and didn't interact well with Helium-3 and other such elements. Furthermore, nucleons interact with the building blocks of other nucleons and have neighbor-neighbor interactions building upon one another to create 3 or even 4 levels of freedom for interaction.

The three symbols of A18, UIX and δυ in the chart below are the three methods used by Akmal. Using variational chain summation with many-body correlation effects, Akmal melded these into EOS. A18 is the model of NN interaction that is used. It is more updated from the previous A14 in regards that it fits nucleon curves better. UIX, sometimes referred to as TNI, is the three nucleon interaction that is used. Its full name is the Urbana three nucleon interaction model and, as stated earlier, it drastically improved estimation of nuclear material interactions when it was released. Optimally Akmal would like something along the lines of a NNNN interaction formula, but such a thing is beyond accurate estimation for now. The final component used was a relativistic boost correction that was applied to NN interaction. Akmal words it as a simplified equation of $\upsilon(P_{ij}) = \upsilon_{ij} + \delta\upsilon(P_{ij})$, where P is the momentum between two nucleons and v is the interaction levels for when P is 0. The omega function is the boost correction, but only applies when P is not 0. The i and j are the correlating nucleons in the interaction. The result of this boost is that when momentum is not 0 a changing of binding energy in symmetric nuclear matter occurs and the repulsive force in NN interactions is altered. The contributions are not trivial from this boost and the fraction of contribution increases with pressure. This would make sense as the matter becomes more relativistic the greater the pressure becomes.

Using these results Akmal applies this to T=0 npeu matter. It is put through a fitting procedure that tries to smooth functions of transition and density. Once again, components of FPS and Sly are used in some of the calculations. The table below this section shows the states that have been calculated. The A18 interaction is the softest, with the next softest being the A18 with the boosted formulation. The next EOS gains significant repulsive forces form the NNN interactions and as a result has a much higher maximum mass. The model with all three at the very top of the chart actually has a lower mass because the boost at this point makes it so that there is less repulsion rather than more. Akmal seems to favor this model, which is the APR model displayed in the main chart at the beginning of the EOS section.

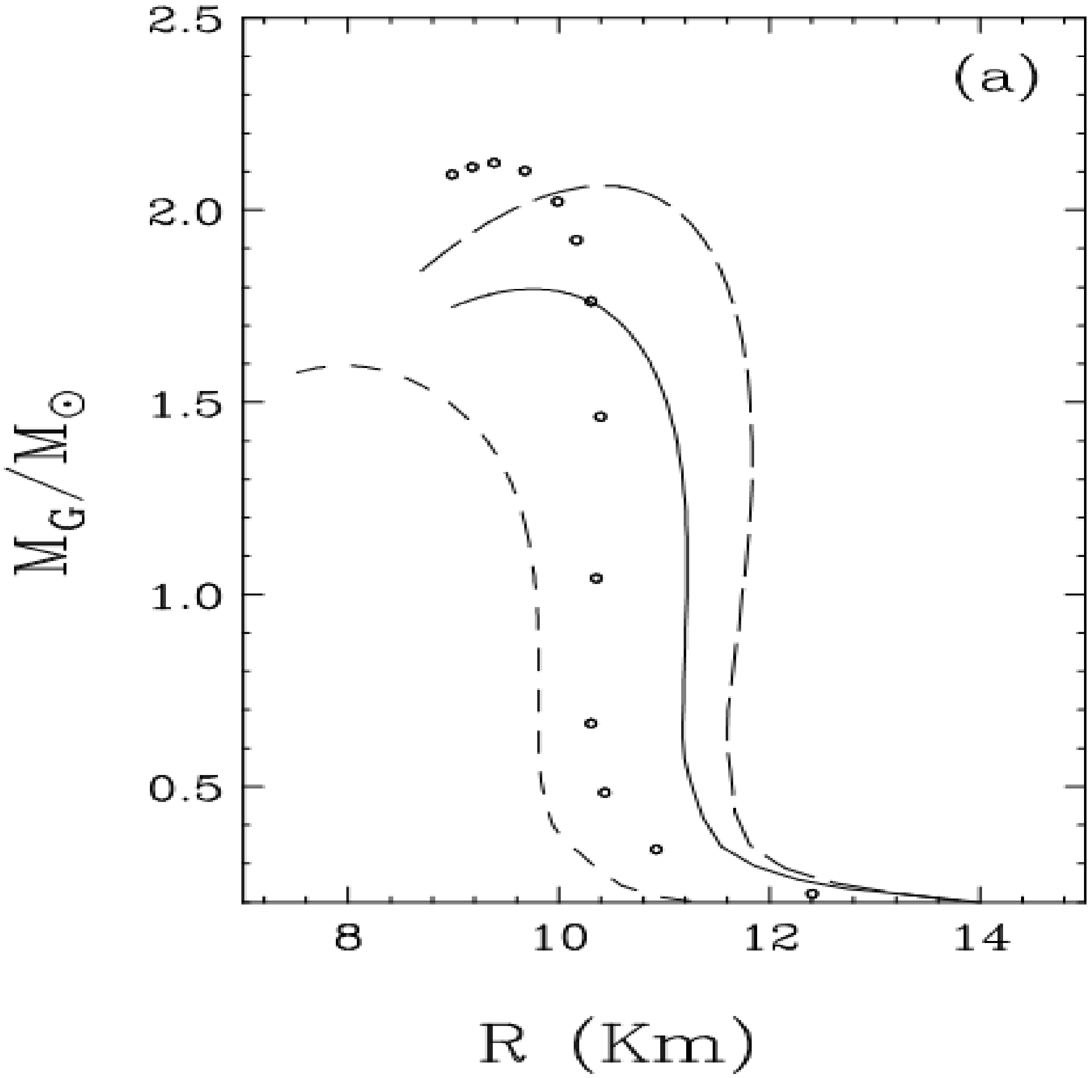
Other notes by Akmal relate to the thoughts of strange states and the absolute maximum mass of neutron stars. Akmal admits that if a mass above 2 solar masses was determined for a neutron star it would rule out models without NNN interaction. However, several states can still exist above that. Superluminality in a neutron star is an issue when matter becomes more and more compressed. There must be a maximum level of compression or else the speed of sound would overtake the speed of light at some point in the matter. Akmal assumes the speed approaches the speed of light at its maximum. It turns out that the effect is relatively small, as current theories put this matter in only the heaviest of stars near the center. Not much of this matter exists in the star so it doesn't add much to the EOS. If however, some exceptions allow it to occur lower, a significant portion of the star could consist of this incompressible matter. Akmal puts this absolute maximum mass barrier right around 2.5 solar masses. In addition to superluminality, Akmal also does address pion and kaon condensation to a very brief degree, and expounds upon quark states to a significant degree. In regards to hyperons, Akmal treats them as more exotic particles and does not include them in the below table.

