

Research Proposal: Averaging Submanifolds and Computational Applications

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1 Introduction

I propose the problem of averaging nearby submanifolds in a Riemannian manifold. Specifically, I will be looking at the special case of averaging curves on a manifold. The goal of this research will be twofold. First, the techniques developed for curves should hopefully be extensible for solving the problem of averaging arbitrary submanifolds, a result important to fixed point theory. Second, an analytic approach to averaging curves allows for the possibility of efficient and accurate computational interpolation schemes useful for computer aided geometric design and motion planning.

2 Proposed Research

For my thesis, I propose to carry on this work for the special case of averaging two curves on a manifold. In an investigation already begun by Dr. Gu, a number of methods for defining such an average curve in the plane have been proposed. Among these routes are techniques using convex bodies, conformal mapping, and more general Riemann maps. The last of these techniques is novel and will require substantial investigation. Possible results from such research may be proving the equivalence of different definitions of average curves. Also, the various methods will be weighed from a computational point of view as I will seek efficient algorithms for finding such curves.

Extensions beyond this work will be looking to finding average curves on manifolds. Specific manifolds I may investigate include certain Lie groups such as $SO(3)$ and $E(3)$ as these manifolds are useful for formulating questions of motion planning. Time permitting, questions of higher order submanifolds such as surfaces might also be investigated.

The core program of my intended research is outlined below:

1. Find the average curve (henceforth called the ‘midcurve’) for two convex closed curves which bound a simply connected region using a convex body technique.
2. Find the midcurve using techniques of conformal mapping.
3. Find the midcurve for closed curves whose interiors intersect in a simply connected region.

4. Find the midcurve for arbitrarily complex closed curves, appealing to techniques of Riemann mapping.
5. Extend the techniques to higher dimensional cases, both for the parent manifold and the submanifolds.
6. Apply the results to questions in computer aided geometric design and motion planning, perhaps implementing the results on an example.

3 Prior Research

I plan on engaging in summer research with Dr. Gu to look at the case of averaging curves in the plane and exploring preliminary problems surrounding this topic, and this work will naturally assist my progress for the coming year.

As for coursework, my geometry background consists of two classes in differential geometry and one in computational geometry. I also have taken point-set and algebraic topology. Additionally, I have taken a class in algorithms and have a strong computer science background. Finally, I will be taking an independent study in computational geometry this fall.

Finally, my last summer was spent at UPENN in the computer science department doing research in combinatorial and probabilistic algorithms, so I have familiarity with mathematical algorithms.

References