Senior Thesis Proposal:
Minkowski Actions of Quaternion Sets and Their Applications

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1   Background

The Minkowski product of $\mathcal{A}, \mathcal{B} \subset \mathbb{H}$ of quaternions is defined as

$$\mathcal{A} \otimes \mathcal{B} = \{\, ab \mid a \in \mathcal{A}, b \in \mathcal{B} \,\} \qquad (1)$$

The product of sets allows us to describe many complicated surfaces as the product of two simpler curves. For an example of a practical application, this enables the computer-aided graphical design of surfaces which were previously inachievable. In their paper *Minkowski Geometric Algebra of Quaternion Sets,* Ian Weiner and Weiqing Gu extended their definition of the Minkowski product of sets of quaternions to define the Minkowski *action of $\mathcal{A}$ on $\mathcal{X}$* for a set $\mathcal{A} \subset \mathbb{R} \times \mathbb{H}^3$ and a set $\mathcal{X} \subset \mathbb{H}$. In their paper, they have explored certain basic examples. There are many other examples which have not yet been investigated.

2   Plans for Original Research

- First, I will extend this research by generating and describing a wider class of surfaces through the Minkowski action of quaternion sets on curves. To achieve that, I will use certain curves that I have learned in Differential Geometry.

- Second, I will investigate the applications of such an action to Laguerre geometry. The Laguerre geometry is is an important tool for the design of geometric objects in computer science.

- Third, I will give a wide range of examples of canal surfaces and ruled surfaces which often occur in computer-aided geometrical design. I plan to use Maple to not only draw all the above surfaces I will obtain, but also to help me in conducting my research, by giving me intuition to form conjectures and by verifying them.

3   Intended Reading

I studied quaternions when I took Abstract Algebra and am currently enrolled in Advanced Abstract Algebra and Advanced Differential Geometry, which gives

me appropriate background for investigating the geometry of surfaces generated by quaternion sets.

To prepare for this research, I am reading *Minkowski Geometric Algebra of Quaternion Sets,* by Weiqing Gu and Ian Weiner. I intend to read *Minkowski Geometric Algebra of Complex Sets,* by Farouki, Moon and Ravani, and *Minkowski Roots of Complex Sets,* by Farouki, Gu and Moon. During the summer, Professor Gu will let me read *Quaternions and Cayley numbers : algebra and applications,* by J. P. Ward.