Final Exam

Robert Dawson
Saint Mary's University - Canada

Final Exam [1]

Robert Dawson

`rdawson@cs.smu.ca`

"Hi, Professor Simmons! Are you making up our exam for tomorrow?"

Don Simmons looked up from his laptop. It was exam week, and a river of students was flowing through the atrium into the library, with a counter-current exiting by the far door. The girl who had spoken to him – short and solid, her round Slavic face framed by dark brown hair – was standing right in front of his café table. Her right hand was pulling, as if to anchor him in the current, on the left hand of a boy whose T-shirt bore the caption "I'm With Stupid". Olga and Keith had been sitting together in his classes since their first year, and they were now in his third year analysis course. "No, I got yours finished and printed up on the weekend. Think you're ready to write it?"

Olga put her forefinger on her pursed lips, pretended to ponder the question for a few seconds; then she smiled and nodded. "Yes."

He could believe it. Marking Olga's exams had always been like watching the numbers come up, one at a time, on a winning lottery ticket. Each answer would be there, evenly-spaced rows of neat handwriting, mistakes crossed out with a ruler. Each near-perfect score would fall into place on the grid, culminating in the triumphant addition and the recording of the grade, often over 100% if there was a bonus question. At the end, he would be nearly as proud of her grade as she would be herself.

Keith's smile was a little tauter. His exams had always been less routine to grade; questions done well often alternated with unexplained dead ends and careless errors. Keith would be on tenterhooks until the grades were posted, awaiting them with terrible seriousness; but his midterm had been strong, and his assignments had suggested a fairly good understanding of the course.

He smiled. "I think you are too."

"Well, anyhow, we are off to study some more. We'll see you there to-morrow at nine!" They rejoined the moving throng, heading for the library, still hand in hand. Almost tenderly, he watched them leave. It occurred to him that when they came back in September for their final year he would not be there to teach them; and with that thought his impending retirement was shockingly real and immediate.

Another familiar face in the crowd: crew-cut hair and black leather varsity jacket. Colin MacKay, first year calculus and football, mainly football. He knew Colin mostly from office hours, where he was a regular and occasionally exasperating visitor; in class he was silent, desperately copying every word and symbol off the blackboard, with no time to think about what they might mean. Colin, he knew, would be contented with a C, maybe even a D. Grading exams like his could be tense, like watching live coverage of a close election, the outcome uncertain until the last number was in.

And there were the others, whose papers sank to the bottom of the pile, half the questions unattempted, branded with a scarlet F. Each of them had his or her own story too. Some were hoping against all evidence to get into medical school or engineering; others might have done well but for a failed romance, too many nights in the pub, or a term sacrificed on the black altar of Dungeons and Dragons. Some would tearfully recount their tales in his office, suddenly individuals after three months of invisibility. Others would vanish leaving no trace except for a name, student number, and letter in his record book.

A girl in a pink hoodie turned into the café area. Her hair was spiky and bleached, with dark roots. Her ears were pierced several times, and she wore a ring in her lower lip. After a moment he placed her: intro calculus, sitting – when she was there at all – about eight rows back by the left wall, texting or whispering into her phone. She walked past his table and glanced at his computer, but did not make eye contact. He smiled tightly and turned back to the screen.

What did she want from his course, for instance? Clearly not knowledge. He had joked last year to one of his colleagues that if, on the first day of class, he were to offer a no-questions-asked C– to anybody who left the classroom and never came back, he would be trampled in the stampede. But it wasn't funny. Too many students really seemed to want as little education for their money as possible. Absenteeism and cheating were rampant. Twenty years ago cheaters had been expelled; now, if the administration even accepted that

a fee-paying "education consumer" (in the loathsome jargon one of the vice-presidents had used last year) could do such a thing, the perpetrator would be asked to attend a two-evening seminar on academic integrity. Whatever that meant today.

The disrespect had been getting worse for years; but he would take it no longer, he would fight back, with what weapons he had. The idea had come to him two years ago, on another cool spring day like this; and today, with retirement staring him in the face, he was putting it into action. He looked down at the words on the computer screen.

CALCULUS 101. FINAL EXAM.

