Harvey Mudd College
Mathematics

Senior Thesis
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Research Proposal: Combinatorial Analogues of the Lefschetz Fixed Point Theorem

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1 Introduction

For my senior thesis, I plan to continue work on a research project in combinatorial topology that began this summer. The goal of this project is to find a combinatorial analogue to the Lefschetz Fixed Point Theorem, just as Sperner's Lemma is a combinatorial to the Brouwer Fixed Point Theorem.

2 Proposed Research

To begin the research project I will investigate some specific cases of the Lefschetz theorem. For example, the Lefschetz theorem applies to continuous maps of a space into itself. If the map f is simply the identity map, the Lefschetz number $\lambda(f)$ is equal to the Euler characteristic of the space. The Euler characteristic can be determined in a combinatorial fashion by examining a triangulation of the space and finding the value $V-E+F$ where V, E, and F represent the number of vertices, edges, and faces of the triangulation, respectively. Thus, we are able to determine $\lambda(f)$ combinatorially. Perhaps there is also a way determine $\lambda(f)$ combinatorially in the more general situation where f is a simplicial map.

Results from combinatorial topology also show that Sperner's lemma is related to the Euler number of a ball, and Tucker's Lemma relates to the Euler number of a sphere. Thus, another goal of this project is to find another analogous "lemma" that relates the Lefschetz number of a map to the space on which the map acts. If they are found, the results of this project could be extremely interesting.

3 Prior Research

My prior preparation for this project includes a semester course in homology theory with Professor Su in which we proved the Lefschetz theorem and some theorems in combinatorial topology. Also, I spent a summer doing research in combinatorial topology with Professor Su where we wrote a paper about applying the Polytopal Sperner Lemma to fair division problems. During this summer research I read several papers in combinatorial topology including F. E. Su, *Borsuk-Ulam Implies Brouwer: A Direct Construction*; F. E. Su, J. De Loera, and E. Peterson, *A Polytopal Generalization of Sperner's Lemma*; F. E. Su, *Rental*

Harmony: Sperner's Lemma in Fair Division. These papers provide a solid background in combinatorial topology.

To familiarize myself with the Lefschetz theorem, I may consult J. R. Munkres, *Elements of Algebraic Topology*; R. F. Brown, *The Lefschetz Fixed Point Theorem*; and perhaps Lefschetz's original paper.