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The Language of Mathematics: A Quantitative Course for a General Audience

Stephanie F. Singer
Haverford College

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Stephanie F. Singer
Department of Mathematics
Haverford College
Haverford, Pennsylvania 19041

The purpose of this article is to convince anyone with the inclination to design a mathematics course for a general audience that it's worth the effort and to provide some concrete examples of successful activities and assignments.

The course I designed and taught this year, "The Language of Mathematics," was the most enjoyable class I have taught to date. The atmosphere was relaxed. The students plunged whole-heartedly into the in-class problem-solving in groups of three or four. They were forthcoming in the class discussions, willing to share emotional responses as well as insights from their own experiences and other studies. I attribute the excellent atmosphere (which I long to recreate in my more traditional classes) to the daily small group activities, the absence of a specified body of material to be mastered, and the fact that students were not graded individually on their problem-solving abilities.

The several goals of the course were: 1) to give students the opportunity to do some mathematics in a pressure-free, fun environment; 2) to teach students to be agressively critical of quantitative facts presented to them (e.g., by the media); and 3) to help students recognize and become conversant in the mathematical mode of language use. This third goal merits some explanation. There is a particular mode of language use essential to the discussion of mathematics (as well as other endeavors, such as the sciences and the law.) The primary feature is lack of ambiguity. While this mode of language use is as natural as breathing to mathematicians, it is extremely foreign to many people. We love theorems for their infallibility, but many other people believe that to every rule there is an exception. We feel free to use any word we like for any concept, as long as we define the word clearly, but most people learn most words from context and from experience. No wonder, then, that mathematics is viewed as a foreign language by many students—not only is the vocabulary unfamiliar, but even the process by which one learns the vocabulary is different! This problem has been well documented by Rin, who studied linguistic difficulties experienced by linear algebra students. The third goal of the course is to help students recognize that this other mode of using language exists and is useful.

The class met for one and a half hours twice each week. There were twenty students from Haverford College, almost evenly split among freshmen, sophomores, juniors and seniors. Fifteen were women. No student was a physical science or math major, and many described themselves as "not good at math." They were graded not on individual problem-solving ability

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but on their class participation and their written work, described in more detail below. The drawbacks of this grading system (e.g., that students could get away with failing to understand some concepts) were well worth the benefits for these students,as these unsolicited comments attest: "After so many bad math/science experiences, this was the first class of that sort which I enjoyed;" "Thank you so much for wiping away a stereotype and an insecurity and a paranoia concerning math—one that's been around for me since the very beginning when I was told that I 'didn't think mathematically;'" and "It was great to look at math at a different angle and not to have all the pressure that I usually associate with math classes."

The first assignment of the semester (suggested by Victor Donnay of Bryn Mawr College) was to write about a good math memory—a memory of noticing a pattern. I described mowing a rectangular lawn and noticing the rectangle of uncut grass in the center switch from skinny to squarish and back again. Students will have no trouble finding delightful memories if you give them encouragement and an example.

To accustom students to doing mathematics in a relaxed environment, each class began with a group exercise, either a word problem, a definition exercise (e.g., find a verifiable definition for "vertical") or an estimation problem. This practice had the added advantage of focusing the students' attention on the class—waking and warming them up. (A tip: when supervising several groups, always have something up your sleeve for the groups who finish early.) Some problems came from John Harte's excellent book, *Consider a Spherical Cow: A Course in Environmental Problem Solving*. One could easily build a course around this book alone, which attacks serious problems with algebra, estimation and a sense of humor. Working through just the first fifteen pages provided much food for thought and discussion: orders of magnitude, using physical units, exponentials and logarithms. Another rich source of problems was the Shell Centre for Mathematical Education (University of Nottingham, NG7 2RD, England). Their examination modules for secondary schools have

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(among other things) thoughtful and well-prepared worksheets for group activities. I used the module called "The Language of Functions and Graphs." Most important of all is to take problems from your own local environment. Ask students to estimate, e.g., how much money is deposited in the nearest soda machine each semester. Have them make up their own estimation problems. You can also throw in some of the classic old puzzles about coins, flies, water jugs and people with twin children.

There were two assignments designed specifically to encourage aggressive criticism of quantitative facts presented in the media. The first concerned sneaky ads: *Find two advertisements in the press, on the radio or on television that couch not-so-impressive quantitative statements in a misleadingly impressive way. Turn in a copy of each advertisement (if you choose to do a radio or TV ad you must turn in a tape or an exact transcript) with a brief explanation of (1) the false conclusion the advertisers hope you will draw and (2) the true meaning of the statement.*

Students found ads ranging from the heinous (boldface "0% interest" with fine print detailing 21% interest to be refunded 1 year later in the form of store credit) to the silly ("100 free wigs to 100 women who send for our free catalog"). The second similarly solicited critiques of quantitative statements and graphs in newspaper or magazine articles.

