Journal of Humanistic Mathematics

Volume 2 | Issue 2 July 2012

A Workshop to Introduce Concepts of Moral Math

Sarah Voss

A Workshop to Introduce Concepts of Moral Math

Sarah Voss[1]

http://www.PiZine.com/
sarahvoss@cox.net

Synopsis

"Moral Math" refers to the study of ideas drawn from mathematics which can positively impact moral decision-making and social behavior. This essay describes a workshop designed to introduce these ideas to interested individuals of varying degrees of mathematical and theological sophistication. Created by a retired minister and former math professor, the workshop details five sets of interactive exercises culled from game theory, theoretical complexity, fuzzy logic, basic algebra, and simple arithmetic. Exercises are user-friendly, interactive, and easily related by analogy to various social issues.

I am a retired Unitarian Universalist minister who loves mathematics; I even used to teach it. When I gave up my position as the head of the math program at an undergraduate women's college to attend seminary, I took my interest in math with me. Subsequently I wrote about the relationship between math and religion for my doctoral thesis, work which was later published as *What Number Is God?*, a volume in SUNY's series in Western Esoteric Traditions [22]. Then I began offering "math" sermons from the pulpit (being always very careful not to scare away mathaphobes with the "m" word); I also penned a number of articles for various journals about "matheology" and related topics. Still later, I became interested in the relationship between math and social behavior, and out of this interest I developed a workshop designed to help individuals explore this connection.

In the past twelve months, I have presented this "moral math" workshop four times in quite diverse settings. One of these was at a conference for

[1]Sarah Voss is an author/speaker whose work focuses on the relationship between religion and math/science. She believes math is an ideal language for modern-day mystics. See [23] for more.

women and faith held in San Francisco, and another at a class in public administration held at the University of Nebraska, Omaha (where I reside). Another time I offered the workshop to a group of mediators affiliated with the Concord Mediation Center in Omaha, and yet another time it was a part of the adult religious education program at the church where I am currently filling in as pastoral care minister during a period of transition. Up to a dozen participants were present in each of the workshops, which was a relief because I always worry that I'll not have enough people present for the interactive part of the program to be effective. Each workshop drew participants who self-identified with varying levels of math expertise, from low to high, and an even greater relief was that each group developed an obvious enthusiasm for the subject, regardless of the math skill the participants claimed.

I admit to some bias about the worth of this work on math and social behavior. Why is it important? My assumptions are three: that many math-related discoveries have been made in recent years which impact our collective understanding of social equity, justice, conflict resolution, and other issues demanding moral decision-making; that the potential effect of these discoveries are important to *all* people, not just to those who are mathematically proficient; that many of these ideas may be presented through the use of interactive exercises which demonstrate the math involved in such a way that most people can "get" them without actually doing any mathematics.

These assumptions have led me to design a series of five sets of interactive exercises culled from game theory, theoretical complexity, fuzzy logic, basic algebra, and simple arithmetic. The exercises are fun and easy. Two of them I created myself, but the others I pulled straight out of math literature. After presenting each exercise, I relate them by analogy to selected examples of social issues. The expectation is that these ideas mined from math can encourage positive social behaviors. I have found that two-hour sessions, with one break, are just about right to lead the participants through the material in a meaningful way. "Meaningful way" means that the attendees gain an appreciation of how math can be used to stimulate new thinking about solutions to various social ills. People hold different social and religious beliefs. The differences are important because they sometimes lead to conflict and sometimes to learning and regeneration. Everyone believes something, although not everyone always has full awareness of their beliefs. Sometimes two or more people believe pretty much the same things, such as that two plus two equals four or that there's a higher power that influences the universe. When people share beliefs, they automatically form bonds.

Individuals often believe differently, too, at least over time. When I was a child, I believed I was a child who lived with two parents and two siblings on a farm in Ohio. At some level I still hold that belief, but now it is supplemented by an additional understanding of life as an illusion. My beliefs about myself have changed as I've aged and also as I've experienced a mystic spiritual path.

