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Foundational Studies and Mathematics Teaching

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Due to sudden increase in governmental direction of education at all levels, universities in Britain are currently trying to respond to demands for a greater variety of teaching committments than is traditionally encountered. We expect to deal in the near future with a wider catchment area for our student intake and with demands for greater flexibility in form and content of courses. So far as mathematics is concerned, we need to encourage greater inter-relation with other departments and to respond to new applications of mathematics, arising from modern technologies and activities. - eg. computer science, actuarial science etc.

It remains, I believe, the main task of higher education to make links between advanced study, which I take to embrace research, and teaching. Part of this project is critical evaluation, which is, unhappily, almost totally absent from the concept of mathematics held by the majority both of its creators and its users. Critical evaluation depends upon a firm bedrock of foundational principles; I prefer to think of these as relatively, rather than absolutely, constant, since I cannot conceive of any real thing which is totally static. In the climate of change we are facing, to secure a critical position against the vagaries of fashion, we must dig deeper into our subject's foundations.

There are, thus, direct links between foundational studies and the practise of mathematics teaching. These, while present within our conventional presentations, have been rendered invisible by familiarity; our material abounds in ossified attitudes derived from old disputes - axiomatics from formalism, set-theoretic foundations from logicism, insistence on rigor and the demise of geometry from attempts to construct secure foundations.

The current inadequacy of established teaching styles and contents to satisfy contemporary users of mathematics, shows clearly that even the deepest foundational discussions can acheive, at best, only temporary stasis. But now, the role of foundational studies in permitting critique of un-reflective practise takes on a fresh urgency. There are more false ideas about mathematics, both within and without that community, than in any other subject. Without too much exaggeration, we may refer to these ideas as myths. Some of the more potent of these are:-

(i) The myth of individualism: maths is the product of individual minds and the history of mathematics is the story of those individuals.
(ii) The myth of elitism: maths is the product and ability of a special, even rare, kind of mind; mathematicians walk a lonely track.
(iii) The racial myth: maths, at least since the renaissance, is a history of white, western discovery, with the inevitable implication that prior mathematical history was a tale of error or inadequacy.
(iv) The sexist myth: women have contributed nothing to mathematics.
(v) The mechanistic myth: maths is cast-iron and irrevocable in its methods of proof.
(vi) The absolutist myth: maths deals with truths which stand secure above the flux of real things.

Of course, deliberately self-conscious mathematicians will not avow these views explicitly, but students of mathematics, if questioned, exhibit precisely these views, derived unconsciously from the material they are taught. The above myths need to be countered by new paradigms of mathematics which emphasise its human-ness, its normality, its rootedness in real, social practise. Something akin to an anthropology of mathematics - an examination of psychological and social behaviour of mathematicians - should precede, and be the true basis for, a new philosophical statement about foundations.

It is quite remarkable that less than a decade ago it would be almost impossible to find much discussion of the nature of mathematics which began from its status as a <u>human</u> activity. Abstract discussions of truth assignment, formalised symbol manipulation, platonic ontologies abounded. The only major exception - the work of Piaget - came from outside the field. Now things are freeing up; it is intriguing that articles on social paradigms of mathematical truth and social histories of mathematical discoveries are emerging at the same time as renewed efforts to popularise the practise of mathematics.

The impact of computers has been profound, through the revelation of impoverishment in much algorithmic manipulation which we teach as mathematics. More benignly, they have offered a tool which restores geometric insight, stimulates the study of discrete systems and algorithms and presents afresh an impetus to the constructive view of mathematics. The relation of these impacts to the task of "de-mythifying" needs study.