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Classroom Voting: Active Learning in Differential Equations

Kelly Cline

David Lomen

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Kelly Cline
Carroll College

David Lomen
University of Arizona

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Imagine that you could create a class period where every one of the students in the entire class is actively engaged in the lesson, each one working through the material and in the process individually discovering each new idea. Educational research consistently shows that students learn best when they are doing the mathematics themselves, rather than passively following the instructor's work. Calling on individual students and asking them questions will create a more interactive classroom, turning the monologue into a dialogue. The only problem with this is that we can only hear from one or two students at a time, while the others passively observe the conversation. Instead, it would be better if we would ask our questions and receive a reply from every single student, requiring them all to grapple with the key issues. The more deeply that students are involved in the lesson, the more they will understand, and they more they will retain.

This strategy is the key idea behind classroom voting, a teaching technique where the instructor incorporates a series of multiple-choice questions, often called ConcepTests, into almost every lesson plan. These questions can be posed using transparencies on an overhead projector, using PowerPoint, or by handing out printed packets of numbered questions. After each question is posed, the students are given a few minutes to work through the problem, to form an opinion, and then discuss the question in small groups before all students vote on the right answer. These votes may be cast using an electronic, hand-held "clicker" so that they can be received by a computer, tabulated by commercial software, and instantly displayed as a histogram. A non-technological method involves holding up colored or labeled index cards so that the instructor will get a good understanding of the results just by glancing around the room. After the vote, the instructor can then call on various students, asking them to explain their vote and the reasoning behind their conclusion.

Students at both of our schools have responded to this teaching method very favorably. When surveyed about this teaching technique, students regularly report that they have more fun in classes doing mathematics with voting rather in traditional classes (see e.g. [1]). A set of end-of-semester surveys conducted at Carroll College in Spring 2007 demonstrated very consistent results among eight different courses at various levels taught by three different instructors. 93% of all students agreed that classroom voting is fun, 87% agreed that classroom voting helps them engage in the material, 76% are comfortable being called upon to explain their votes, and 74% would choose a section of a class using voting over one without. This is important because we are creating an effective learning environment in which the students take pleasure in doing substantial mathematics.

A recent study at Cornell provides evidence that this technique can be significantly more effective than traditional lecture methods, but only if it is used to motivate the students to participate in small group discussions before a vote [2]. These small group discussions can be very powerful for several reasons: The students learn from their peers, who understand the common mistakes that may no longer be obvious to the instructor. In addition, students learn to "talk math" with each other on a regular basis, verbally expressing their own mathematical reasoning, and learning to evaluate the reasoning of others. Votes are recorded individually, so there is a real motivation for each person to determine the correct answer, and not just copy the votes of their peers.

You might expect the main purpose of this method is for assessment and it is. However, it is used for formative assessment only, and not used to contribute to a student's grade. One way we implement this technique is to first present new material and examples and then pose a ConcepTest, so the results tell us whether or not the students understand. If they do, we move ahead. If not we can have more discussion or more questions on this topic. We can also ask questions while presenting new material, challenging the students to figure out how to apply new ideas for themselves as a part of the teaching process, rather than using them as a post-lesson evaluation tool. For example, we can introduce a new idea with a question, as in the following ConcepTest, which was recently asked in a class before the word "bifurcation" had ever been discussed.

Example 1. A *bifurcation* occurs if the number of equilibria of a system changes when we change the value of a parameter. For the differential equation $df/dx = bf^2 - 2$, a bifurcation occurs at what value of b?

(a) $b = 0$
(b) $b = 2$
(c) $b = -2$
(d) $b = 2/f^2$
(e) Not enough information given.

After several minutes of discussion, 33% of the class voted for (a), 44% voted for (d), and 22% voted for (e). If this question was being used for assessment, those results would be disastrous. But instead, the purpose of the question is to have students start thinking about a new idea, to get their hands dirty, and to see how much they can figure out. The post-vote discussion is usually most fruitful if the instructor is very Socratic, refusing to confirm or deny the accuracy of anyone's reasoning, and instead asking the students to work things out in the process of the discussion. After the vote on this question, one student explained that since this was really about equilibria, we should set the derivative equal to zero, and this allows us to solve for b, thus (d) is the right answer. It was only after several people had shared different ways of viewing this problem that a student was able to clearly explain that if b was negative then there would be no equilibrium, while if b was positive there would be two equilibria, and thus that must be the bifurcation point. After the students had mostly agreed on this idea, then we could confirm that they were correct, and proceed to generalize and emphasize the key points.

While ConcepTests can be used to check student mastery of "drill" problems, really effective ConcepTests often focus on conceptual, rather than computational issues, emphasizing the most difficult ideas and the most common misconceptions. Learning how to formulate a differential equation to model a specific system is one of the most conceptually difficult, yet vitally important tasks that students need to master, and thus this is very fertile ground for classroom voting. For example, the following question was asked in a recent lesson on systems of differential equations.

Example 2. Tank A initially contains 30 gallons of pure water, and tank B initially contains 40 gallons of pure water. A solution containing 2 pounds/gallon of salt is pumped into tank A at a rate of 1.5 gallons/minute. The mixture in tank A is stirred constantly and flows into tank B at a rate of 1.5 gallons/minute. The mixture in tank B is also stirred constantly, and tank B drains at a rate of 1.5 gallons/minute. If $A(t)$ is the amount of salt in the tank A at time t and $B(t)$ is the amount of salt in tank B at time t, which initial value problem represents this scenario?