I am a mystic, a former mathematics professor, a retired minister, an "interests-based" mediator, and a shaman. The "shaman" part is a new claim, and it only came about after I read Anne Fadiman's mind-opening book *Spirit Catches You and You Fall Down* about cultural clashes between the Hmong immigrants and much of our U.S. society [11]. Toward the end of this book Fadiman quoted renowned anthropologist Dwight Conquergood, who pointed out that:

> Shamans are, first and last, quintessential mediators. They are threshold-crossers, endowed creatures who can go between the earth and the sky. Grand articulators, shamans' special gift and mission is to bring opposites together – to bring the physical and moral worlds into meaningful conjunction. That is why they are identified with archetypal connectors such as images of ladders, bridges, ropes, and cosmic trees that sink roots into the earth while branching towards the sky ... [11, page 267]

For me, "shaman" has long connoted holy men and women, often with indigenous roots, who enter into trance-like states from which they perform healing rituals. As far as I know I've never entered into a trance-like state (other than the one commonly deemed "human"), but Conquergood's description nonetheless fit me like an old, comfortable glove. For all of my professional ministerial career, I used mathematics to *bridge* and bring together two seeming opposites – science and spirit – in the hope that my efforts might help heal those suffering from some sort of spiritual alienation. My success rate? Alas, my successes have not always been obvious. Nonetheless, here is a story about one that was well-affirmed.

> It was 2005 and I had prepared a presentation on moral math as a tiny part of a three-day UNESCO-type event in Seattle. My "track" was on trustworthiness and justice, and, right before I presented, a middle-aged gentleman shared his personal story about the terrible difficulty he'd had growing up Jewish in Austria

> after World War II. Another man in our group was also from Austria, and his family had been part of the resistance movement. This second man suggested that the first person's story was one-sided, and the conversation took off from there, with the result that very raw emotions were ignited and one of the facilitators was even choking back tears. Being unable to bring resolution to the situation, the facilitators finally decided just to move forward with the agenda, which was fine with me except that the next item on the agenda was my highly-cognitive presentation on moral math. Oh, no, I groaned inwardly. What on earth was I supposed to do next?
>
> Winging it, I told the group a little about my interest in healing and how I found mathematics a fruitful place to glean fresh, out-of-the-box ideas. Then I led them through an experiential exercise called the dollar auction which I'd found in the literature on mathematical game theory and, after that, through a net-theory exercise which I'd developed to illustrate the way self-organization can emerge from the apparent randomness of chaos theory. To my surprise, I not only conveyed how math might work to promote trust, but I also showed them that it did. As one observer put it, genuine healing took place in the room.
>
> Even the Jewish presenter told me later that he'd found my presentation powerful and personally healing, particularly the part which dealt with the random nature of self-organized emergence, which he related to the situation of the Holocaust. His feelings were echoed in various ways by other participants so that, all in all, it was extraordinarily exciting for me. I had assumed that my work could lead to healing, but this time I actually saw it happen [23].

After this exhilarating experience I felt certain that other opportunities would open up for me to further this work, but none did. It was disappointing. Meanwhile, I grew older, and now I have reached a point where I wish to share my ideas with others in the hope that some younger person will want to pick up and further explore this material, developing it into age-appropriate curricula for formal and informal education.

To that end, I now share:

1. A list of online videos

These videos, I believe, demonstrate how mathematics can be used to illuminate social behavior. Whenever feasible, I send this list to pre-registered participants before the workshop actually begins, with a brief note:

> *How can math help render moral issues more accessible? For a jump start, check out these online videos before you come to the seminar:*

- Visualizing Obama's Budget Cuts: This site uses visuals to enhance our comprehension of large numbers and help increase our insight into social issues around the national budget.[2]
- Immigration, World Poverty, and Gunballs: This video offers another visual demonstration regarding number-size; this one attempts to clarify issues linking immigration to reducing world poverty.[3]
- Teaching Game Theory with Video Clips: This is a blog that introduces various resources across the net, including five movie clips, which can be used to gain understanding of real business situations.[4]
- Direct Network Flow Problem on Numb3rs: This video is an example of how the 6-season television series known as *Numb3rs* portrays mathematical topics as useful tools for solving various crimes.[5]
- How Algorithms Shape Our World: In this TED talk, Kevin Slavin shows how algorithms of Wall Street can be digitally reshaped to shed insight on the vicissitudes of the Dow Jones index and how the topography of algorithms offers a 'metaphor with teeth' to rethink the role of contemporary mathematics.[6]

---

[2] http://www.wimp.com/budgetcuts, accessed 06/11/12.