The language issues entered the class explicitly in two ways. First of all, we spent considerable time on the concept I have come to call "verifiable definition." A verifiable definition of a word is a description of the meaning of the word giving a **category** of objects to which the word may apply and an unambiguous **criterion** for deciding which members of the category may be truthfully described by the word. For example: a person having two X chromosomes is called *female*. Another example: a *function* is a collection of pairs of numbers with the following property: if (a,b) and (a,c) are both in the collection, then b = c. The categories in these examples are "people" and "collections of pairs of numbers," respectively. This concept is a variation on Arons' "operational definition," described in *A Guide to Introductory Physics Teaching*, with more emphasis on lack of ambiguity than on the nature of the criterion. Students had plenty of practice finding verifiable definitions (e.g., for "area", "speed" and "to count"). We discussed the concept explicitly in class and finally students wrote essays on what verifiable definitions are, when they are useful and when they are inappropriate. Second of all, we focused on mathematical texts as texts, i.e., as words written down by one human being in an attempt to communicate. Over a period of weeks, students read and compared four very different treatments of the topic of functions. Any sufficiently varied treatments would work; I used an old-fashioned book (Griffin), a product of recent reform (Hughes-Hallett et al.), a common cookbook approach (Shenk) and a text stressing rigor (Spivak). The first assignment of a series was to react to the texts: *First Reaction: skim all four texts. What do you notice? What do you like? Dislike? How do they make you feel? How does the writing differ from other kinds of writing? Which has the best pictures? The best typeface? Criticize! Annotate! Compare and contrast! Submit a reaction in the medium of your choice (e.g., words, music, drawing). Be creative!*

This first assignment compelled examination of the texts without requiring comprehension. The second assignment, designed to teach students a method for approaching a difficult text, was assigned repeatedly, with reinforcement from classroom work and discussions on functions: *Detailed Reaction: Choose one of the four texts. For each of the first four new ideas you read, either (1) give an explanation in your own words and an illustrative example or (2) describe what it is you don't understand.*

The final assignment allowed students to synthesize and me to evaluate what they had learned: *Compare and Contrast: read the definition of "function" in each of the four texts. How do the definitions differ? Are all of the texts describing the same idea? Discuss the relative merits of each definition.* Judging from student feedback, a final, unifying class discussion of this series of assignments would have been helpful.

Other major, graded written work included an essay on proofs, truth and validity, a proof that the square root of 3 is irrational and an explanation of where an analogous attempt to prove the square root of 4 would break down (after discussion of a proof that the square root of 2 is irrational) and a few write-ups of problems. There were numerous minor, ungraded writing assignments, mostly reactions to various readings and write-ups of problems discussed in class.

In lieu of an exam, students wrote a final 3-5 page paper. There were three choices: to investigate an endeavor that, like mathematics, has developed a language of its own, to relate ideas learned in the class to topics of their own choosing, or to write up several mathematical proofs (e.g., from exercises in Herstein and Kaplansky). Students wrote on a variety of interesting subjects. One paper discussed the stylized language of African talking drums. Another proposed an experiment to determine if students of science and students of the humanities

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perform linguistic tasks (such as giving directions or describing objects) in measurably different ways.

One student chose to write about his personal experience in the class; anyone doubting the worth of such a class should consider this excerpt:

> Not only have I been given a chance to understand math, but I was encouraged to question and explore it. Aspects of math which had before seemed inconceivable or illogical began to make sense. Numbers which I had always assumed were infallible were now open to question and approachable. Imaginary numbers and non-Euclidean geometries, I found, were not esoteric inventions of distant brilliant mathematicians, but actually logical systems which could be explained concretely with a pencil, ruler, and a piece of paper, or maybe with an orange. For me, any number with over three digits was large and inconceivable, but through practical estimations and critical reading of quantitative information I have learned the discrepancies beween large numbers. I have also become aware of how easily they can be manipulated to fool people like me who when they see more than

three digits do not even take the time to read the number. The realization that numbers can be used deceptively and that they can reveal essential information has been very beneficial to me. I am more comfortable, inquisitive, and alert when handling quantitative information for my other classes, not to mention in my daily encounters with numbers. I can even feel comfortable reading my bank statements.

Anyone wanting further information should not hesitate to contact me.

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