Harvey Mudd College
Mathematics

Senior Thesis
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Research Proposal: Fractal Attraction Basins and Numeric Solution Methods

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1 Introduction

Although one of the simplest methods for finding zeros of a differentiable function, Newton's Method offers a doorway into a world of infinite complexity. When applied to complex polynomials of large enough order, Newton's Method (both discrete and continuous) has been shown to give rise to a chaotic mapping of the complex plane. The domains of attraction for individual solutions divide the plane into beautiful Julia-like sets with fractal boundaries.

2 Proposed Research

The thesis I propose will be largely experimental in nature. I will investigate the behavior of solution methods other than those of Newton, when applied to analytic mappings of the complex plane. Specifically, I would like to try to formulate sufficient conditions for chaos and fractal sets to arise, as well as investigate the structure of any fractals that do. I will use computer simulations, in conjunction with analysis of the complex dynamical system that each solution method gives rise to, in order to characterize the domains of attraction for each fixed point of the system.

3 Prior Research

In preparation for this thesis, I have completed courses in Dynamical Systems, Complex Analysis, and Numerical Analysis. As a result, I feel fully capable of analyzing a dynamical flow in multiple dimensions, as well as creating a numerical solution algorithm to be used in computer simulations. A preliminary search through the literature has found that Peitgen, Prüfer and Schmitt investigated the behavior of Newton's Method for some select models in [2], while in [1], Neuberger gives a nearly complete description of the domains of attraction for the continuous Newton's Method for polynomials.

References

[1] J. W. Neuberger. Continuous Newton's method for polynomials. *Math. Intelligencer*, 21(3):18–23, 1999.

[2] H.-O. Peitgen, M. Prüfer, and K. Schmitt. Global aspects of the continuous and discrete Newton method: a case study. *Acta Appl. Math.*, 13(1-2):123–202, 1988.