[3] http://www.numbersusa.com/content/resources/video/educational/immigration-world-poverty-and-gumballs-updated-2010.html, accessed 06/11/12.

[4] http://thefilter.blogs.com/thefilter/2008/10/teaching-game-theory-with-video-clips.html, accessed 06/11/12.

[5] http://www.youtube.com/watch?v=_fitfJfF46Q, accessed 06/11/12.

[6] http://www.wimp.com/algorithmsworld/, accessed 06/11/12.

2. A workshop overview

I distribute this overview (see Figure 1 below) to participants at the beginning of the workshop. This single sheet of paper lists the experiential exercises we will do, along with the math source I've drawn from for the exercise and the moral issue it addresses, plus a list of relevant examples of social behavior that this mathematical understanding might influence.

Moral Math Workshop	**EXERCISE SUMMARY**	SarahVoss@cox.net
Math Source/Moral Issue	Experiential Exercise	Social Behavior
1. Nonzero Sum Game Theory (Cooperation & Trust)	*Dollar Auction*	Nuclear arms race
	Chicken for Groups	Genovese 1964 murder Cuban missile crisis WWII troops escaping flood
cf: Alternative Reality Games (ARGs)		
2. Chaos Theory/Emergence (Best Trends &Attitudes)	*The Contact Process*	Rich get richer Segregation *Pig of Happiness*
3. Fuzzy Logic (Fairness)	*Fuzzy Tax Form*	Class-free decision making
	Crime and Punishment	DWI: levels .081 vs .079
4. Algebra (Caring for Others)	*Equal Cake Cutting*	Division of estates & territory
5. Math for Kids (Generosity & Kindness)	*One Grain of Rice*	Growth of AI (Singularity) Environmental pollution

Figure 1: A single page handout provided to workshop participants as an overview of the whole workshop.

3. A bibliography

This is a list of my favorite resources for each of the topics covered at the workshop. I pass this (see Figure 2 on the following page) out at the beginning of the workshop and refer to it as we go through the exercises. I also have additional handouts for several of the topics which I have not included here, but which I distribute as we take up the topic.

———o———0———o———

Now, I would like to briefly describe the exercises I do in this workshop. I usually begin with the *Dollar Auction*, a nonzero-sum game in which a dollar bill is auctioned off to the highest bidder. This exercise works like a normal auction except for one small difference: the runner-up bidder also pays the auctioneer whatever he/she bid, but receives nothing in return. In my experience, people usually hesitate when the bidding reaches about $1 (a complete wipe out, except, of course for the second bidder who is about to lose close to $1). The bidding normally stops around $1.40, when someone realizes that the game has changed from the opportunity to get a great bargain to the need to minimize their potential loss, and then to the recognition that there is no way out of the game except to stop it. At this point, I bring up the similarities between this game and, say, a nuclear arms race. The obvious conclusion, and the hope, is that education might have at least some kind of modifying effect on whether or not people even engage in such a process.

The second exercise, like the first, is drawn from nonzero-sum game theory, but this one is an example of a social dilemma where one of the players is a "group." *Chicken for Groups* is a short exercise where everyone gets a chance to win either $1.00 or 10 cents, simply by writing that amount down on a piece of paper. If at least one person writes 10 cents, then everyone gets what he or she wrote. However, if everyone writes $1.00, no one wins anything. This is a Volunteer's Dilemma, where it is desirable for one person to volunteer for the common good. A classic example is the 1964 murder of Kitty Genovese, who was stabbed to death in the courtyard of her New York City apartment while thirty-eight neighbors watched and heard her cries, but did nothing to assist her. Another is the 1962 Cuban missile crisis where the Soviet Union (via Krushchev) and the United States (via President Kennedy) were engaged in an increasingly dangerous "chicken" race to nuclear war until one of them (Krushchev) publically "volunteered" to back-off.

