Harvey Mudd College
Mathematics

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Research Proposal: A Treatise on Subtropical Algebra

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1 Introduction

In recent years, the study of the tropical semiring has seen resurgence due to applications to the study of algebraic geometry, tropical geometry, plane curves, combinatorics, phylogenetic trees, and various other fields. The tropical semiring $(\mathbb{TR}, \oplus, \odot)$ has two equivalent variants, $(\mathbb{R} \cup \{\infty\}, \min, +)$ and $(\mathbb{R} \cup \{-\infty\}, \max, +)$, the inclusion of ∞ or $-\infty$ serving to provide an identity for the $\oplus$ operation min or max, respectively. Francis Su is interested in a delightful degeneration of tropical algebra, which he calls "subtropical algebra." This is the algebraic structure described by $(\mathbb{R}, \wedge, \vee) = (\mathbb{R}, \max, \min)$. Because the concepts of min and max are so frequently useful in the study of game theory, it is believed that any progress in the study of subtropical algebra could have direct applications to the study of game theory.

2 Proposed Research

With the above motivations, my research will endeavor to derive some results in subtropical algebra analogous to recent results in tropical algebra. For instance, what is the correct notion of a polynomial, or polynomial root in subtropical algebra? What is the geometric analogue of these notions: convex hull, polytope, line, plane? Are there generalizations of classical theorems from convexity theory in this setting? Are there applications of these ideas to minimax theorems in game theory?

3 Prior Research

Due to the algebraic nature of this topic, pertinent coursework might include Abstract Algebra I/II and Galois Theory. It is straightforward to show that min and max distribute over one another, which is analogous to the distributivity of intersections and unions over one another. For this reason, Set Theory and Topology may provide insight into manipulation of the min and max operations. Finally, since subtropical algebra studies the real line, Mathematic Analysis I may provide further general intuition.

References

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