Harvey Mudd College
Mathematics

Senior Thesis
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Research Proposal: A Spatially Heterogeneous Mathematical Tumor Model with Angiogenesis, and Immune Resistance

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1 Introduction

For my senior thesis, I plan to investigate models for cancer cell growth and angiogenesis – formation of new blood vessels in the tissue surrounding the tumor. The goal of this project is to create a mathematical model and numerical simulation for cell growth and response to three modes of therapy: anti-angiogenesis therapy, immunotherapy, and chemotherapy. We would then validate the model against clinical data to assess the model's success.

2 Proposed Research

Significant work has been done modelling cancer cell growth under the assumptions of spatial homogeneity, but only preliminary simulations exist for spatial heterogeneity. We hope to extend this work by refining the spatially heterogeneous models and observing its response to the three modes of drug therapy.

The goals for this project are:

- To Develop or adapt a 3-Dimensional spatially heterogenous model for cancer cell growth and angiogenesis.
- Write an effective numerical simulation of the model. Cellular automata or another grid method may be used to discretize the tumor domain.
- Observe the response of the model to different treatment protocols for antiangiogenesis therapy (stop blood vessel formation), immunotherapy (boost the immune system), and chemotherapy (destroy cancer cells or limit growth) in isolation and in conjunction.

3 Prior Research

My prior preparation for this project has included two courses on Partial Differential Equations (Math 180 and Math 182), and a course on cell biology and genetics (Bio 52). These courses have enabled me to understand spatial dynamics in systems of differential equations and how they relate to biological systems. Also I have competed in the yearly COMAP

Mathematical Competition in Modeling twice. This has given me experience in creating and assessing mathematical models.

In addition to my course work, I spent the summer of 2001 working with lab biologists at the Marine Biological Laboratory in Woods Hole, MA. During that summer I sharpened my communication skills. I feel confident reading biological papers and asking research biologists questions about their work even though my work concerns computation.

I have also completed a preliminary survey of the current work done towards modelling cancer cells with the help of L.G. dePillis. We have identified key papers and authors, notably M. Chaplain and my advisor, L.G. dePillis, who remain fairly active in this arena.

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