"Moral Math" Workshop Related Resources

Dollar Auction/ Chicken for Groups
Barash, David P. *The Survival Game* (Times Books, 2003)
McGonigal, Jane. *Realty Is Broken: Why Games Make Us Better and How They Can Change the World* (Penguin, 2011)
Wright, Robert. *NonZero: The Logic of Human Destiny* (Pantheon Books, 2000)
http://www.ted.com/talks/robert_wright_on_optimism.html

Look to a future in which games once again are explicitly designed to improve quality of life, to prevent suffering, and to create real wide-spread happiness.
-McGonigal, p10

Emergence and Networks
Barabasi, Albert-Laszlo. *Linked* (Penguin, 2003)
Buchanan, Mark. *Nexus: Small Worlds & Groundbreaking Theory of Networks* (Norton, 2002)
Callahan, David. *The Cheating Culture* (Harcourt, 2004)
Capra, Fritjof. *The Web of Life* (Anchor Books, 1996)
Devlin, Keith. *The Numbers Behind NUMB3RS: Solving Crime with Mathematics* (Plume, 2007)
Evans, Dave. *Social Media Marketing* (Sybex, 2008)
Gladwell, Malcolm. *The Tipping Point* (Little, Brown 2000)
Gleick, James. *Chaos: Making a New Science* (Penguin, 1988)
Monkton, Edward. *The Pig of Happiness* (Andrews McMeel, 2007)
Wheatley, Margaret. *Leadership and the New Science* (1st ed. 1992)
http://www.gametheory.net/popular/film.html

Self-organization has emerged as perhaps the central concept in the systems view of life, and like the concepts of feedback and self-regulation, is linked closely to networks. -Capra p83

Fuzzy Logic
Bishop, Bob. *Shade of Reality* (Glenbridge Publishing, 1998)
Kosko, Bart. *The Fuzzy Future* (Harmony Books, 1999)
Shermer, Michael. *The Science of Good and Evil* (Times Bk, 2004)
Watts, Duncan J. *Six Degrees: The Science of a Connected Age* (W.W. Norton, 2003).

Preferential attachments: small differences in ability or even purely random fluctuations can get locked in and lead to very large inequalities over time.
-Watts, 109

Fair Cake Division
Eastaway, Rob and Jeremy Wyndham, *Why Do Buses Come In Threes?* (Barnes&Noble, 1998)
Hill, Theodore. "Mathematical Devices for Getting a Fair Share" in *American Scientist* (88: July-August, 2000, p325-31)
Stewart, Ian, "Your Half's Bigger Than My Half!," *Scientific American* (Dec., 1998, p 112+)

In the past 50 years, a number of mathematical devices have been discovered that offer elegant, practical and often surprisingly simple resolutions to many problems of fair division. Hill, p325

Math Fable
Demi. *One Grain of Rice: A Mathematical Fable* (Scholastic Press, 1997)
Kurzweil, Ray. *The Singularity Is Near: When Humans Transcend Biology* (Viking, 2005)

Figure 2: The bibliography handout for the Moral Math Workshops. Complete bibliographic information for each of the items in this list is provided in the references to this note.

After doing these two exercises, we discuss participants' reactions, and I distribute a handout with additional explanation about the theory behind non-zero-sum games, including an introduction to the classic Prisoner's Dilemma and some brief examples of what happens when the relative weights of the options (Temptation, Reward, Punishment, and Sucker payoffs) are switched around, plus a list of some of the conclusions that have been drawn from nonzero-sum game theory regarding cooperation. All of this is written in user-friendly language, so that those with little math expertise will be inclined to "stay with it." I wind this section up by referring to the list of resources on the bibliography I distributed earlier, and by talking a little bit about how some alternate reality games are potential sources to raise the quality of life and to help ordinary people by curing disease, stopping climate change, spreading peace, ending poverty. The way to change the future is to play with it first. (See, for example, http://vimeo.com/26827357, accessed June 7, 2012.)

