Harvey Mudd College
Mathematics

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Research Proposal: Cesaro Limits of Analytically Perturbed Stochastic Matrices

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1 Introduction

Let P_0 be a complex $n \times n$ matrix. An *analytic perturbation* of P_0 is a power series

$$P(\varepsilon) = P_0 + A(\varepsilon) = P_0 + \varepsilon A_1 + \varepsilon^2 A_2 + \cdots$$

in which A_1, A_2, ... are all complex $n \times n$ matrices as well. I am interested in investigating the hybrid Cesaro limit

$$\lim_{\varepsilon \downarrow 0} \frac{1}{N(\varepsilon)} \sum_{k=1}^{N(\varepsilon)} P^k(\varepsilon),$$

where $N(\varepsilon)$ takes on positive integral values and satisfies $N(\varepsilon) \to \infty$ as $\varepsilon \downarrow 0$, when P_0 is a stochastic matrix with eigenvalues on the unit circle other than 1 and the perturbed matrix $P(\varepsilon)$ remains stochastic for all sufficiently small positive ε.

2 Proposed Research

In [3], Filar, Krieger, and Syed characterize the above Cesaro limit in the case that P_0, the unperturbed stochastic matrix, has no eigenvalues on the unit circle other than 1 (note that all eigenvalues λ of a stochastic matrix satisfy $|\lambda| \leq 1$—see [1], Chapter 2). I would like to explore how the Cesaro limit may be affected if P_0 *does* have such eigenvalues (for example, -1 or i).

I have reviewed some of the background literature regarding perturbation theory, in particular [2] for its exposition on reducing the generators of perturbed Markov chains. I now plan on investigating the reduction process as applied to perturbed stochastic matrices for eigenvalues on the unit circle other that 1. This will involve constructing examples of perturbed stochastic matrices whose eigenvalues split in specific, desired ways when the reduction process is performed. Such examples should shed light on the different types of behavior possible for components of the overall hybrid Cesaro limit that are associated with the unit-circle eigenvalues. More specifically, these examples might indicate more about whether the hybrid Cesaro limit always exists: corroborating examples could suggest methods for proving the existence of the limit in certain situations, whereas a counterexample would provide fertile ground for studying the specific conditions necessary for divergence.

In addition, I have located in [4] an inverse eigenvalue result that may prove useful to my analysis. The result characterizes the set of all possible eigenvalues for an $n \times n$ stochastic matrix; if this set possesses certain properties, it may be possible to adapt some of the proofs in [3] for eigenvalues on the unit circle other than 1.

References

[1] Abraham Berman and Robert J. Plemmons.
Nonnegative Matrices in the Mathematical Sciences.
Academic Press, 1979.

[2] François Delebecque.
A reduction process for perturbed markov chains.
SIAM Journal on Applied Mathematics, 43(2):325–350, 1983.

[3] Jerzy Filar, Henry A. Krieger, and Zamir Syed.
Cesaro limits of analytically perturbed stochastic matrices.
Linear Algebra and its Applications, 353:227–243, 2002.

[4] Henryk Minc.
Nonnegative Matrices.
John Wiley & Sons, 1988.