NM models	Max. mass beta-stable	Max. mass PNM	Max. I beta-stable	ρ_I of INC Models	Max. mass beta-stable	Max. I beta-stable
A18+δv+UIX*	2.20	2.21	115.	0.32	2.92	261.
A18+UIX	2.38	2.39	143.	0.48	2.46	157.
A18+δv	1.80	1.81	67.	0.64	2.26	123.
A18	1.67	1.68	55.	0.86	2.19	115.
FPS	1.80	-	73.			
NM+QM models	**B**	**Max. mass beta-stable**	**Max. I beta-stable**	**B**	**Max. mass beta-stable**	**Max. I beta-stable**
A18+δv+UIX*	122	1.91	96.	200	2.02	107.
A18+δv	122	1.74	66.	200	1.76	67.

Reference [18] (Physical Review C pg. 1823)

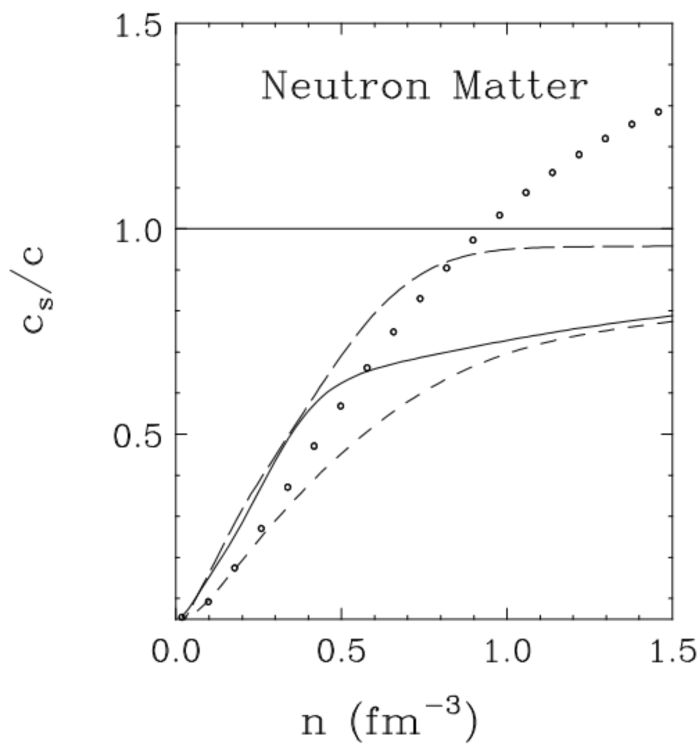
BBB1 and BBB2

The initial BBB1 model was constructed by Baldo, Burgio, and Bombaci. They used a microscopic EOS with asymmetric matter as their assumed neutron star material. It is a standard npeu construction that is held in equilibrium with weak interactions. Rapid cooling regardless of superfluidity in core is assumed due to Urca Potential levels and NNN interactions are introduced in both the BBB1 and BBB2 model. The Urca Process consists of neutron and proton/electron interactions that produce neutrinos that are free to leave the star. In addition to this aspect, the three body forces in the BBB1 model are not taken from one model, but combine various models in order to meet the correct saturation point and binding energies of particles in the neutron star core. Baldo, Burgio, and Bombaci did not have a single NNN potential that fit all these requirements so several of them are melded together at certain points. The interaction models they use are called BFH followed by a number based on interaction levels and they were specifically designed for EOS approximations of symmetric matter. They used Urbana XII interactions and A14, which were slightly older versions than Akmal used, but as stated they did not stick to just one model so it is not a definite conclusion that Akmal has a better model than BBB1 and BBB2. Their final resulting model gave a maximum mass of 1.794 solar masses and is shown below. They compared various other models on the same graph to display how the adding or subtracting of certain forces would interact with their model.



Reference [19] (pg. 14)

The main BBB1 model is the solid line that peaks at 1.794 solar masses and uses a BHF3 (3 body model) and the dashed one below that uses the BHF2 resulting in a much softer matter due to the exclusion of any NNN interactions. The higher dashed line follows the solid line at less dense levels and uses a model called DBHF, otherwise known as the relativistic Dirac-Brueckner model. Unlike the BHF3 model it is repulsive at all densities in the star, whereas the



Reference [19] (pg. 12)

BHF3 includes both attractive and repulsive forces. The uppermost circular model is a variatonal model, but due to previously mentioned restrictions in sound speed it would not work. The model to the left shows how with increasing density the variatonal model's sound speed crosses the light speed barrier, making it impossible. The other models are not likely due to their incomplete nature of NNN interactions. The main BBB1 model eventually was increased by about .1 solar masses in the BBB2 model due to the inclusion of Paris NN interactions and other adjustments to the NN interaction levels resulting in more repulsion.

BGN and Balberg and Gal

First, one may notice that BGN has four separate indicators on the table listed at the beginning of the EOS section in this paper. All of these are produced by Balberg and Gal in 1997 and offer valuable insights to the variety of maximum mass within a similar equation. In fact, these EOS cover almost the whole range of proposed maximum neutron star masses. It is important to note that the models have ranges and listed above are only the maximum ends of the estimations since those are the values of interest. Both of the BGN models that have a H1 following it in the table at the beginning of the EOS section are the modified versions that include strange bosons, namely hyperons. Because hyperons are a much more accepted assumption than other superdense particle phases such as quarks, kaons, and pions, the hyperon model is included under the more typical grouping of neutron stars. It can be noted that the softening on the EOS can be seen via addition of hyperons. Both BGN models have lower maximum masses than their counterpart non H1 EOS. Current understand holds that due to the Pauli Principle involving neutrons and electrons, that at a certain threshold pressure it will be favorable to form hyperons. This energetical favorability results in a lot more massive particles in the form of hyperons. These hyperons are also much slower and as a consequence result in this softening. It is thought that Σ^- hyperons would form first, as the conversion promotes negative charged hyperons, but EOS often vary in their assumptions due to the poor understanding of NH interactions. Balberg and Gal discuss much of this in their paper, *An Effective Equation of State for Dense Matter with Strangeness [5].*

As pressure increases in a neutron star NN interaction increases and neutrons are smashed together. Eventually, this buildup of pressure combined with the presence of leptons has to transform to something else. At the same time, charge neutrality must be maintained in the core composition. At outer points this is maintained by electrons and muons with their negative charges, but if they are to be replaced it must be with an appropriately charged particle as well. Unfortunately Balberg and Gall still have uncertainty in their EOS due to the fact that NN interactions dealing with nucleon incompressibility are not well known. Because hyperons are born from weak interactions that result due to compressed baryons and then are maintained via strong interactions this presents an area where estimates must be made. In addition to this, HN interactions are also still not well known so it is not possible to entirely guess what will happen when a species of hyperon is introduced to high pressure nucleons.