Next, I introduce the *Contact Process*, which is an interactive exercise I developed to illustrate concepts from self-organizing emergence and network theory. This exercise is based on a few simple action rules which involve an element of randomness. To begin, I distribute the handout I've prepared for this exercise, which includes a little background about relevant ideas from complexity theory (starting with the Butterfly Effect, which many of the participants will have already encountered in popular culture) and an overview of the exercise, along with the game "rules" we'll use:

Action Rules for the Contact Process:

1. Arrange chairs as on a checkerboard grid. Participants choose where to sit so that at least some of them are close together (neighbors). The group has a stockpile of red and green cards.
2. Each participant chooses a red *or* a green card from the stockpile, and then picks a number between 1 and 6 (or 1 and 4 for a small group) which will be that individual's special number for the duration of the play.
3. The leader rolls a die, and everyone having that number stands. Those standing will have either a red card or a green card.
   (a) Each participant standing *who holds a red card* trades it for a green card from the group store.
   (b) Each participant standing up *who holds a green card* does one of the following:

   i. Trades it for a red card if *one or more neighbors have a red card* (shifting probability).
   ii. Keeps the green card otherwise.
4. Participants wave cards to see the patterns developing, then repeat step #3. Continue indefinitely.

It only takes a few repetitions and a little imagination (which participants always bring in abundance) for the group to visualize how the red or green patterns shift in the room. Most are initially astounded when clusters of red or green appear and sometimes even "tip over" at some point so that one color essentially "emerges" as dominant in the room. I then ask such questions as "what happens if a newcomer to the room entered with a preference for, say, "red"? Where would they choose to sit? In the ensuing discussion, we eventually summarize *Selected Outcomes of Self-Organizing Emergence*:

- In inanimate systems (such as economics): the rich become richer
- In health systems: disease spreads in epidemics
- In engineering systems: electric power grids collapse like dominoes
- In computer systems: the Internet gets viruses
- In cellular systems (such as the neural network of the brain and life itself): self-making occurs
- In social systems: fads prevail, racism develops, cooperation spreads

The third set of exercises is drawn from fuzzy logic, and once again, I begin with a handout. This one includes two "forms" drawn directly from sources in the bibliography, one an income tax form that would allow tax payers to have partial say in how their taxes are spent (from *The Fuzzy Future*, by Bart Kosko [16, page 60]) and the other a suggestion of recommended prison terms for abortion based on number of weeks that have elapsed since conception (from *Shades of Reality*, by Bob Bishop [3, page 187]), thus illustrating partial punishment depending upon "degree" of crime. This notion is more easily appreciated when applied to DWI offenses, where our current system of testing for drunkenness suggests that one person might be deemed totally drunk and a second not drunk at all (and punished accordingly) when the alcohol readouts in their respective systems are only a fraction different.

I give a little background on the mathematics involved in fuzzy logic, but time limitations usually mean that I can't do much more than offer these examples of how fuzzy logic can be an aid to social decision-making. Participants are not necessarily convinced that these particular ideas are the ones they want to champion, but they quickly see that fuzzy math can lead to out-of-the-box thinking, with the possible outcome being more fairness and equity in social behavior.

The fourth exercise is drawn from an algebraic algorithm showing one method of cutting a square cake into an odd number of equal pieces (from *Why Do Buses Come In Threes*, by Rob Eastaway and Jeremy Wyndham [9, pages 82–83]). I am quick to point out that there are other, more sophisticated mathematical devices for dividing material things up fairly. The math in this exercise, however, is simple – simple enough that I am always tempted to go through the algebraic proof with the group, but past experience with anxiety on the part of those who are less mathematically sophisticated has taught me to restrain myself. The math details are all there on the handout for anyone interested, and, of course, I'm happy to help them through it at some other time. Instead, I demonstrate the actual steps involved in locating the places to cut the cake, and then I bring out the cake – or cakes if there are a lot of people – and send them off in odd-numbered groups to duplicate the procedure. I use frozen, name-brand cakes that come in 5" squares, and I'm the extra person who ensures that the number of participants cutting each cake remains odd. This exercise is always a popular one, and there's never any cake left to lug home, either.

