2022

Using Short Bursts to Optimize Redistricting in Georgia

Vedika Vishweshwar

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May 2022

Abstract

Identifying extreme outliers in large state spaces is a difficult problem. I consider this problem in the context of finding political districting plans that maximize the number of districts in which the majority of the population is from a minority group, such as African Americans. Since the set of all possible districting plans is enormous and unfeasible to examine in practice, this paper proposes a sampling method to find these outlying plans. Specifically, this paper experiments with short bursts in the context of minority voting rights in Georgia. Short bursts are a type of Markov Chain in which an unbiased random walk is performed for a small number of steps, and is then restarted from the plan with the most majority-minority districts. This paper shows that short bursts successfully find outlying legislative house, legislative senate, and congressional plans.

Keywords voting rights, redistricting, Markov chains, short bursts

Acknowledgements This work was inspired by Cannon et.al. (2020), which applied short bursts to redistricting in Louisiana. The author would like to thank Professor Sarah Cannon for her mentorship and guidance throughout this project.

1 Introduction

In the 2020 election cycle, traditionally Republican-leaning Georgia cemented its status as a swing state on the federal stage. It elected not one, but two democratic senators to Washington, namely Senators Reverend Raphael Warnock and Jon Ossoff. They are both the first Democratic senators to be elected in the state since 2000, which changed the makeup of the Senate by giving the Vice President the tie-breaking vote. Interestingly, both Warnock and Ossoff won by extremely thin margins. Warnock won 51% of the vote [18], while Ossoff won 50.6% of the vote [19]. Their elections reflect both the growing influence of black voters around Atlanta and the success of minority voter registration drives. The closeness of their elections coupled with their far-reaching federal consequences amplify the importance of ensuring fair voter representation in Georgia for both federal and state elections.

One method to achieve fair representation is to implement political districting plans that comply with the Voting Rights Act. Although U.S. Senate elections do not use districting plans, they still impact the outcomes of U.S. House, legislative House, and legislative senate elections. Implementing fair plans is particularly important in Georgia the removal of preclearance. Georgia has historically engaged in minority voter suppression, and before 2014 required preclearance from the federal government to change its electoral process. Preclearance was intended to ensure that Georgian voting legislation complied with the Voting Rights Act by maintaining or improving minority voter strength. Redistricting was included within the scope of 'preclearance', and was thus subject to federal oversight in Georgia before 2014.

However, in 2014, the Supreme Court effectively struck down preclearance [9], allowing Georgia to independently legislate its voting process. As a result, Georgia did not obtain federal approval before implementing new districting plans in 2021, negatively impacting minority voters. After enactment, three racial gerrymandering lawsuits against the congressional map separately alleged that minority voters were not provided sufficient political opportunities. These suits suggest that removing preclearance was detrimental to Georgia's VRA compliance. Combined with the rising federal importance of Georgian elections, the absence of preclearance necessitates finding other strategies to preserve minority voting rights.

One such strategy involves filing lawsuits under Section 2 of the VRA, which mandates that districting plans cannot dilute minority voting power. These lawsuits can be filed in response to gerrymandered districting plans that do not adhere to Section 2. A critical component of these lawsuits is finding plans with more majority-minority districts than the current or proposed plan. Majority-minority districts are districts where racial minority groups comprise the majority of the district's population. This paper proposes using short bursts to find districting plans where the number of majority-minority districts are maximized. Given that the state space of all potential districting plans is extremely large, examining every plan to find the one that contains the global maximum of majority-minority districts is unfeasible in practice. Alternatively, one could sample the state space to generate a large collection of plans to draw conclusions from. Employing this latter strategy, short bursts begin at an arbitrary starting redistricting plan and perform b steps of an unbiased random walk. Then, the plan with the most extreme value (i.e. the most majority-minority districts in this case) is identified, and the next burst of length b is restarted from this plan. While this paper experiments with short bursts in the context of maximizing majority-minority districts, short bursts can also be applied to optimize other relevant redistricting criteria such as compactness and competitiveness.

2 Literature Review

Here, I discuss relevant background information on Georgia's redistricting process and the application of Markov Chains to redistricting problems.

2.1 The Redistricting Process

Redistricting is the process of redrawing legislative districts from which federal and local political representatives are elected. Districting plans divide states into connected districts of equal population. These districts are typically composed of census blocks, which are similar to city blocks and considered the smallest census geographical unit. The Georgian state legislature is responsible for drawing the state's congressional and legislative districting maps. Congressional districts for the U.S. House of Representatives are redrawn every decade after new census data is released and seats in the US House of Representatives are reapportioned. After the 2020 census, Georgia was apportioned 14 seats [28], which is the same number as the previous decade. The legislature also redraws state legislative maps at this time, of which its state House of Representatives is made up of 180 districts and the State Senate of 56 districts [20].

