

Research Proposal: The Geometry and Topology of the Minkowski Product

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1 Background

The Minkowski product of two complex sets A and B is defined as the set containing all complex numbers of the form ab , with $a \in A$ and $b \in B$. For my thesis, I intend to investigate the geometric and topological properties of the Minkowski product; specifically, I would like to study the way the geometry of two sets is linked to the topology of their Minkowski product.

2 Plans for Original Research

- It is relatively easy to verify that, if A is homeomorphic to A' and B is homeomorphic to B' , the Minkowski product of A and B need not be homeomorphic to the Minkowski product of A' and B' ; thus the topology of the Minkowski product of two does not depend solely on the topology of the two sets. I would like to begin my research by examining precisely what information about a pair of sets A and B is needed to determine the topology of their Minkowski product. Ideally, I intend to search for a geometric equivalence relation, which is as general as possible, and such that if A and A' are equivalent, then the Minkowski products of A and A' with any arbitrary B will be homeomorphic.
- Second, I will examine the condition of topological invariants under the Minkowski product. For example, I intend to analyze the conditions on two sets A and B which are necessary to determine the homology of their Minkowski product.
- Third, I plan to examine the above results in spaces other than the complex numbers. The Minkowski product, and the questions I have posed above, are equally meaningful in the quaternions, the Cayley numbers, or indeed any Lie group. I would therefore like to examine the possibility of answering these sorts of questions in a more general setting.

3 Reading

- Gu, Weiqing and Ian Weiner. “Minkowski Geometric Algebra of Quaternion Sets.”
- Farouki, Rida, Weiqing Gu and H.P. Moon “Minkowski Roots of Complex Sets,” *Geometric Modelling and Processing 2000*, 287-300 (2000).
- Ward, J. P. *Quaternions and Cayley numbers : algebra and applications*.
- Steenrod, N. *The Topology of Fibre Bundles*.