The fifth and final exercise shows how ideas adapted from math and applied to social behavior can be introduced to children at very young ages. To accomplish this, I have the participants act out the children's story *One Grain of Rice* [7], to which end I provide appropriate props and garments to clothe two volunteers. One volunteer plays the role of the Raja, a mighty but unwise leader in the East who rules his country with an iron fist and little generosity. The other plays Rani, a young girl under the Raja's rule.

When the country suffers a bad drought, the Raja keeps the court storehouses filled with rice while the people go hungry. Rani, whom the Raja takes to be well-intended but not very clever, works out a deal with him to give her a single grain of rice on the first day of the month, two pieces on the second, four pieces on the third, eight on the fourth, and so on for one entire month. By the end of the month the Raja has completely emptied his stockpiles and would be ruined except for the generosity of the young girl,

who gives the rice back to him upon his promise that he will be a good ruler henceforth and will make sure to care for his people properly.

The two volunteers act out the story as I read it, and also read their own parts aloud (from a script I give them) whenever I give the proper clue. Rani pulls out of a bag (the store house) varying sizes of lightly checkered cloth to represent the various amounts of rice she's given. The final one is lengthy enough for Rani to carry it out the door. The rest of the workshop participants (i.e., the "people") make "grumble" sounds whenever something goes wrong for them and "joyous" sounds when things get better. Although some groups have initially been somewhat reserved, by the time the story is finished everyone is having a great time, and the only thing lacking is the (real) children.

Although there are many other such stories that could be substituted, *One Grain of Rice* is a lovely note on which to conclude the workshop. Two social activities that I introduce that are representative of this exponential-like growth include modern computer technology, particularly artificial intelligence, and environmental pollution. The handout I send home after this exercise is an excerpt from *The Singularity Is Near*, by Ray Kurzweil [17, pages 7–9]; it includes a lovely parable about a lake owner who wants to stay home and tend the lake so that it doesn't become totally covered with lily pads, which are said to double every few days. He waits for months and months without anything significant happening to the lake, then takes a vacation only to find that the last seven doublings have completely smothered the fish in the lake. Kurzweil then applies this mathematical parable to information-based technologies, thus setting up the possibility that "by the end of this century, the nonbiological portion of our intelligence will be trillions of times more powerful than unaided human intelligence" (page 9). It's a nice moral issue to ponder after the workshop is finished.

At my most recent workshop I solicited written feedback, two of which I quote below with permission. One woman said, "I love the social implications in which these mathematical models can 'normalize,' 'naturalize,' and 'neutralize' particular social behaviors ... This was fun!" By some unbelievable coincidence this woman is a ministerial colleague less than half my age who also holds an undergraduate degree in mathematics, thus "proving" (at least to me) that there are others who can marry math and spirituality. Another participant, a computer programmer, wrote, "The pacing of the talk was excellent. We covered two hours of material in two hours, not too fast and not too slowly. It was the right level for the audience, and rich enough

so both beginners and people experienced in math and complex systems got something from the presentation ... For each area of math you covered, the triple of a little theory, plus exercises, plus real-world examples was very effective."

———o———0———o———

Overall, my experience with this moral math workshop has been highly challenging and extremely rewarding. Nearly everyone who has been through the session leaves it with an "aha!" attitude, and I leave it with great gratitude for the social wisdom we can gain through mathematical tools.

References

[1] Barabasi, Albert-Laszlo. *Linked: How Everything Is Connected to Everything Else and What It Means*, Plume, Penguin Group (USA) Inc., New York, 2003.

