Harvey Mudd College
Mathematics

Senior Thesis
Fall, 2005

# Research Proposal: Negative Continued Fractions

Alex Eustis

Faculty Advisor: Professor Arthur T. Benjamin

## 1 Introduction

A continued fraction is an expression of the form

$$a_0 + \cfrac{b_1}{a_1 + \frac{b_2}{a_2 + \frac{b_3}{a_3 + \dots}}}.$$

In a *regular* continued fraction, the $a_i$ are positive integers and the $b_i$ are all 1, but other forms are also commonly used. Continued fractions are a historically rich subject with applications and connections to many other subjects including diophantine equations, infinite series, and rational approximations [1, 2]. Combinatorial interpretations of continued fractions are known as well [3].

## 2 Proposed Research

According to the mathematician Richard Guy, the theory of continued fractions can be developed equally well or better by using $b_i = -1$ or equivalently, reversing the $+$ signs to $-$. This modification is not completely trivial and approximates a given number with different convergents than the regular version. However, references on using continued fractions this way are scarce, and the first part of the project will be to review what is known about this kind of continued fraction and to decipher Richard Guy's conversational remarks. This will lead to an exposition of known results from a new angle, and hopefully new results as well. Since regular continued fractions have a known combinatorial explanation, the negative continued fractions should have one as well, perhaps relating to alternating sign identites. I plan to research the combinatorial implications of using continued fractions this way.

## 3 Prior Research

Although there is a wide variety of theorems concerning continued fractions, the particular selection of $b_i = -1$ appears to be a fairly unresearched area. I am currently still searching for published results.

For me, the area of continued fractions is new. Luckily it does not require more than high school algebra to learn the basics, but there are many connections to more advanced branches of mathematics. Learning more about this subject will be a rewarding experience.

## References


[1] Claude Brezinski, *History of Continued fractions and Padé Approximants.* Springer Verlag, 1991.

[2] Rockett, Andrew M. and Peter Szüsz, *Continued Fractions.* World Scientific Publishing Co., 1992.

[3] Benjamin, Arthur T. and Jennifer J. Quinn, *Proofs that Really Count: The Art of Cominatorial Proof.* The mathematical Association of America, 2003.