

Research Proposal: Explaining the Universe: Exploring the Structure of G_2

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1 Introduction

According to the latest results from string theory our universe is an 11-dimensional manifold with the structure of $\mathbb{R}^{3,1} \times M^7$. We call M^7 a G_2 -manifold, meaning that M^7 has G_2 as its holonomy group. Here G_2 is an exceptional group.

2 Proposed Research

We will attempt to gain information about the structure of M^7 by exploring the algebraic and geometric structures of G_2 . It is already known that G_2 is a Lie Group. Beyond this there are many other facts known about G_2 . Before proceeding to do research on G_2 , Prof. Gu and I will determine which problems concerning G_2 are still open. We anticipate that we will be using representation theory in our exploration. Once we have new information about G_2 , we will be able to apply those results to further understand the structure of M^7 .

3 Prior Research

In the past few semesters I've taken algebra I and II, differential geometry and advanced differential geometry, as well as two courses in quantum mechanics and one in theoretical mechanics. Algebra II has given me a background in representation theory which should be helpful in exploring the structure of G_2 . In advanced differential geometry I learned about Riemannian manifolds and Lie groups, both critical to understanding the problem. My physics background should be valuable in interpreting our results physically, as well as giving some intuition into how to proceed. In the coming months I will need to explore some ideas from algebraic topology, such as

the holonomy group. I will also need to become very familiar with G_2 . Currently, I am reading Robert Bryant's paper *Manifolds with G_2 Holonomy* as preparation.