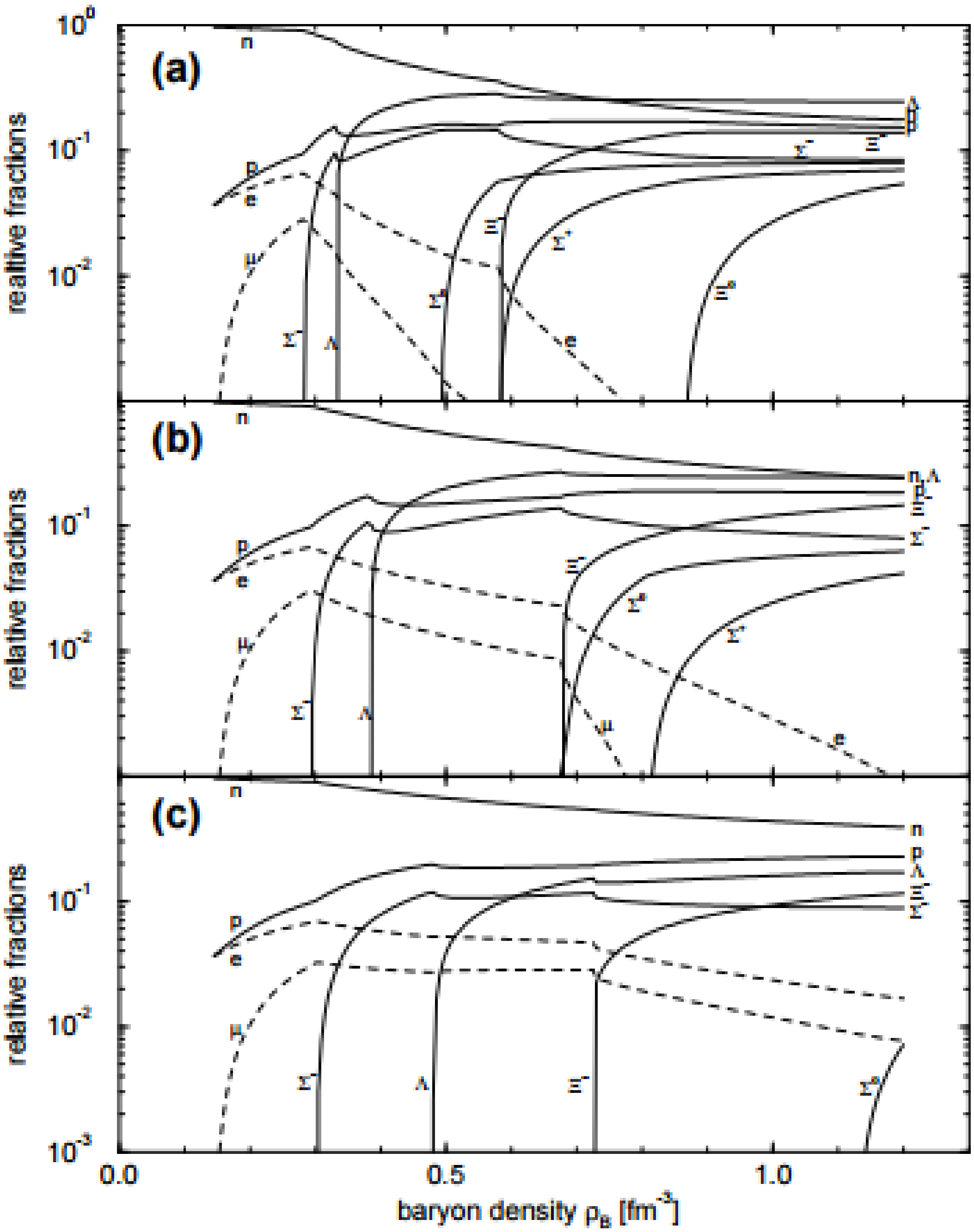
As a result, several models were constructed by Balberg and Gal and not all of them are listed in main EOS table for this section. Many of the differences in hyperon models are due to which species appear first and in what quantities they can appear if they appear at all. For example, one large divide between hyperon models is the inclusion of Σ^-. It is a promising candidate for a first appearance due to its light hyperon mass and the fact that it can maintain charge neutrality by replacing electrons and muons with its negative charge. The only problem is that some studies have shown compelling evidence that this particle is possibly repelled in nuclear matter in the neutron star core. Balberg and Gal considered this important enough to create a divide in charts. If one notices above the BPAL charts do not even have this particle listed in the main EOS table. In those models it was deemed that Σ^- could never be formed in neutron star cores. Other models may just decide to push it up in density appearance instead of just getting rid of it. If the model does maintain the particle though then the Σ^- will appear and this will coincide with deleptonization, a process which rapidly lowers the levels of leptons in the star. There are a variety of adjustable variables when it comes to hyperons and nucleons and depending on their value saturation levels can change. This means that Σ^- levels will not keep on growing until all the leptons are gone. It can only compose a certain portion of the matter before the positive charged version of it must appear due to charge dependent interactions. However, these new particles also reach saturation levels and new species of hyperons must appear to accommodate charge neutrality and neutron excess. The next particle Λ appears until repulsive multibody forces cause them to reach saturation levels. Production of Ξ^- particles then take over because they are still considered favorable due to negative charges. Unlike Σ^- however, Ξ^- does not have charge dependent interactions and at higher densities it will overcome the fraction of Σ− in the nuclear matter. In the non- Σ− EOS it skips this first production phase. Nucleon repulsion plays a large role and if it is weak then less hyperons will be produced. If it is strong then more are produced and if HN interaction is very powerful then Ξ- can appear first and more neutral particles can be produced earlier.