In the absence of Constitutional requirements, federal guidelines on redistricting are shaped by legal precedent. Supreme Court rulings such as Baker v. Carr (1962) [4] and Thornburg v. Gingles (1986) [10] require states to adhere to both the Equal Populations principle and the Voting Rights Act rules on race. The former mandates that all districts be of equal population, while the latter establishes criteria for whether a majority-minority district should be created. Specifically, Thornburg v. Gingles (1986) requires that a majority-minority district be drawn if

1. "The racial or language minority group is sufficiently numerous and compact to form a majority in a single-member district

2. The minority group is politically cohesive
3. The majority votes sufficiently as a bloc to enable it... usually to defeat the minority's preferred candidate"

These standards intend to prevent minority vote dilution by 'cracking' or splitting up minority voters across several districts to reduce the likelihood of the minority representative being elected. Besides cracking, these standards also inadvertently restrict packing minority voters across regions into a single district when they are sufficiently numerous to form multiple, local blocs. In this way, Thornburg v. Gingles (1986) firmly rejects racial gerrymandering. Put together, the federal guidelines on the Equal Populations principle and Voting Rights Act rules on race provides a baseline standard of fairness for the redistricting process.

In addition to following these federal requirements, states can also impose their own redistricting requirements. Georgia requires that all legislative and congressional districts are contiguous, or that all parts of a district connect [8]. Constituents of a district should live as near together as practicable, and it must be possible to travel between any two points in a district without crossing into another district. This requirement prevents the legislature from pairing constituents from far-flung regions into a single district. Moreover, the Georgian legislature can consider compactness, political subdivisions, and communities of interest [8], or other relevant redistricting principles, when drawing districting plans. Thus, in addition to following federal requirements, Georgian districting plans must also be contiguous.

2.1.1 Redistricting Controversies in Georgia

Georgia has long been embroiled in gerrymandering lawsuits that accuse the state of failing to protect minority voting rights. Most recently, the American Civil Liberties Union of Georgia (ACLUGA), five independent voters, and a coalition of civil rights organizations including Georgia NAACP, filed three separate lawsuits against Georgia's newest redistricting plan ratified in December 2021 [22]. All three lawsuits contend that the 2021 plan does not accurately reflect Georgia's demographics. The U.S. 2020 census reveals that Georgia's black population grew by 16% since 2010 and currently makes up 32.6% of the state's population [31]. In the same time period, Georgia's white population declined 10%. These trends suggest that the state will likely be majority non-white by the next census [31]. However, Georgia's 2021, Republican-drawn congressional map retains the same number of white-minority districts from 2010 despite this explosive growth in the Black population, consequently triggering the aforementioned voting rights lawsuits.

The lawsuits contend the following: Firstly, the ACLUGA lawsuit (Alpha Phi

Alpha v. Raffensperger) argues that Georgia's legislative house and senate maps combined fail to include seven black-majority districts that are supported by recent black population growth [3]. The plaintiffs argue that these new districts could be drawn in metro Atlanta and southwestern Georgia among other regions, but thatthe 2021 maps only include black-majority districts in areas that already elect black representatives, such as eastern Atlanta. Consequently, they allege that the map does not grant black voters the new political opportunities they deserve.

Similarly, the lawsuit filed by five independent citizens (Pendergrass vs Raffensperger, 2021) [7] addresses racial gerrymandering with respect to Cobb and Douglas counties specifically. The suit argues that the Georgia's congressional house plan splits Cobb and Douglas counties across five districts, when the black population is sufficient to justify an entirely new majority-minority district. They allege that the plan simultaneously 'packs' some Black voters from the counties into the Atlanta metropolitan area, which is heavily democrat-leaning, while 'cracking', or splits up, other Black voters across Republican-leaning, white-majority districts. The plaintiffs thus seek an additional majority-minority district under the VRA.

Lastly, the suit filed by the NAACP and others (Georgia NAACP v. Raffensperger) [5] is the most comprehensive and discusses the 2021 plan's infringement on the voting rights of other minorities in addition to blacks. It claims that Georgia's legislative and house maps 'pack' and 'crack' Hispanic and AAPI populations to retain the same number of white-majority districts as last cycle, conflicting with population growth trends for those groups. The plaintiffs not only seek revised, racially equal legislative and congressional plans, but also seek an order requiring Georgia to obtain preclearance before implementing any voting changes for the next decade. This second request acknowledges Georgia's history of minority voter suppression and aims to minimize it in the future. Taken together, the lawsuits sound the alarm on the erosion of minority voting rights in Georgia and beg the question of whether more racially-equitable, valid redistricting plans can be drawn.

2.1.2 Recent Supreme Court Action

Likewise, Alabama also immediately encountered racial gerrymandering lawsuits after its 2021 districting plans were ratified. In the congressional map, 1/3 of black Alabamians are packed into one district, while other blacks are cracked across four white-majority districts [32]. 2020 Census data reveals that blacks constitute 27% of the population [30], yet they currently only comprise a majority in one of seven congressional districts. Thus, three groups of plaintiffs sued, claiming that the map violated the VRA by under-representing blacks. The plaintiffs sought a new districting map that includes an additional majority-minority district. In January 2022, the district courts enjoined the three cases into one (Merrill v. Milligan, 2022) and ruled in favor of the plaintiffs [6]. They ordered Republicans to revoke the existing map and draw another that included the additional majority-minority district.