(a)
$$A'(t) = 1.5 - \frac{A}{30} \qquad A(0) = 0$$
$$B'(t) = 1.5 - \frac{B}{40} \qquad B(0) = 0$$

(b)
$$A'(t) = 1.5 - \frac{A}{30} \qquad A(0) = 0$$
$$B'(t) = \frac{A}{30} - \frac{B}{40} \qquad B(0) = 0$$

(c)
$$A'(t) = 3 - \frac{A}{30} \qquad A(0) = 0$$
$$B'(t) = \frac{A}{20} - \frac{B}{40} \qquad B(0) = 0$$

(d)
$$A'(t) = 3 - \frac{A}{20} \qquad A(0) = 0$$
$$B'(t) = \frac{A}{20} - \frac{3B}{80} \qquad B(0) = 0$$

(e) None of the above.

This time, the students clustered around two of the options, with 61% of them voting for (c) and 39% of them voting for (d). The post-vote discussion was very fruitful because the student groups had each developed a mini-consensus, so everyone had an idea and had something to share. This is very important and it often helps if on the first day of voting you make it very clear that this is part of the learning process for their benefit: No points will given and no penalties assessed for right or wrong answers. The only unacceptable response after a vote is to say "I don't know" or "I just guessed." In this discussion students quickly focused on the strategy of using units, pointing out that both $A(t)$ and $B(t)$ should be in pounds, that $A'(t)$ and $B'(t)$ should be in pounds per minute, and thus that all the terms on the right hand side of these equations must have units of pounds per minute as well. However, there are still several ways to get the correct units. The discussion wasn't resolved until one student pointed out that the rate that salt flows out of tank B must be 1.5 gallons/minute times B (the number of pounds), and divided by the total number of gallons in tank B (40 gallons), so therefore (d) must be the correct answer.

Here is another set of ConcepTests with a focus on reasoning.

Example 3. Each of these four graphs is a solution of

$$\frac{dH}{dt} = -k(H - H_o) \tag{1}$$

for various initial conditions and equilibrium values (from [3] - See Figure 1). The graphs show the temperature of a yam being heated in an oven. Which graph corresponds to the...

(a) warmest oven?
(b) lowest initial temperature?
(c) same initial temperature?
(d) largest value of k?

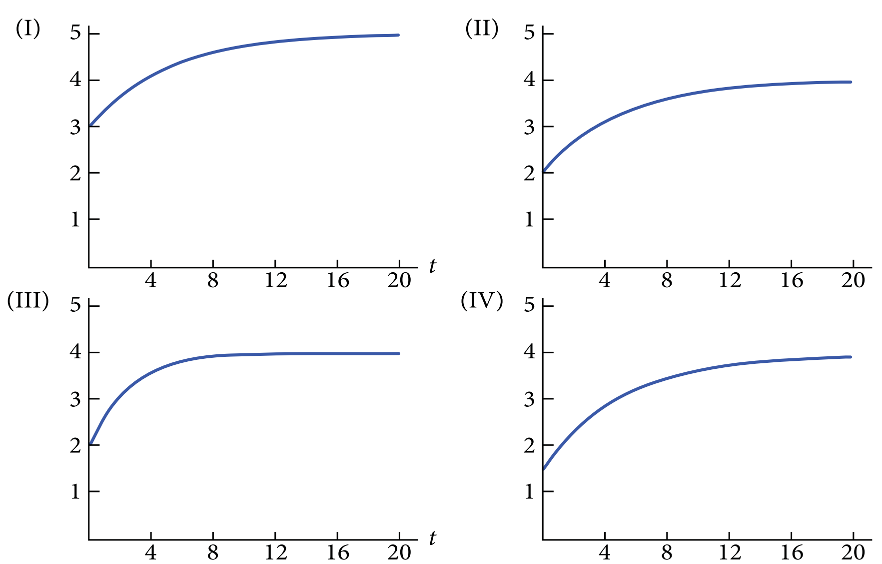

Figure 1: Solutions of Equation (1) for different initial conditions and equilibrium values

Part (d) is the most challenging, as both the value of k and the difference between the initial temperature and the equilibrium temperature determine the initial slope of the temperature vs. time curve.

Classroom voting often takes five to ten minutes for each question, including time for both pre and post vote discussions, so even adding one or two of these questions can quickly consume a substantial portion of the class period. A differential equations class is usually very busy, with an extensive set of topics to be covered in a short period of time. Is it possible to include all this voting without making substantial cuts to the syllabus? We have succeeded in using this teaching method in our courses quite extensively, teaching the same topics and giving the same types of exams. The key seems to be not using voting as an add-on, something extra to be stuffed into an already busy course, but instead to use it as a new way to present the same material. We do fewer examples on the board, using the voting questions to have the students work through the examples themselves. We use the questions less for practice, and more for discovery, to apply ideas for the first time, to provoke common issues, special cases, and misconceptions, and then to resolve them in the resulting discussion. The majority of practice is relegated to homework assignments.

It is a difficult challenge to write really good multiple-choice questions that will bring out the important issues, produce useful discussions, and then guide the students to a solid understanding of the subject. However, there are some good resources available. The Project MathQUEST website (http://mathquest.carroll.edu) contains a library of over 300 ConcepTests for differential equations. Each question has accompanying teacher's comments, as well as past voting statistics, showing the results that this question produced from different classes in the past. These statistics can make it a lot easier to select the right questions for your own students.

Successfully using classroom voting for the first time can be a real challenge, but need not require any special technology. It takes some time and a commitment to restructure your class in a more student-centered way, which will actively engage every single student in all parts of the lesson. They'll have more fun, and they'll probably learn more mathematics as well.

References

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