Regardless of which model is used, Balberg and Gal clearly note that at higher densities NN interactions are beyond acceptable values and partial densities must be lowered somehow. While they do mention pion condensates, kaon condensates, quark stars, strange states, and even more abstract concepts they do not produce full models and mention them in only moderate detail. In later papers they expound upon such states, but detailed EOS are not graphed out because HN equations are not very well known, let alone HH interactions. Such models were not worth detailing due to such high levels of uncertainty, but it is worth speculating what changes they can cause in further sections.

Overall, a baseline EOS is needed for the addition of hyperons, and the way hyperons effect the EOS is particularly interesting in the fact that it is mainly dependent on species. First of all, it is important to remember the softening effects that hyperons have on the EOS. As kinetic and potential energy is transferred in the form of mass to the creation of hyperons the partial density lowers and so does the repulsion force in the nucleon matter. More importantly, hyperons are described by Balberg and Gal as “pressure control”. They play this role in an area where they can exist in a stable state, which is nuclear matter in dense conditions. The pressure plays a major role in equation of state particularly because EOS is sometimes described as the balance of internal star pressures that push outwards and gravitational pressure. The order of this pressure relief is dependent on species interaction and nucleon interaction. The best way of putting it is that as pressure reaches its maximum bounds a species of hyperon must appear to dissipate the imbalances in the matter and depending on when or how much is allowed to form it will affect the amount and formation density of next hyperon species. These factors act as delay mechanisms in the appearance of the next species. The good news is that most hyperon species effect the pressure control somewhat similarly and therefore effect the EOS to the same degree. So while each species does have its own EOS interactions, the pressure control system is actually mainly species independent. This leads to EOS formulations that do not deviate in extreme degrees, but do still have their differences based on previous appearance patterns that have been described. The previous listed properties of neutron stars, involving, but not limited to strong magnetic fields, superconductivity vortexes, superfluid behavior, and crust properties are all fields that Gal and Balberg admit must be studied to understand hyperon composition.

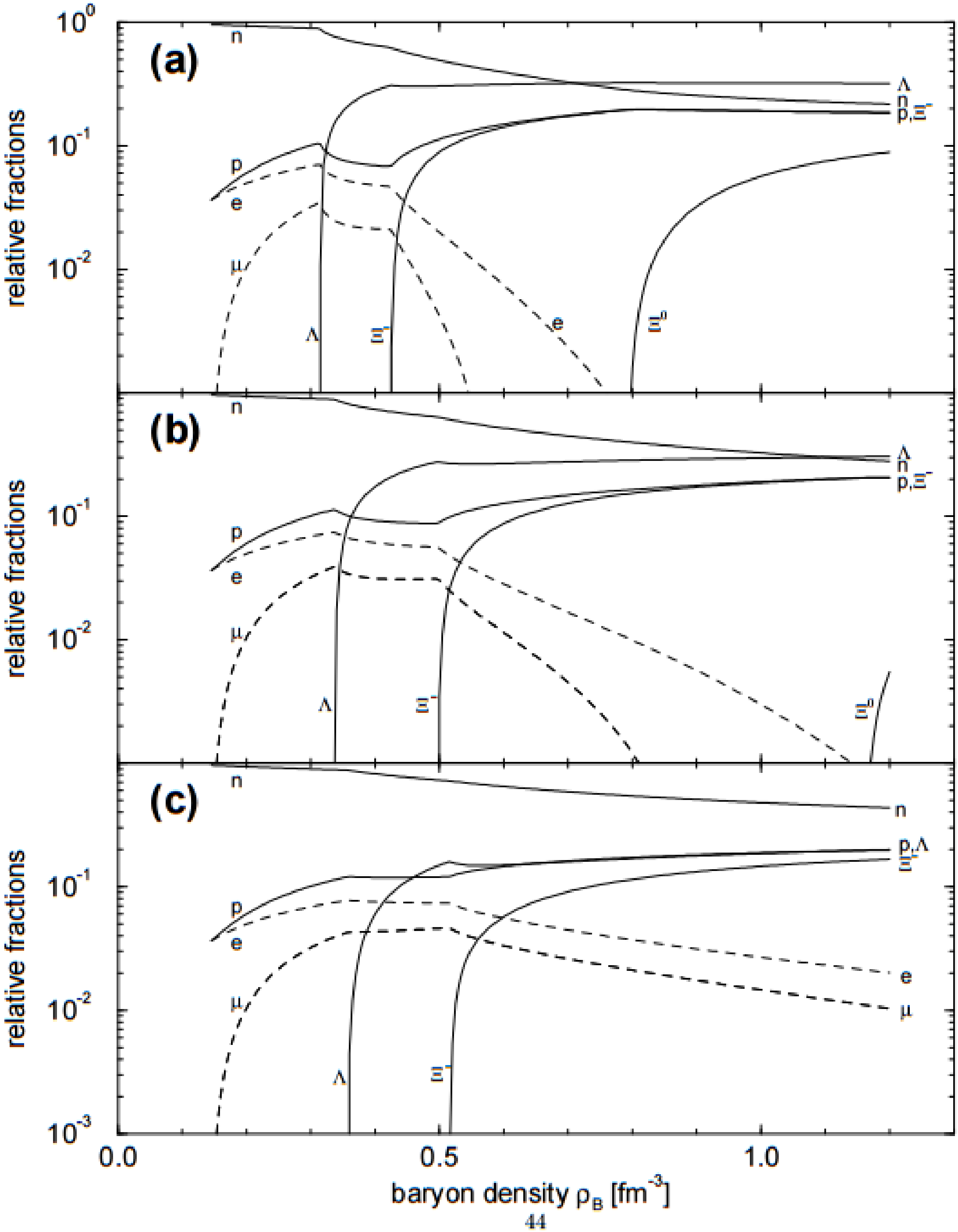
Two models are displayed on the next pages that graph the fraction of hyperon particles at various densities. The hyperon levels can be adjusted to get different maximum mass levels from stiffened and softened EOS. The first page has Σ^- interaction while the second has none. Note the drop of e and u levels when deleptonization appears in both of the graphs. The drop off levels in muons and electrons occur the most whenever a new negative hyperon is introduced to the core material in order to keep charge neutrality. As particle appearances are shifted upward in pressure some are pushed off the chart. The appearance levels of these different hyperon species is what controls the EOS formulations. Of course, actual levels could be between these levels anywhere. These are just levels picked by Balberg and Gal that represent levels near the minimum, maximum, and intermediate appearance levels. Each particle still reaches a saturation level, and although they seem relatively equal keep in mind the graph is exponential so they actually have a fair amount of discrepancy, but not to an extremely large degree. Appearances of new hyperon species and rapid deleptonization levels can also cause spikes in fractions which are also displayed in the graphs.