In response to this decision, Alabama's attorney general immediately appealed and filed a motion to stay the ruling. In February 2022, the Supreme Court granted the motion and stayed proceedings until after the 2022 Midterm elections. In other words, the Court blocked the district court's ruling from taking effect until the case is fully litigated, which will only occur after November 2022. Meanwhile, the Court allowed Alabama's initial congressional map to be reinstated. They reasoned that changing the maps would be too disruptive towards the state's primary elections, for which absentee voting is scheduled to begin on March 30th 2022. Thus, by refusing to amend districting maps before an election, the Supreme Court temporarily upheld a map that both conservative-leaning judges and Democrats acknowledge as racially gerrymandered.

The Supreme Court's decision on Merrill v. Milligan (2022) set a vital precedent for similar cases in Georgia and Texas. On February 28th, a district court upheld Georgia's legislative and congressional maps for the upcoming midterms despite acknowledging that they potentially violate the VRA [21]. Similarly, lower courts stayed Texas' Republican-drawn map that several plaintiffs, including the Biden administration, claim violates Hispanic and black voter rights [21]. Further legal proceedings on both cases are postponed until after the midterms. Lower courts in other states facing similar racial gerrymandering cases are also expected to follow the precedent set in Merrill v. Milligan (2022), potentially affecting the midterm election results.

Moreover, given that redistricting is guided by judicial precedent rather than constitutional law, the Court's decision on Merrill v. Milligan and the ensuing decisions of lower courts have long-standing consequences for future redistricting guidelines. In ruling to stay the Alabama decision, the Court prioritized the Purcell principle [26], the idea that courts should not change electoral procedures close to an election, over equal voting rights. Their ruling indicates the court's growing skepticism of race-based challenges to congressional maps and suggests that proving racial gerrymandering does not necessitate immediately redrawing maps. Fundamentally, these decisions signal the waning influence of the VRA to a court that has frequently ruled to limit federal reach.

Once fully litigated, cases such as Merrill v. Milligan (2022) and Pendergrass v. Raffensperger (2021) will matter immensely to the future relevance of this research. This paper uses short-bursts to maximize the number of majority-minority districts in redistricting plans, which enhances VRA compliance.

Should the Courts rule to maintain the Georgia and Alabama maps until the next redistricting cycle, they will essentially erase the relevance of the VRA to redistricting. Their decision would override Thornburg v. Gingles and invalidate the requirements for drawing a majority-minority district. The voting power of minorities would be left to the mercy of partisan strategy, likely exacerbating existing racial gerrymanders and reducing the likelihood of electing minority candidates. Then, the only federal redistricting requirement with judicial teeth would be the equal populations principle. Within this constraint, states would have full discretion over the shapes of their maps. Alternatively, should the Court rule in favor of the plaintiffs in both cases, they would reinforce the relevance of the VRA to redistricting. Then, this research could tangibly help uphold minority voting rights by identifying more racially equitable plans. Regardless, by staying both maps for the midterms, the Court has already indicated that the VRA does not always precede the Purcell principle.

2.2 Markov Chains

A Markov chain is a stochastic process with the Markov, or memoryless, property. Essentially, it is a random walk on a pre-determined state-space where the next state in the chain only depends on the previous state. The next state is entirely independent of chain length, chain behaviour predating the most recent state, or any other features of the chain. Rather, the chain follows fixed probabilities of transitioning from state to state in a time-independent manner. Refer to [27] for additional detail on Markov chains.

An example of a basic Markov chain is the unbiased random walk on a line. Let X_t denote the state of the chain at time t. Here, Ω is $\mathbb{Z}$, or the integer state space, and the chain starts in $X_0 = 0$. It moves to the right (to a larger integer) and to the left (to a smaller integer) with probability 1/2 each. Since the chain is equally likely to move either up or down, this random walk is unbiased. Then, conditioned on $X_t = a$ at time t, the state X_{t+1} at time t+1 satisfies:

$$\mathbb{P}(X_{t+1} = a+1) = 0.5, \mathbb{P}(X_{t+1} = a-1) = 0.5, \mathbb{P}(X_{t+1} = b) = 0 \text{ if } b \neq a+1, a-1$$

While biased random walks do not have equal probabilities of moving left and right, these probabilities are constant for all X $\in$ Ω. Once again, the walk starts in $X_0 = 0$ and has bias p in (0,1), where p is the probability of moving to the right. If p > 1/2, then the walk is biased rightwards towards larger integers, and vice-versa. Then, for a biased random walk on the line conditioned on $X_t = a$ at time t, the state X_{t+1} at time t+1 satisfies:

$$\mathbb{P}(X_{t+1} = a+1) = p, \mathbb{P}(X_{t+1} = a-1) = 1-p, \mathbb{P}(X_{t+1} = b) = 0 \text{ if } b \neq a+1, a-1$$

Markov chains are frequently used for simulations to generate random samples in a process called Markov chain Monte Carlo (MCMC). This process utilizes the Ergodic Theorem, which states that finite and aperiodic chains converge to a long-run stationary distribution on Ω after enough steps. Specifically, finite Markov chains are ones for which $\mathbb{E}[X_t] < \infty$, or the expected value of the state of the chain at any time is finite. A chain is aperiodic if it does not move through states in a predetermined pattern (i.e. the unbiased random walk on integers has period two because it returns to any state after an even number of steps). The Ergodic Theorem is extremely powerful because it implies that after enough steps, we can essentially predict the behaviour of finite and aperiodic Markov chains. The current state simply becomes a random sample of the stationary distribution, which is an extremely useful property when Ω is large and difficult to enumerate.