Out of habit, he saved the document, putting the date into the file name so that it would not get confused with other files. Though, he thought, nobody who saw this exam would be likely to mistake it for anything else that they had ever seen.

First, some integration by parts. How about $\int x^3 \sin(x) e^x dx$? Yes, that would do. There was a comparatively easy way to do that one using undetermined coefficients, but for some reason most textbooks didn't mention it. It hadn't been covered in his course either. Twice, years ago, he had tried to introduce it; both times, students had complained that he was going away from the textbook. Well, too bad. He was doing it again.

On a whim, he copied the problem into Mathematica. There was only the slightest pause before the answer appeared on the screen, but the number of terms was gratifyingly large and he could guess from experience how many pages of calculation it would take, working by hand, using the standard method.

Now for a trigonometric integral. Perhaps $\int \sec^7(x) dx$? No, make that $\int \sec^7(-13x) dx$. After a few rounds of integration by parts and solving, that constant ought to be getting rather painful. He toyed briefly with the idea of making the integral definite as well, and decided against it. Not yet.

The first step of the next integral was obvious. Just replace $\sin(x)$ by the new variable u and all the trigonometric functions would disappear, leaving an innocuous-looking square root of a polynomial. Another substitution would simplify that further, except... It was like a wrinkle in the carpet: wherever you pushed it down, up it popped again somewhere else. The only way out of the labyrinth was to introduce an elliptic integral. He did not expect that any of them would ever have seen such a thing.

The code flowed onto the screen, transformed every few minutes into neat professional glyphs. Do you know what an error function is, boys and girls? Oh, I'm so sorry. Here, you might have an easier time with this one. A few people got it twenty years ago on the Putnam contest. There's a really pretty trick to it, but I don't suppose you'll find it. And this one? No, actually I can't do it either, but then you see I don't have to.

All right, that's enough integrals. See if you can find the area between these curves. First you need to find where they cross. Here, look at the screen. Watch Mathematica do it. That's odd, Mathematica seems to be stuck. Never mind, you're all so clever, you'll come up with something. How are you doing so far?

Nothing at all? Well, don't just sit there, move on and try another. Here's an arc length problem. Remember how I told you in class that when we set a problem like this there's always some little coincidence built in that makes it possible to integrate that square root? Well, there's an exception to everything, isn't there? But maybe you'll find one somewhere if you look hard enough. If you do, let me know. And how about this differential equation? Just a little harder than the ones we did in class. Just a little.

Can you sum this series? Oh, it's definitely possible. Ramanujan did it about ninety years ago. The notebook he recorded it in was lost for a long time, but it was found in the seventies. Fascinating story, but we don't have time for it now. No, I won't be scaling the marks. No, there is no makeup exam. Yes, I know you need this course to stay in university. Oh, do stop crying. It won't help, you know.

<div align="center">♠</div>

Half an hour later he read the file through appreciatively, and sent it to the public network printer, just inside the library doors. Three copies, collated and stapled. His coffee had gone cold. He put his laptop into its shoulder bag, stood up, slung the strap over his shoulder, and went over to the counter for another cup. He walked past the girl in the hoodie, still tapping away at her cellphone. She did not look up; by the time his coffee was served, she had vanished.

He sat back at his table and waited for a few minutes before he joined the stream of bodies entering the library, followed it through the doors, and turned off into the alcove containing the printer. Even from a few paces away, he could see that one of the three copies was not there. His fingers told him the same story as he picked them up. He did not look around, but returned

impassively to the atrium. As he emerged, he saw a flash of pink near the far door, and he allowed himself a brief feral grin. If his observations around the time of the last midterm were correct, a lot of cheaters were about to have a very interesting week.

He sat down again, took out his laptop, and turned it on. After a few minutes, a tall boy with a thin beard stopped by his table. Jonathan Monk, first year calculus. Asked some clever questions in class; should have done better on the midterm. "Hi, Professor Simmons! Made up our exam yet?"

"No, Jonathan, I'll be doing the one for your class this afternoon. The one I just made up was for some other students."

"Cool. Be kind to us, eh?"

He smiled benevolently. "I'll try. You go and study. And make sure you get a full night's sleep before the exam. I think you pulled an all-nighter before the midterm."