One final note on Balberg and Gal is that some models may actually be able to fit proto-neutron star material better especially regarding ones that don't involve hyperon material. The volatile actions in proto neutron stars and their opaqueness to neutrinos are not good environments for hyperons so it is more likely that npeu models of Balberg and Gal could be related to earlier neutron star matter. Granted, proto-neutron stars do not last a long amount of time, and while somewhat valuable to the determination of maximum neutron star mass, they are hard to obtain more concrete data on as they are in the middle of a stars supernovae material.



Reference [5] (pg.43)

figure 4



Reference [5] (pg. 44)

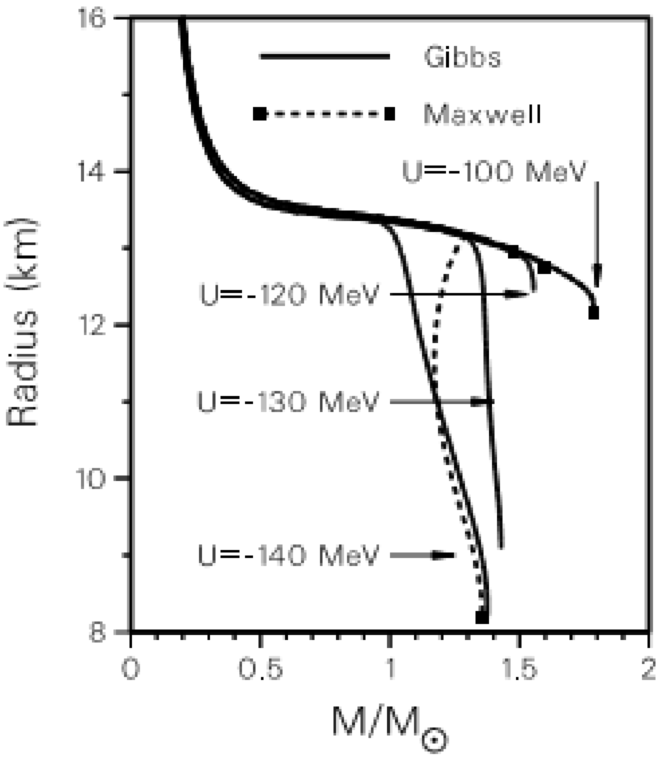
The Pion Core and Pion Condensation

Pions consist of a quark and an antiquark. They are defined as mesons and because they are built out of very light up, down, and anti-up/down quarks they are considered the lightest mesons, and hadrons. Under normal situations a charged pion, indicated by a π^- or π^+ , lasts for about 10^-7 seconds with a neutral pion lasting several powers less than that. They are assumed to play crucial roles in the strong force so having a lot of these in the core with any nucleons present is going to cause some huge interactions.

When pressures rise in the stellar body chemical potential levels of electrons begin to increase. As this grows greater than the effective mass of negative pions, pions are created and begin to form into Bose-condensation consisting of pions. It is important to note that s-wave interactions involving pions and nucleons can increase the effective mass while pressure is increasing, which is one of the causes of disputes as to if or where this creation of large amounts of pions would occur. In this situation s-wave interactions are simply referring to short-range repulsion forces and long distance attraction forces. Despite the dispute over density occurrence, the large amount of bosons in the form of pions will arrange themselves to occupy a single mode and with the addition of p-wave interaction forces will form a Bose-condensate. This results in massive particles that are all added to a state of low momentum. In turn this softens the EOS and results in a lower maximum mass. The most likely candidate of pion is a negative pion consisting of a down quark and anti-up quark. They are typically produced with a proton pair through neutron-Nucleon formations. It is also possible, but less likely that neutral pions could form as well. Very few EOS have been done for a pion condensate neutron star and its presence would soften the EOS to around 1.7 solar masses, but can vary depending on how much is present.

The Kaon Core and Kaon Condensation

The concept of a kaon (K-) core is relatively new due to the fact that they are so much more massive than pions. Pions were considered the most likely candidate for a condensate core in part because of their light properties. Kaons are still mesons and bosons just like kaons and do fall under the consideration of a single state condensate just as pions do. However unlike pions their effective mass is lowered instead of heightened due to strong baryon interactions. This pushes them into a potential candidate range for a core. Depending on attraction levels of K- particles it could be the most efficient particle to appear and maintain charge neutrality. The negative, or antikaon, is the most likely particle to do this out of all kaon particles, but neutral charged kaons have been mentioned as a possible candidate as well. As soon as the effective K- potential is equivalent to the electrochemical potential interactions will begin to form kaons. Negative kaons can be created from interactions between electrons and a partner nucleon or a neutron with a partner. In the case of an electron interaction the charge neutrality is kept with the production of the negative kaon. If it is formed from a neutron interaction the charge neutrality is maintained with a proton and negative kaon. The ending K- particle consists of a strange quark and an anti-up quark.



Reference[6] (pg.18)