The Ergodic property of Markov chains lends itself particularly well to redistricting problems. Running a chain on the districting space for enough steps will generate an ensemble of plans that converge to plans randomly sampled from the long-run stationary distribution. Then, it is possible to judge proposed or current plans as outliers with respect to this ensemble.

2.3 Using Markov Chains to Improve Redistricting

Markov Chain Monte Carlo methods are frequently applied to solving redistricting problems, in which Ω represents the discrete state space of all possible districting plans. Here, Ω is extremely large. For example, Minnesota has 6,156,73,718,225,577,984 potential Senate Districts that are derived from pairing its 134 House Districts [15]. Moreover, this number does not even capture the various valid State House districting plans. Thus, we cannot determine the size of Ω, which makes examining every single plan impossible in practice.

However, the Ergodicity of Markov Chains allows us to sample from the stationary distribution, which is assumed to represent all potential plans. A collection of such valid districting plans is known as an *ensemble* [29], which is obtained from Ω by taking Markov Steps from a *seed plan* and collecting all plans that adhere to some predetermined criteria. In the literature, this ensemble is used to construct a baseline of typical redistricting plans against which current or proposed plans are evaluated. If a current plan is an outlier with respect to this baseline, it indicates gerrymandering. Thus, Ergodicity allows statisticians to generate an unbiased and representative random sample of districting plans from a large state-space, providing a basis for comparison.

Technically, an ensemble is collected from taking Markov Steps on a dual graph. Each atom of the plan (usually a census block) forms a node and edges between nodes represent geographic adjacency. Then, districting plans partition a dual graph along edges into d connected parts. The plan is only valid if all districts are contiguous and have equal populations. Adherence to the Voting Rights Act requirements are evaluated post-facto. Here is a graph of Georgia's block groups where geographic adjacency is indicated by the distance between points. The nodes are color-coded according to their black voting age population (BVAP) as measured by 2010 census data.

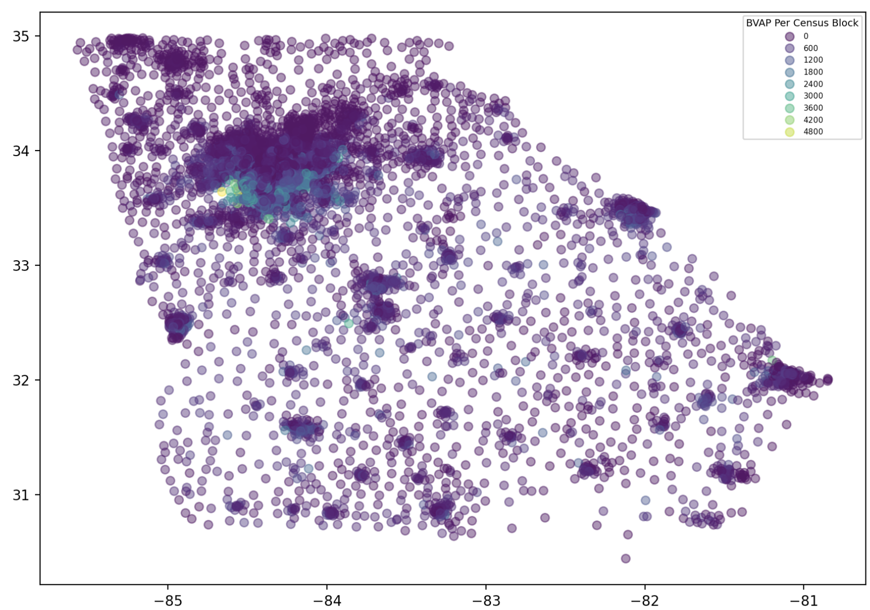

Figure 1: Dual graph of Georgia's census block groups color-coded according to BVAP

However, MCMC methods can also be used to generate extreme plans rather than baseline ones, which are also useful for challenging current plans. Specifically, these extreme plans could maximize or minimize a desired statistic, such as the number of majority-minority districts or predicted party performance. For example, to demonstrate a violation of the Thornburg-Gingles criteria, plaintiffs show that more majority-minority districts could have been formed than those in the current plan [16]. Thus, generating extreme plans through MCMC can also help show alternative plans to gerrymandered ones.

Cannon et.al. (2020) identified a highly effective MCMC method called **short bursts**, which finds extreme plans more effectively than biased or unbiased random walks. Working with Louisiana geographical data, they found that short bursts yielded plans with more majority-minority districts than the current plan and those generated by biased and unbiased random walks alone. Mechanically, short bursts begin at an arbitrary seed plan, and then perform

b steps of an unbiased random walk. The chain then identifies the most extreme observed plan (i.e. the one with the most majority-minority districts) and re-starts another unbiased random walk of length b from that plan. They note that each unbiased random walk of length b is called a short burst and that they are performed repeatedly.

