Harvey Mudd College
Mathematics

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Research Proposal: Rupture in Thin Fluid Films

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1 Introduction

For a thin fluid film in one dimension whose height is given by $h(x,t)$, we can model the film by the thin film equation

$$h_t + (h^n h_{xxx})_x = 0, \tag{1}$$

where the physical values for n are 1, 2, and 3. We can study the behavior of this equation for various boundary conditions using numerical methods and functional analysis of the energies of h. For instance, we require that the mass $\int h dx$ of the film be conserved. We may also find it useful to consider the arclength of the film and other higher-order energies that have less in the way of physical interpretation.

If h experiences a singularity, we say that the film has *ruptured*.

2 Proposed Research

In order to examine the thin film equation, I must first gain working knowledge of lubrication theory and of the body of literature on the subject. I will then attempt to characterize the behavior of the film for certain types of boundary conditions using numerical simulations with L. Zhornitskaya and A. L. Bertozzi's work in [2] as a guideline, and using the energies of the film. Finally, I will search for new energies of the film that may provide further insight into its behavior.

3 Prior Research

It has been theoretically demonstrated that for $n \geq 3.5$, finite-time singularities (ruptures) cannot occur in Equation 1. It appears that there may be a critical value n_* of n below which rupture may occur and above which it may not. However, this value and any dependence it may have on boundary conditions are unknown. [1]

References

[1] A. L. Bertozzi, *The Mathematics of Moving Contact Lines in Thin Liquid Films*, Notices of the AMS, 45 (1998), pp. 689-697.

[2] L. Zhornitskaya and A. L. Bertozzi, *Positivity preserving numerical schemes for lubrication-type equations*, SIAM J. Numer. Anal., 37 (2000), pp.523–555