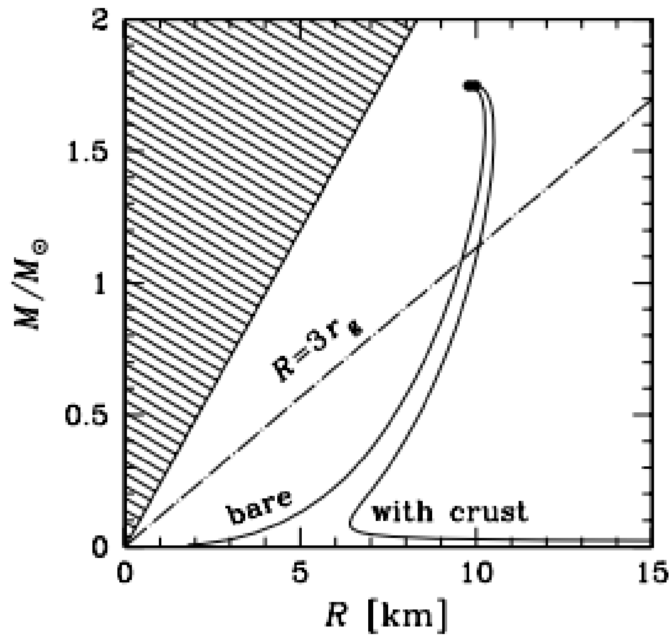
The chart to the left shows possible radius limits with mass of a kaon condensate star. Several likely potential levels are listed that encompass a range of 40MeV. The majority of these are carried out with the Gibbs method as the Maxwell one creates some inconsistencies. While the low estimate gives 1.35 solar masses the higher end gives about 1.8 solar masses. There are not many models that estimate a much higher maximum mass than this if the star is a pure kaon-condensate star. The kaon-condensate softens the EOS in a similar manner to the pion condensate and results in a lower maximum mass than a npeu neutron star. Once again, though, it is important to reiterate that Kaon-Nucleon (KN) interactions are poorly understood and are more complex than NN or NNN interactions due to kaons instability outside of neutron star matter. It may be that these potentials are never reached due to these interactions and kaons do not form in the core. Furthermore, it is also possible that kaons exist with a hyperon transition as well. Few reliable EOS exist for this combination due to the fact that it would require two EOS with already unknown factors to combine and include Kaon-Hyperon(KH) interactions. Hyperonization, or the appearance of hyperons, would undoubtedly effect the rate of production of kaons as there would be less materials for kaons to deal with due to deleptonization and the continual charge neutrality actions performed by hyperon production.

The Quark Core and Strange Stars

The fundamental building block of nucleons are quarks and at a threshold density the nucleons will not be able to support their combined quark state. At this density the nucleon must break apart into separate up and down quarks, or whatever other components it may consist of. The main issue is that the exact density is not known to an accurate degree besides the fact that it is at least on the order of $10^{14}\left(\frac{g}{cm^3}\right)$ at the lowest. Lab conditions have been unable to test such an exotic state of matter and current understanding of superdense matter is not accurate enough to give a reasonable estimate. It is not a question as to whether or not quark states or pion condensate exists, but rather under what conditions and pressures it exists at. If the point where nucleons break down into quark materials is at a higher pressure than when condensate states occur then it will not happen. It is possible to have strange quarks present as well, but any heavier variety of quark would not be stable in the neutron star core.

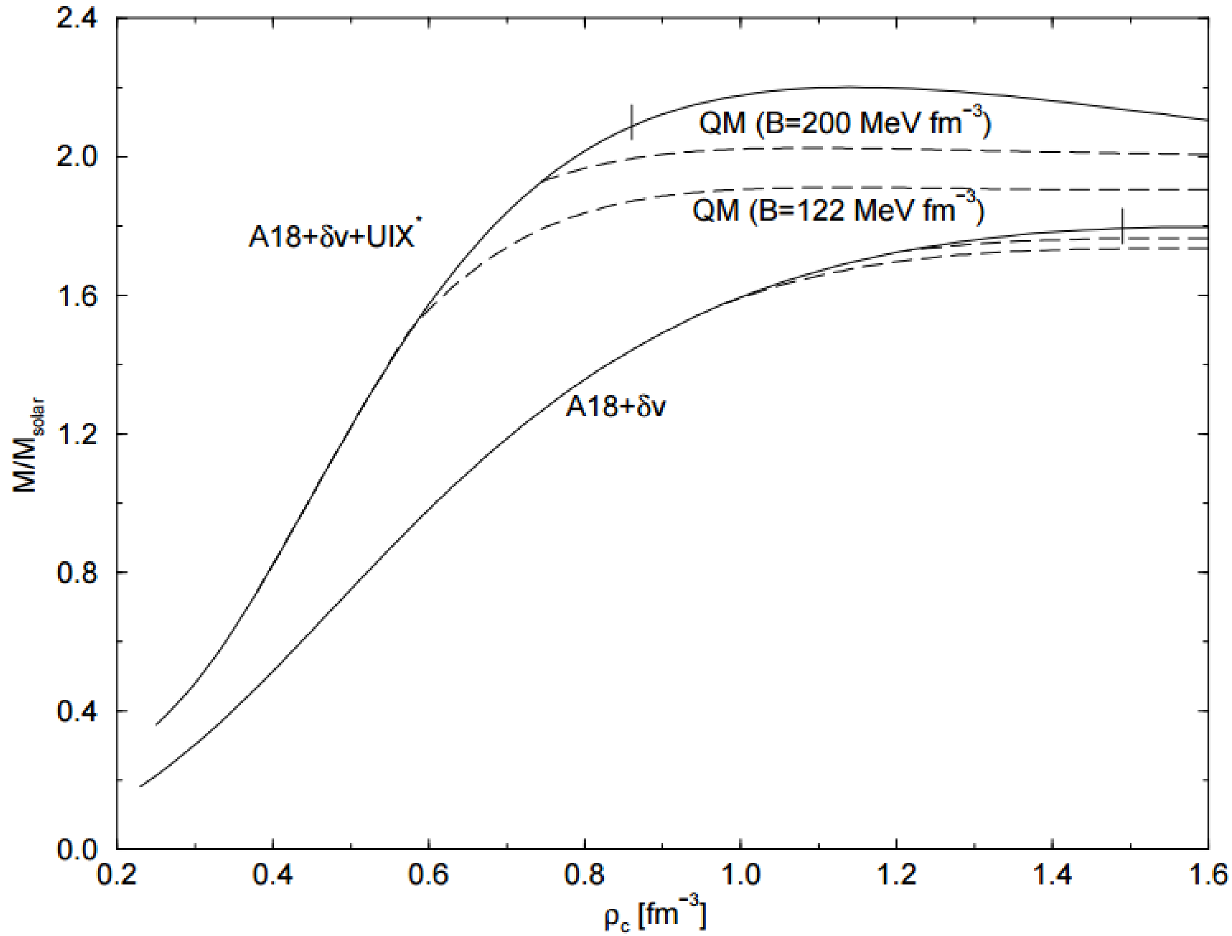
The somewhat expanded version of the quark core, but also strikingly different with the inclusion of strange matter is often called a strange star. Unlike the other proposed materials, strange matter and strange stars are mainly hypothetical. They are still relevant however, as many texts that discuss neutron star structure and mass include them. Whereas states such as quark cores and pion condensate can actually exist, strange matter is simply a proposed state in neutron stars.