[2] Barash, David P. *The Survival Game: How Game Theory Explains the Biology of Cooperation and Competition*, Times Books, Henry Holt and Company, New York, 2003.

[3] Bishop, Bob. *Shades of Reality: How the New Fuzzy Philosophy Will Change Your World View*, Glenbridge Publishing, Lakewood, Colorado, 1998.

[4] Buchanan, Mark. *Nexus: Small Worlds and the Groundbreaking Science of Networks*, W. W. Norton & Company, New York, 2002.

[5] Callahan, David. *The Cheating Culture: Why More Americans Are Doing Wrong to Get Ahead*, Harcourt, Orlando, 2004.

[6] Capra, Fritjof. *The Web of Life: A New Scientific Understanding of Living Systems*, Anchor Books/Doubleday, New York, 1996.

[7] Demi. *One Grain of Rice: A Mathematical Folktale*, Scholastic Press, New York, 1997.

[8] Devlin, Keith J. and Gary Lorden. *The Numbers Behind NUMB3RS: Solving Crime with Mathematics*, Plume, Penguin Group (USA) Inc., New York, 2007.

[9] Eastaway, Rob and Jeremy Wyndham, *Why Do Buses Come In Threes? The Hidden Mathematics of Everyday Life*, John Wiley and Sons, New York, 1999.

[10] Evans, Dave. *Social Media Marketing: An Hour a Day*, Sybex, Wiley Publishing, Indianapolis, Indiana, 2008.

[11] Fadiman, Anne, *Spirit Catches You and You Fall Down: A Hmong Child, Her American Doctors, and the Collision of Two Cultures*, Farrar, Straus and Giroux, New York, 1998.

[12] *Game Theory in Film*, available at: http://www.gametheory.net/popular/film.html, accessed June 7, 2012.

[13] Gladwell, Malcolm. *The Tipping Point: How Little Things Can Make a Big Difference*, Little, Brown and Company, Boston, 2000.

[14] Gleick, James. *Chaos: Making a New Science*, Penguin, New York, 1988.

[15] Hill, Theodore. "Mathematical Devices for Getting a Fair Share" in *American Scientist* (88: July-August, 2000, pages 325–331)

[16] Kosko, Bart. *The Fuzzy Future: From Society and Science to Heaven in a Chip*, Harmony Books, New York, 1999.

[17] Kurzweil, Ray. *The Singularity Is Near: When Humans Transcend Biology*, Viking, New York, 2005.

[18] McGonigal, Jane. *Reality Is Broken: Why Games Make Us Better and How They Can Change the World*, Penguin Press, New York, 2011.

[19] Monkton, Edward. *The Pig of Happiness*, Andrews McMeel Pub., Kansas City, Missouri, 2007.

[20] Shermer, Michael. *The Science of Good and Evil: Why People Cheat, Gossip, Care, Share, and Follow the Golden Rule*, Times Books, New York, 2004.

[21] Stewart, Ian, "Your Half's Bigger Than My Half!," *Scientific American*, December 1998, pages 112+.

[22] Voss, Sarah, *What Number Is God?: Metaphors, Metaphysics, Metamathematics, and the Nature of Things*, SUNY Series in Western Esoteric Traditions, State University of New York Press, 1995.

[23] "Mathematics, Ministry, and Mediation: An Interview with Sarah Voss", Holos: Forum for a New Worldview, Vol. 5, No. 2 (2009), available at http://www.holosforum.org/v5n2/voss.html, accessed June 7, 2012.

[24] Watts, Duncan J. *Six Degrees: The Science of a Connected Age*, W. W. Norton & Company, New York, 2003.

[25] Wheatley, Margaret. *Leadership and the New Science: Learning About Organization From an Orderly Universe*, Berrett-Koehler Publishers, San Francisco, 1992.

[26] Wright, Robert. *NonZero: The Logic of Human Destiny*, Pantheon Books, New York, 2000.

[27] *Robert Wright on Optimism*, TED Talk, February 2006, available at: http://www.ted.com/talks/robert_wright_on_optimism.html, accessed June 7, 2012.