Specifically, short bursts in this paper are based on the Recombination Markov Chain, which is an MCMC method performed on districting plans. Informally, the chain randomly selects two districts from a seed plan, merges them together, and then splits them in half (population-wise) once again. This represents one step of the random walk. The Chain doubly ensures that the number of districts remain the same and that all districts are contiguous. In each step of the walk, the algorithm randomly selects another pair of districts and repeats the process. For a formal explanation of the Recombination Markov Chain, see DeFord et.al. (2020)

In this paper, I apply short bursts to Georgian geographical data and aim to find districting plans with more majority-minority districts than currently in place.

3 Short Bursts

Cannon et. al. (2020) formalize short bursts as follows [16] . I adopt the same formalization. Let Ω be a discrete state space and let $\mathcal{M}$ be a Markov chain on Ω. Let s: $\Omega \rightarrow \mathbb{Z}$ be a score function where if σ_1 and σ_2 are two states of Ω that differ by one transition of M, then $|s(\sigma_a) - s(\sigma_b)| \leq 1$.

In this context, Ω represents all possible districting plans for Georgia and σ_i represents various districting plans, or the states of the chain. $\mathcal{M}$ is the Recombination (Recom) Markov chain, which is a family of chains frequently used in redistricting, and is defined as a "large step random walk on the space of graph partitions" [17]. The chain either begins from a given redistricting plan or a randomly generated seed plan. Given this initial plan, one step in the Recom chain involves randomly selecting two adjacent districts, fusing these two districts together, and then randomly re-partitioning them in a manner that maintains the population balance within ϵ of the ideal population. The ideal population is calculated as the total population of all districts divided by the number of districts. Re-partitioning the districts within ϵ of the ideal population mathematically reflects the Equal Populations principle of redistricting. Lastly, s(σ) represents a one-dimensional score function, which is the number of majority-minority districts in a given plan, which we want to maximize.

While one could find the σ that maximizes s(σ) by listing every state in Ω

and calculating the score for each, this is unfeasible given the size of Ω. For this reason, identifying the true maximum of $s(\sigma)$ on Ω is extremely unlikely. Rather, this paper hopes to find the state with the largest score possible.

Short bursts work as follows: in each burst, b steps of a Markov Chain $\mathcal{M}$ are performed from the starting state. Of the states visited, the state σ with the highest $s(\sigma)$ is identified. If multiple states visited during a burst had the same $s(\sigma)$, the most recent of those states is selected. The next burst begins from this σ and this process is repeated. The final parameter of interest is total steps, which is the total number of transitions in the Markov chain. It is equal to the number of bursts multiplied by the burst length.

This paper aims to find the optimal burst length and number of bursts for Georgia's congressional and legislative maps respectively. Georgia's congressional map consists of 14 districts, while its legislative senate map consists of 56 districts and its legislative house consists of 180 districts. Optimal is defined with respect to the score function and refers to the burst length and number of bursts that yield the highest number of majority-minority districts per run.

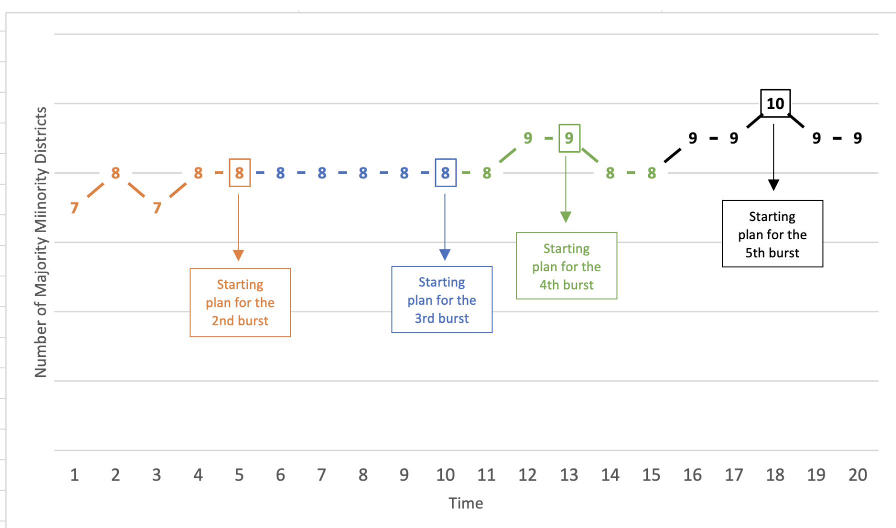

Figure 2: Sample Short Bursts of Burst Length 5 on Georgia's Legislative Senate Plan

Figure 2 displays a sample of the bursts I ran on Georgia's legislative senate maps. Here, I show four bursts of length five. The initial partition contains seven majority-minority districts, and the plan with the most majority-minority districts in the first burst becomes the starting plan for the second burst, and so on.

4 Demographics and Methods

I begin by discussing basic demographic and districting information about Georgia and then discuss the results of the short bursts for each type of districting plan.