Similar to other quark material in less hypothetical neutron stars, strange stars would still not contain the heavier quarks. Under theories of strange matter it is possible for down quarks to convert half of the amount of down quarks to strange quarks. This works via weak force interactions whereas the decomposition of nucleons to up and down quarks happens via strong force interaction. Although strange quarks are inherently unstable under normal conditions they may be able to remain present in neutron stars and, depending on how strange matter exactly works, could compose the majority of the star rather than just the core section. This is due to the fact that normal quark matter would have a very high Fermi Energy making it stable only in the core of the star. With the conversion of many of the quarks to strange quarks the Fermi energy could be sufficiently lowered so that most of the star consists of the bizarre quark matter. It is not of extreme interest to delve into detail for the support for and against this theory, but it is compelling to point out that there are at least a few arguments that point toward strange stars being a credible theory in at least some oddly behaving neutron stars. Therefore, it is at least worth mentioning in a role of neutron star masses, but it is also important to consider that, although they may be born from Type II supernovae, they may exist independently from other neutron stars. This holds in a sense for neutron stars with a quark core as well. They may just be unique classifications and some astrophysicists may not consider them neutron stars while others will. Below is an EOS model for strange stars with and without crust. The without crust model is just in case strange matter can extend all the way to the surface of the star. In this case the star is labeled as a bare star. Modeling was done by MIT and the image is provided yet again by Haensel, Potekhin, and Yakovlev. While a variety of books were used in theory discussion, *Equation of State and Stucture [1]* has superior graphs to many other books so I have used them in many of the EOS displays. As can be seen below, the maximum mass of these models approaches the upper boundaries of the higher EOS models for neutron stars. There are a variety of other models provided, but at the highest boundary they approach about 2 solar masses.



Reference [1] (pg.423)

Below this chart is an example of how Quark matter can effect Maximum Mass. This chart in particular is from Akmal's EOS papers [18]. In addition to his equations for normal npeu material, he also analyzed quark material. In this model, up, down, and strange quarks exist in a 3-color state and are mixed within nuclear matter. The negatively charged quark matter off balances the positive nuclear matter. Rather than forming a density equilibrium, the particles form a charge neutral equilibrium. According to Akmal, this is a much more stable state than having pure quark matter or charge equilibrium material. Depending on the BAG constant that is used, the EOS will soften more or less. Lower BAGs result in lower maximum values and Akmal evaluates this at values of 120 and 200. The previously noted studies of Balberg and Gal also addressed quark material, and while they did not provide detailed charts or EOS in their 1997 paper they did make assumptions using a BAG value of 200 as Akmal uses.

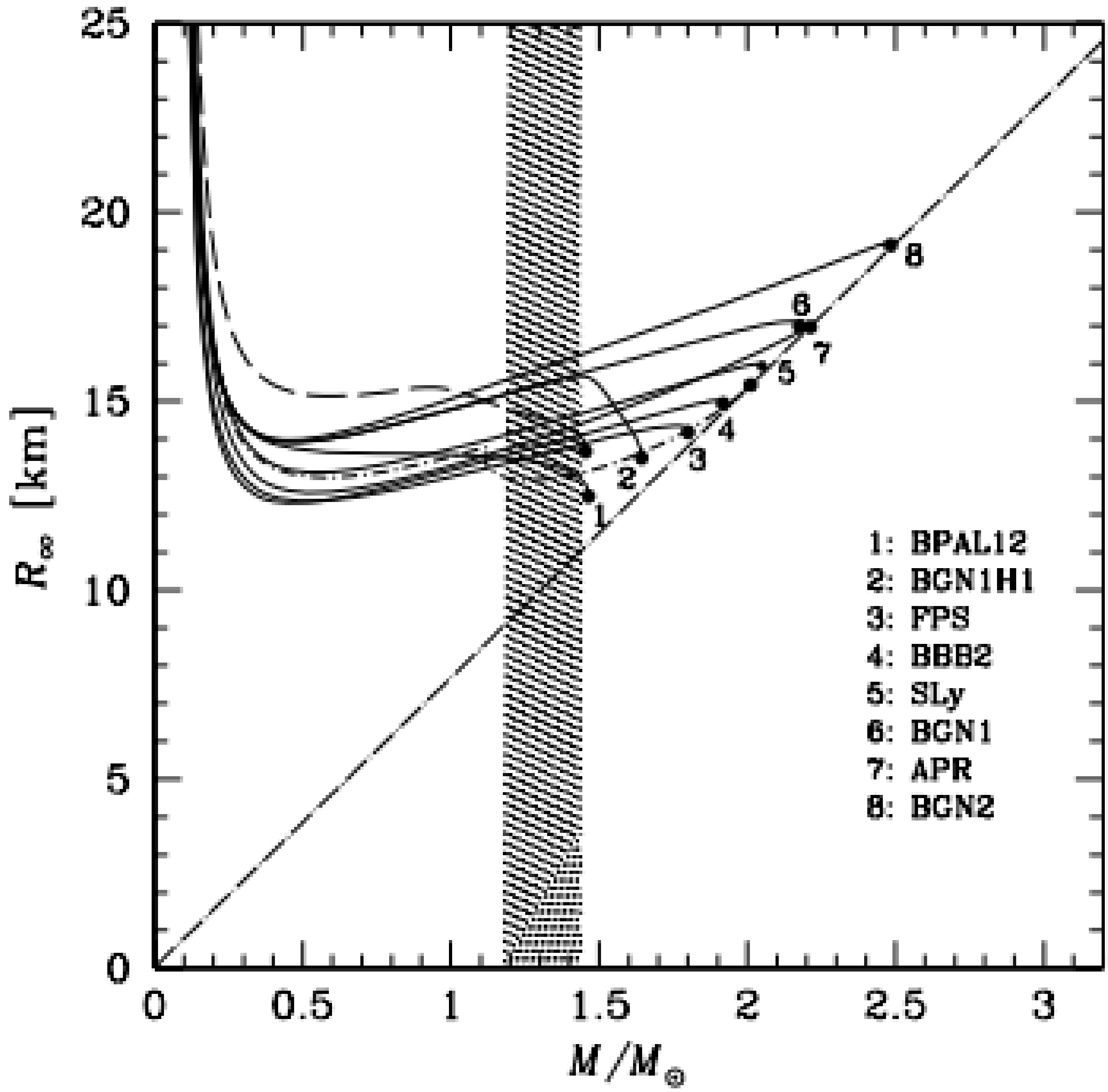


Reference [18] (Physical Review C pg. 1823)

