

Research Proposal: Combinatorial and Algebraic Representations of Juggling Patterns

Matthew Macauley
Faculty Advisor: Prof. Orrison

1 Introduction

A juggling pattern where one ball is thrown on each beat can be represented as a string of numbers using siteswap notation. Each digit refers to one throw. Larger numbers represent higher throws. Also, an odd number means the ball goes from one hand to the other, while an even number means the ball doesn't switch hands. Any reasonable juggling pattern is going to repeat after a certain number of throws. For notational purposes, the only one cycle of a sequence is usually only written. For example, 534534534534 just becomes 534. There are several natural properties of siteswap notation. First of all, the average of the digits must be an integer, and that integer is the number of balls required for that particular pattern. Also, no two balls can land at the same time. This means that given that one digit is n , the $(n+i)$ -th digit cannot be $n-i$.

2 Proposed Research

Suppose we juggle a valid n -ball siteswap pattern as we walk forward. The paths of the balls will trace out a braid in 3-space with n strings. A braid can be represented algebraically as a braid group. For my thesis, I will examine the properties of the braid groups of valid siteswap patterns. I will try to answer questions like what braid groups yield juggling patterns, whether we can define a correspondence between braid groups and juggling patterns, and whether we can prove theorems about siteswap notation using braid groups that have only previously been proven using combinatorial methods.

3 Prior Research

Siteswap notation was invented in 1985 independently by three mathematicians. They were Bruce Tiemann at Caltech, Paul Klimek in Santa Cruz, and Mike Day in Cambridge. Tiemann has published two articles published on the notation: "The Physics Teacher" (November 1989) and "Juggler's World" (Summer 1991). To my knowledge, no one has published results of an algebraic representation of juggling patterns.

References