4.1 Georgia Demographics and Districting

I focus on Georgia for several reasons. Aside from its national prominence as a crucial swing state, it has a relatively large and diverse population. According to the 2020 census, Georgia's total population stands at 10.7 million, with 32.6% of the population consisting of blacks and 9.9% consisting of Hispanics or Latinos [31]. This benefits the running speed of my code because census block groups are populous enough for population balance to be achieved easily enough. If Georgia had an extremely small population, I would need to use larger census units such as precincts or counties to achieve population balance. Moreover, Georgia also has a relatively large legislative House of Representatives with 180 seats, which provides good granularity of data.

4.1.1 Minority Representation in Georgia

In the 2020 census, Georgia's voting age population (VAP) stood at 8.22 million, of which BVAP formed 29.9% or 2.46 million [14]. Table 1 reveals Georgia's demographic information from the 2020 census.

Table 1: Racial demographics in Georgia.

Georgia Demographic Information			
Racial Group	Total Population 2020	Percent Change TOTPOP (2010 - 2020)	Percent Change in VAP (2010 - 2020)
Non-Hispanic White	5,570,192	-1.0%	+2.4%
Black or African American	3,492,082	+12.6%	+18.8%
Hispanic or Latino	1,060,478	+31.6%	+37.8%
Asian	471,324	+52.6%	+57.0%
Total Population	10,711,908	+10.6%	+14.2%

Note: TOTPOP is total population and VAP is voting age population. Population figures for racial groups from [11]. Percentage changes in population and VAP from [13]

Table 1 illustrates that blacks are not only the largest minority group in Georgia, but are also responsible for a sizable portion of Georgia's growing electorate. This paper employs short bursts in the context of the black population and majority-BVAP districts. Henceforth, the term 'majority-minority district' reflect districts where the majority of a population is from a minority group, but are not necessarily black. In contrast, the term 'majority-BVAP district' reflects districts where black voting-age citizens form the majority of the voting age population. Given the substantial size of Georgia's Hispanic and Asian populations, the number of majority-minority districts is not always equal to the number of majority-BVAP districts. This distinction becomes important when considering the results of short bursts run on Georgia's congressional maps.

Of Georgia's fourteen congressional representatives, five are black [1]. While majority-minority districts are not guaranteed to elect minority representatives, it is highly likely that these representatives were elected from those districts in Georgia's previous plan. Since the current plan also has five majority-minority districts and Georgia's votes are racially polarized [25], it is likely that the 2022 midterm election will also yield five black representatives. In Georgia's legislature, sixteen of 56 senators are black and 35 of 180 house representatives are black [2]. The new maps create thirteen majority-BVAP districts for the Georgian senate elections and 45 such districts for the Georgian house elections [24].

4.2 Methods

4.2.1 Data

I had population data based on the 2010 census for Georgia's census block groups and precincts. A census block group is formally defined as divisions of census tracts that contain between 600 and 3,000 people [12]. Georgia currently has 5,533 census block groups [11]. Precincts in Georgia are equivalent to election districts, of which Georgia has 2,652 [23]. I initially experimented by running short bursts on plans constructed with both precinct data and block group data. The latter maps yielded results that more closely resembled Georgia's current numbers of majority-minority districts for all three types of plans. Additionally, the smaller size of the block groups allowed better granularity and faster running speeds. Thus, all reported findings in this paper are from running short bursts on maps that use block groups as atoms.

4.2.2 Experiments

This paper aims to understand how short bursts perform in maximizing the number of majority-BVAP districts in Georgia across three maps, namely: congressional, legislative house, and legislative senate.

A subsidiary goal of this paper is to publish short bursts code that is easily accessible to those with limited experience in python and redistricting. All experiments code can be found at this link (https://github.com/vedikavish1/Georgia-Redistricting), which contains documents used to run short bursts and instructions to load and draw the networkx graphs. Working with such graphs is tricky, and this link provides readers with a clear introduction to the topic while also equipping them to run the experiments.

The score assigned to each districting plan is the number of districts where 50% of the voting age population consists of black voters. I ran 10 trials of each of the following burst lengths $b \in \{2, 5, 20, 50, 100, 200\}$, recording the highest score for each run. For each run, I set population constraints such that each district would differ by at most 2% from the ideal population. All plans also meet an additional compactness constraint, which ensures that the number of dual graph edges whose endpoints are in different districts are no more than twice the total number of such edges present in the seed plan.

Each trial begins at a seed plan that is generated with a recursive spanning tree, which is a method that recursively partitions a tree into a given number of parts of a population that are within the population bound. I then took 5,000 Markov steps from this seed plan using the Recom method as discussed above.

5 Results

The following results are from running short bursts on dual graphs affixed with Georgia's 2010 census data. Since Georgia's black population has grown by 16% since 2010 [31], these short bursts are unlikely to find as many majority-BVAP districts as contained in the current plans. However, these results form an important lower bound for current and future redistricting plans in Georgia. In other words, given Georgia's pace of diversification, the current plan should contain an equal number or more majority-BVAP districts than the maximum found in these bursts. Thus, despite using 2010 census data, these results serve as a crucial benchmark to assess the fairness of Georgia's current districting plans.