Conclusion

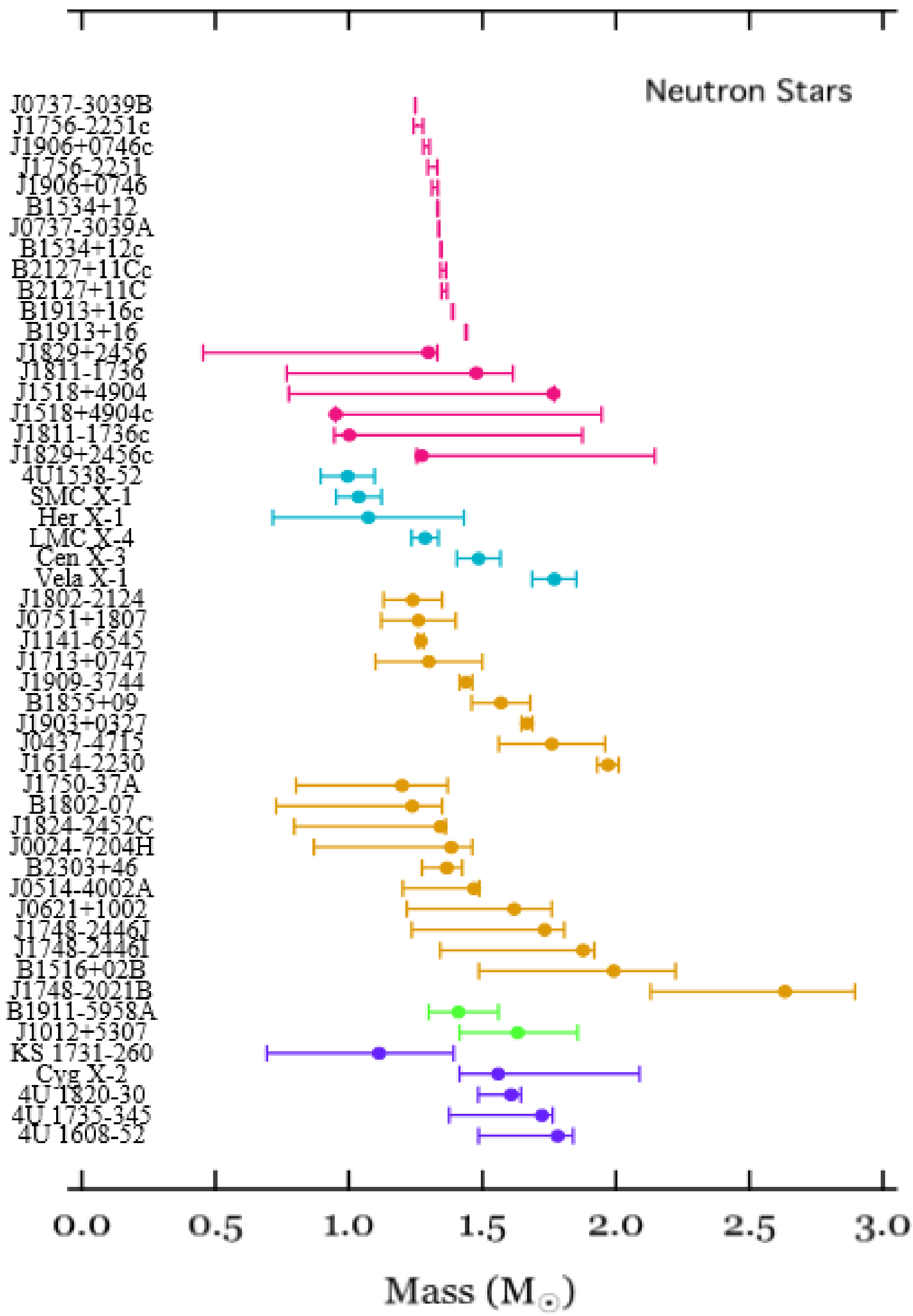
Below is a graph that summarizes many of the EOS covered. The diagonal dashed line going through the center is the point where the radius-mass relationship becomes unstable and the star will collapse. The plain dashed radius-mass line represents a quark-hybrid star and peaks at 1.43 solar masses. The other unlabeled dashed and dotted line is a pure kaon-condensate core star. As seen, it slightly passes 1.9 solar masses, but does not go over the previously mentioned 2.0 solar masses. This chart takes the stiffer potential levels for the kaon state so the mass measurement remains on the high end of the spectrum. The other hyperon and npeu models are all graphed below in addition to these other states.

Particularly, one can see the difference with hyperon addition. BGN1 and BGN1H1 follow the same path until the appearance of hyperon where the H1 model dips down sharply with its radius shrinking rapidly with the addition of mass. Thus it quickly reaches its threshold density. BGN1H2 is not listed below, but it would follow a similar path and would dip downwards as well with the appearance of hyperons. Note that the models only diverge at a set point because smaller mass neutron stars would not sustain sufficient conditions for stable hyperons.



Reference [1] (pg. 321)

The most recent observations of neutron stars can limit some of these models or at least restrict some neutron stars to different classifications. It could be that more than one of these models is correct in the concept that there may just be different species of neutron stars. The best way to narrow down potential EOS is to look at concrete data.



Reference [27] (pg.16)

The above chart shows a recorded list of neutron stars. It is very likely that there are neutron stars of at least two solar masses, if not greater. This certainly rules out all neutron stars being quark-hybrid stars unless they are at their maximum value calculated under Akmal. This also challenges the pure-condensates of pions and kaons. Of course, it is still possible to have a mixture, as adding more npeu material would stiffen the EOS. Also, as previously mentioned, many neutron stars behave oddly and it could be that not all of the neutron stars we have documented are actually all the same class of neutron star. Whether one would want to consider two stars with different compositions as both neutron stars is an entirely different question. However, it does appear that the limiting factor in neutron stars is probably some sort of npeu state, either pure, or mixed with other exotic particles. This is not meant to rule out the exotic states entirely however, as new light may be shed on them that stiffens their EOS. It can be noted that recorded neutron stars rarely approach values that are significantly over 2 solar masses and the one on this graph that is significantly above has great uncertainty. The maximum value of just under 2.5 solar masses seems to still be a safe limiter on maximum neutron star mass barring any discovery of a massive neutron star. It is likely that more massive neutron stars are similar to the maximum npeu mass models with less repulsive forces.

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