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Research Proposal: Cesaro Limits of Analytically Perturbed Stochastic Matrices

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1 Introduction

Let P be a complex $n \times n$ matrix. An *analytic perturbation* of P is a power series

$$P(\varepsilon) = P + A(\varepsilon) = P + \varepsilon A_1 + \varepsilon^2 A_2 + \cdots$$

in which A_1, A_2, ... are all complex $n \times n$ matrices as well. I am interested in investigating the Cesaro limit

$$\lim_{\varepsilon \downarrow 0} \frac{1}{N(\varepsilon)} \sum_{k=1}^{N(\varepsilon)} P^k(\varepsilon),$$

where $N(\varepsilon)$ takes on positive integral values and satisfies $N(\varepsilon) \to \infty$ as $\varepsilon \downarrow 0$, when P is a stochastic matrix and the perturbation $P(\varepsilon)$ remains stochastic for all sufficiently small positive ε.

2 Proposed Research

In [4], Filar, Krieger, and Syed characterize the above Cesaro limit in the case that P, the unperturbed stochastic matrix, has no eigenvalues on the unit circle other than 1 (note that all eigenvalues λ of a stochastic matrix satisfy $|\lambda| \leq 1$—see [1], Chapter 2). I would like to explore how the Cesaro limit may be affected if P *does* have such eigenvalues (for example, -1 or i) —in particular, whether the Cesaro limit still necessarily exists, and if so how the eigenvalues on the unit circle other than 1 may contribute to the limit. This will involve both computation, carried out on carefully constructed examples, and gaining a better understanding of results relating to perturbation theory, especially from [3] and [5].

3 Prior Research

During the summer of 2004, I worked with Professor Hank Krieger on this problem, acquainting myself with some of the matrix theoretic and perturbation theoretic background necessary to understand [4], and obtaining some preliminary results. Additionally, two prior courses in real analysis (Analysis I, Math 131, and Analysis II, Math 132) and some independent reading in complex analysis (primarily from [2]) may prove useful to my understanding of the material I will encounter.

References

[1] Abraham Berman and Robert J. Plemmons, *Nonnegative Matrices in the Mathematical Sciences*, Academic Press, New York, 1979.

[2] John B. Conway, *Functions of One Complex Variable*, Springer-Verlag, New York, 1978.

[3] François Delebecque, *A reduction process for perturbed Markov chains*, SIAM J. Appl. Math. **43** (1983), 325–350.

[4] Jerzy Filar, Henry A. Krieger, Zamir Syed, *Cesaro limits of analytically perturbed stochastic matrices*, Linear Algebra Appl. **353** (2002), 227–243.

[5] Tosio Kato, *Perturbation Theory for Linear Operators*, Springer-Verlag, Berlin, 1980.