5.1 Legislative Senate Maps

Figure 3 shows the spread of majority-BVAP districts found for running short bursts on Georgia's legislative senate maps. The total steps equalled 5,000 in every run.

There are several takeaways from this figure, displayed below. Firstly, b=5 outperformed the other burst lengths: it had both the highest median and maximum number of majority-BVAP districts for the legislative senate experiments. Interestingly b=2 had the highest range, finding both the maximum and mimimum number of majority-BVAP districts. Put together, it seems that shorter burst lengths are more volatile, and therefore more likely to find extreme plans.

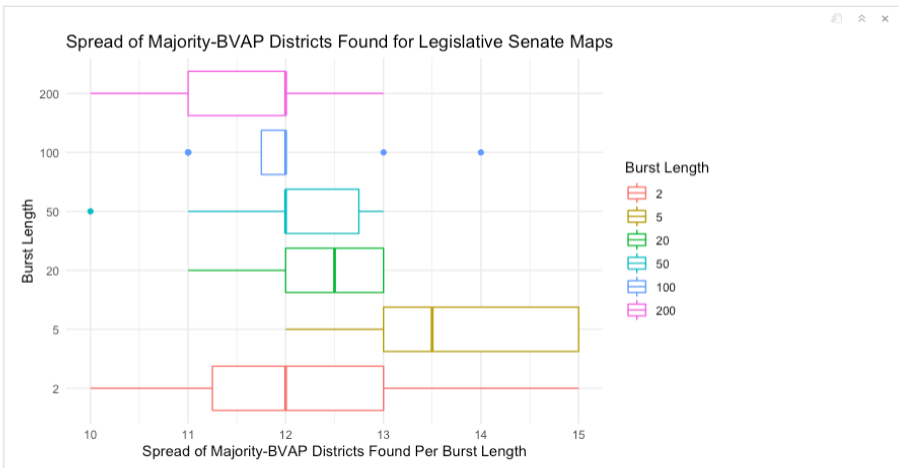

Figure 3: Spread of majority-BVAP districts found for running short bursts for Georgia's legislative senate plans

More importantly, short bursts produced legislative senate plans that contained more majority-BVAP districts than Georgia's current plan. Specifically, burst lengths b = 2, 5, and 100 yielded plans with fourteen and fifteen such districts, while Georgia's current plan only contains thirteen [24]. This implies that Georgia's current senate plan needs at least *two* more majority-BVAP districts to meet this lower bound of fifteen. Since the enacted plan fails to represent Georgia's 2010 black population, it is highly likely that it also under-represents Georgia's 2020 demographics given trends of black population growth. Thus, by generating more racially equitable plans than the enacted one, short bursts are extremely successful with respect to Georgia's legislative senate map and can generate results useful to lawsuits such as Alpha Phi Alpha v. Raffensperger.

5.2 Legislative House Maps

Figure 4 shows the spread of majority-BVAP districts found from running short bursts on Georgia's legislative house maps. Like above, the total steps equalled 5,000 in every run.

Several interesting trends emerge here. Firstly, contrasting its out-performance in the legislative senate maps, burst length b = 5 was least successful in the house maps. It yielded a maximum of 34 majority-BVAP districts, which is fewer than the minimum found by the most successful burst length, b = 20. Experiments with b = 20 showed the highest median and maximum numbers of majority-BVAP districts: 37 and 40 districts respectively. Interestingly, the mixed success of burst length b = 5 contradicts that of burst length b = 2, which was successful on both the legislative senate and house maps. In the house maps, experiments with b = 2 yielded a median of 36 majority-BVAP districts and a maximum of 38 such districts. The conflicting performance of shorter burst lengths on Georgia's legislative maps necessitates repeating these experiments enough times to attain statistical significance. Doing so was beyond the scope of this paper.

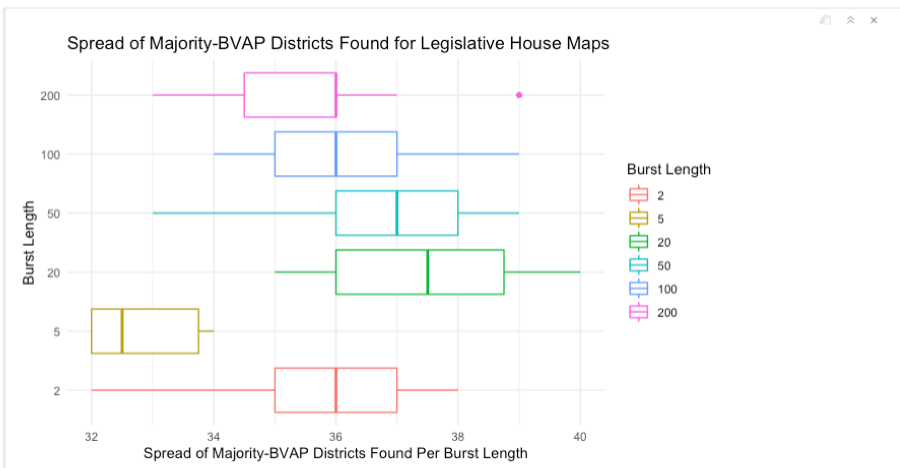

Figure 4: Spread of majority-BVAP districts found for running short bursts for Georgia's legislative house plans

Moreover, these experiments yielded a maximum of 40 majority-BVAP districts for Georgia's legislative house map. Interestingly, there are currently only 35 black representatives in the house, all of whom were elected from the 2010 map. While acknowledging that majority-BVAP districts do not always elect black candidates, the presence of 35 black representatives suggests that there were approximately 35 majority-BVAP districts in the previous plan. Then, short bursts have exceeded this estimate by finding many plans with up to 40 majority-BVAP districts.

