Harvey Mudd College
Mathematics

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# Research Proposal:
# A Representation Theoretical Approach to n-Candidate Voting

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## 1 Introduction

In many candidate races, given data of rankings of candidates is simple, but tallying those votes is a tricky issue. The question is: What constitutes a fair system of tallying votes? Of the many methods, Saari provided geometric reasoning to conclude that a particular positional procedure causes the fewest paradoxes. I wish, however, to provide an algebraic proof of this fact, which I believe could strengthen the theory and add more intuition to the study of voting theory. Hopefully I can bring together the worlds of economics, algebra, and political science.

## 2 Proposed Research

I plan to provide a concise and intuitive decomposition of the profile space for an $n$-candidate vote for any arbitrary $n$. This means examining the $n!$-dimensional $\mathbb{C}S_n$-module with a voting theoretical perspective, and discerning what pieces are important to the maps which "tally" the votes. Moreover, I wish to verify Saari's work that the Borda Count is the most fair method of tallying votes, by using representation theory to prove that it causes the fewest potential voting paradoxes.

## 3 Prior Research

During the summer of 2003, I worked with Professor Michael Orrison at Harvey Mudd developing the tools to translate the problems of voting theory to the world of symmetric group representations. We then solved some of these problems in novel ways, by computationally finding seminormal bases in small dimensions, liberally applying Schur's Lemma, and compairing positional ranking to pairwise schemes.

## References

[1] Donald G. Saari, *Explaining all three-alternative voting outcomes.*, Journal of Economic Theory, 87 (1999) pp. 313-355.

[2] Donald G. Saari, *Mathematical structure of voting paradoxes. I. Pairwise votes.*, Economic Theory, 15 (2000) pp. 1-53.

[3] Donald G. Saari, *Mathematical structure of voting paradoxes. II. Positional voting.*, Economic Theory, 15 (2000) pp. 55-102.