Thesis Proposal: An Eigenspace Approach to Fourier Transforms on Iterated Wreath Product Groups

Nate Eldredge
Advisor: Michael Orrison

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Abstract

For my senior thesis, I plan to extend the research that Prof. Orrison, Ruben Arenas, and myself plan to conduct this summer. During the summer, we will be studying an eigenspace approach to calculating Fourier transforms on groups, specifically the symmetric group. My thesis will apply a similar approach to iterated wreath products of symmetric and cyclic groups.

1 Prior Work

To date, my prior work on this topic is mainly general study of abstract algebra. I have taken Math 171 (Abstract Algebra I) and am currently taking Math 172 (Abstract Algebra II); I expect the upcoming section on representation theory to be especially useful. In connection with these courses, I have read large parts of Dummit and Foote's *Abstract Algebra*. I have also read parts of Clausen and Baum, *Fast Fourier Transforms*.

I have had some general experience with Fourier transforms in courses in differential equations and systems engineering.

2 Intended Reading

During the summer, I will finish reading Clausen and Baum, as well as relevant parts of Dummit and Foote. There are also several papers on the topic by Rockmore and others, of which I will read the most appropriate. Cooley and Tukey's classic paper on fast Fourier transforms would be worthwhile reading as well. I plan to investigate image processing literature to learn about applications of wreath product groups to this field. Finally, I will be learning Matlab in order to implement the computational techniques I develop.

3 Original Research

I will work to extend the eigenspace approach to Fourier transforms that we will study during the summer to iterated wreath product groups. We believe this technique to be relatively novel, and as such its application to any important class of groups should be of interest. Iterated wreath products are used in image processing as they conveniently represent transformations of images that preserve their structure on an appropriately local or global scale, and so are a class of groups whose Fourier transforms have definite applications.

The techniques I will study should lend themselves to algorithms for computing these transforms, and so this project will have a computational aspect as well. I plan to implement these algorithms and investigate their usefulness. Of particular interest is the computational complexity of such algorithms.

Other possible areas of research include examining and possibly implementing some of the image processing algorithms that could use this approach. Special computational techniques such as parallel processing could also be interesting.