Additionally, the number of majority-BVAP districts in the enacted plan far surpasses the lower bound of 40 established by these experiments. Georgia's 2021 plan contains 45 majority-BVAP districts, which implies that it is racially equitable with respect to Georgia's 2010 census data. However, 2020 census data is required to confirm that it also offers Georgia's current black population fair representation.

5.3 Congressional Maps

Figure 5 shows the spread of the majority-BVAP districts found for running short bursts on Georgia's US Congressional House maps. The total steps equalled 5,000 in every run displayed.

Georgia's enacted congressional plan includes five majority-minority districts, but only two majority-BVAP districts [24], [21], which is entirely unchanged from the previous plan. Pendergrass v. Raffensperger [7] alleges that an additional majority-BVAP district can be drawn based on 2020 census data. Running short bursts on congressional maps confirmed that this is possible, as demonstrated by Figure 3.

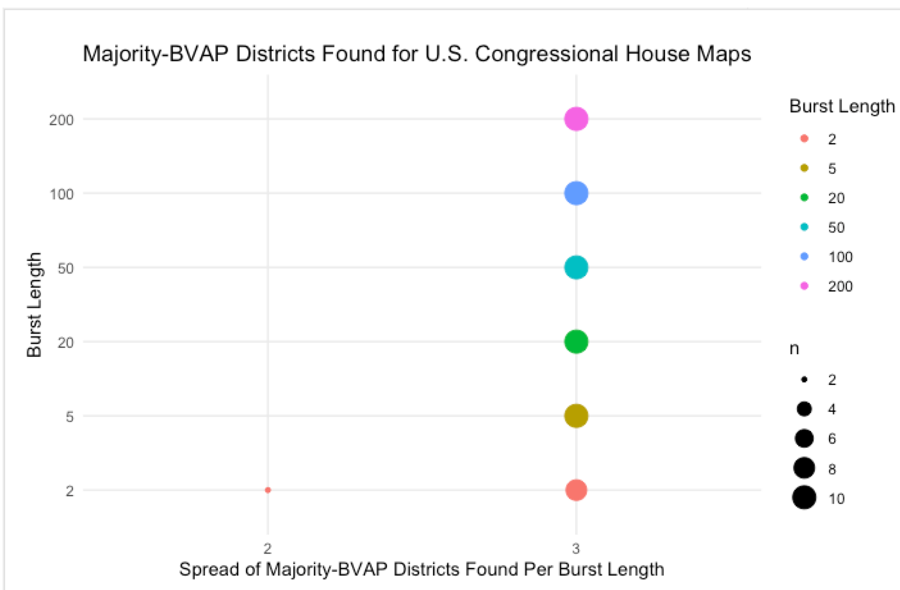

Figure 5: Spread of majority-BVAP districts found for running short bursts for Georgia's US Congressional House plans

Figure 5 shows that every experiment on five out of six burst lengths yielded three majority-BVAP districts. Only burst length b = 2 had two runs that yielded fewer districts. These results imply that the enacted plan under-represents Georgia's BVAP in 2010 and is likely to also under-represent its current BVAP given population growth trends. They also substantiate the plaintiff's case in Pendergrass v. Raffensperger (2021).

6 Conclusion

This paper creates an accessible and available framework to use short bursts to find more racially equitable redistricting plans in Georgia. A key contribution to the literature is disseminating short bursts code that is easily comprehensible to those with limited experience in python and redistricting. The main finding is that Georgia's enacted legislative senate and congressional plans do not offer black voters fair political representation based on 2010 census data. Since Georgia's BVAP has grown by 19% [14] since 2010, it is extremely likely that the enacted plan also under-represents black Georgians today. This finding corroborates the plaintiff's cases in Alpha Phi Alpha v. Raffensperger (2021) and Pendergrass v. Raffensperger (2021) respectively.

A significant limitation to the findings is the use of 2010 census data rather than 2020 census data, which prevents them from being used as evidence in Georgia's recent lawsuits. Unfortunately, affixing 2020 census data to census block groups data was beyond the scope of this paper. Thus, a logical extension of this research would be running experiments on all three plans affixed with Georgia's 2020 census data. These results would be immediately relevant to the lawsuits and could realistically improve political representation for black Georgians today.

Another notable limitation of this paper is the absence of statistical confidence. It was only feasible to experiment with each burst length ten times per type of redistricting plan. Ten is an insufficient sample size to determine which burst lengths are statistically more successful than others with respect to the score function. Therefore, another extension of this paper would be to run these experiments tens of thousands more times to determine which burst lengths are statistically most successful for finding plans with maximum majority-BVAP districts.

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