

# Research Proposal: Eigenvalues of Adjacency Matrices

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## 1 Introduction

When a search engine ranks the pages corresponding to a query by order of relevance, it multiplies a matrix by an initial vector corresponding to the query repeatedly. The matrix that is multiplied repeatedly is the transpose of the adjacency matrix of the graph of the webpages in the search engine's database multiplied by itself ( $A^T A$ ). The adjacency matrix's  $i, j$ th entry is 1 when webpage  $i$  links to webpage  $j$  and a 0 otherwise. One problem discovered with these search engines is that, if the dominant eigenvalue of the adjacency matrix is repeated, the rankings of the webpages are not unique. The Harvey Mudd College Mathematics Clinic Team of 1999-2000 discovered that if matrix  $V = (e^A - I)^T(e^A - I)$  were used instead of  $A^T A$ , the dominant eigenvalue of  $V$  is never repeated. The new problem is that the repeated multiplications of  $V$  and the initial vector results in the ratio of the largest two eigenvalues tending toward 1. In the examination of one particular graph of potential webpage linkages, the broom tree graph with brush width  $b = 1$ , Professor Estelle Bazor has proven that the ratio of the largest two eigenvalues of  $V$  are bounded above by 0.94.

## 2 Proposed Research

I will examine the broom tree graph and see what happens to the largest two eigenvalues of  $V$  when the brush width is  $b = 2$  and  $b = 3$ . It will also be interesting to look into other graphs and the ratio of the largest two eigenvalues of  $V$  for these new graphs.

## 3 Prior Research

This subject was previously studied by Professor Ward [1] for clinic during the school year 1999-2000. This eigenvalue problem was a subsequential issue from clinic. Further research into the behavior of the largest two eigenvalues of a given matrix has potential to be original research.

## References

- [1] Farahat, A., T. Lofaro, J. C. Miller, et al., *Authority Rankings from Hits, Pagerank, and SALSA: Existence, Uniqueness, and Effect of Initialization*, Harvey Mudd College Mathematics Clinic, 2001.