References

[1] Mohammed Abbad and Jerzy A. Filar. Perturbation and stability theory for markov control problems. *IEEE Transactions on Automatic Control*, 37(9):415–420, 1992.

> ANNOTATION: Abbad and Filar study the asymptotic behavior of perturbed Markov decision problems. In particular, they investigate the existence and calculation of certain kinds of optimal strategies for these decision problems. This provides a partial motivation for studying hybrid Cesaro limits of stochastic matrices.

[2] Abraham Berman and Robert J. Plemmons. *Nonnegative Matrices in the Mathematical Sciences*. Academic Press, 1979.

> ANNOTATION: Berman and Plemmons develop the theory of nonnegative matrices in a more sophisticated way than either [6] or [7], beginning with the idea of matrix-invariant cones. They also cover M-matrices and present applications to systems of linear equations, finite Markov chains, and input-output analysis in economics. This text, along with [7], are primarily useful for the more elementary results that can be related to stochastic matrices.

[3] François Delebecque. A reduction process for perturbed markov chains. *SIAM Journal on Applied Mathematics*, 43(2):325–350, 1983.

> ANNOTATION: Delebecque describes a reduction process for perturbed Markov chains. This reduction process gives information about the perturbed eigenvalues of the chain and permits the determination of the perturbed chain's long-term behavior; for example, it can be applied to obtain an approximation of the perturbed chain's invariant measure. The process is used centrally in [4], and variants may prove useful in the more general problem I am considering.

[4] Jerzy Filar, Henry A. Krieger, and Zamir Syed. Cesaro limits of analytically perturbed stochastic matrices. *Linear Algebra and its Applications*, 353:227–243, 2002.

ANNOTATION: Filar, Krieger, and Syed characterize a hybrid Cesaro limit for analytically perturbed stochastic matrices in the case that the unperturbed matrix has no eigenvalues on the unit circle other than 1. Their characterization makes use of the reduction process in [3] via eigenprojections associated with the perturbed eigenvalues of the matrices. This article forms the basis for the work I am doing and may provide insights as to possible approaches to take.

[5] Tosio Kato. *Perturbation Theory for Linear Operators.* Springer-Verlag, 2nd edition, 1980.

ANNOTATION: Kato devotes attention to the basic theory of linear operators on each of finite-dimensional vector spaces, Banach spaces, and Hilbert spaces. He develops perturbation theory for the finite- and infinite-dimensional cases separately, with a large focus on the latter. However it is the former, and in particular the theory concerning analytic projections, Puiseux series for perturbed eigenvalues, and the reduction process, that are important in [3], [4], and my own work.

[6] Henryk Minc. *Nonnegative Matrices.* John Wiley & Sons, 1988.

ANNOTATION: Minc develops the Perron-Frobenius theory of matrices with nonnegative entries, giving extensive coverage of the properties of eigenvalues of these matrices and also delving into more specific classes of nonnegative matrices. This book contains an inverse eigenvalue result for stochastic matrices that may be useful for my work.

[7] Eugene Seneta. *Non-negative Matrices and Markov Chains.* Springer-Verlag, 2nd edition, 1981.

ANNOTATION: Seneta writes about both finite and infinite (countable) nonnegative matrices, though primarily the former. He briefly covers the Perron-Frobenius theory in the finite case and presents applications to, among others, Markov chains and stochastic matrices. The development of the Perron-Frobenius theory is very accessible and clear, which makes it an ideal tool for understanding some of the basic theory surrounding stochastic matrices.

[8] Zamir U. Syed. *Algorithms for Stochastic Games and Related Topics.* PhD thesis, University of Illinois at Chicago, 1999.

Annotation: Syed examines stochastic games, which combine elements of matrix games and Markov decision processes. Specifically, he finds algorithms for determining optimal strategies in certain classes of stochastic games. This provides partial motivation for studying hybrid Cesaro limits of perturbed stochastic matrices, since one way of placing a valuation on strategies for these games involves Cesaro sums.