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A Test of the Theory of Optimal Taxation for the United States, 1869-1989

Gregory Hess
Claremont McKenna College

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Gregory D. Hess*

Abstract—A popular theory of optimal tax policies suggests that tax rates should follow a random walk This paper extends the existing empirical literature in three ways First, the impact on the marginal utility of consumption when the government chooses a tax plan to smooth the distorting impact of taxes is considered Second, exogenous changes in the real rate of interest are incorporated into the government's optimal tax plan Finally, the tax elasticity of output is not constant over time Allowing for these changes, there is evidence that the government discounts the future, attempts to smooth the distorting impact of taxes on the marginal utility of consumption, and that the tax elasticity of output moves predictably during wars

I. Introduction

The theory of optimal taxation requires that the path of tax rates be chosen to minimize the deadweight burden of taxation, given an initial level of government debt and an expected future path for government expenditures. To date, empirical tests of optimal taxation have relied upon simplifications of the theory in order to derive testable models. Sahasakul (1986) derived, in a perfect foresight model, the result that tax rates should be uniform across time if the tax elasticity of output is constant. Bizer and Durlauf (1990) extended this approach by solving a stochastic linear-quadratic model of taxation, which implied that tax rates should follow a random walk. These papers find evidence that lagged business cycle data and lagged government spending improve the explanatory power of tax Euler equations, therefore rejecting the uniform taxation hypothesis. Bizer and Durlauf also discover an eight-year cycle in tax changes which they attribute to the political business cycle.

However, rejections of the random walk hypothesis that are due to lagged business cycle effects and lagged government spending may be attributed to their simplifying assumptions. This paper extends the empirical test of the optimal taxation hypothesis in three ways. First, this model considers the impact from the government's choice of tax plans on the marginal utility of consumption. Incorporating the government's ability to smooth consumption may account for the rejection of the models due to the business cycle. Second, whereas in the existing literature the real interest rate factor is fixed, I allow exogenous changes in the real rate of interest to affect the optimal tax plan.[1]

Finally, the assumption that the tax elasticity of output is constant is relaxed. One would expect, for example, that the labor supply response to tax changes would be less elastic during periods of war than otherwise. Therefore, wars that involve dramatic movements in national resources are times in which the linear-quadratic specification is poor Using data for changes in real defense expenditures. I estimate the effect of wars on the tax elasticity of output. The model's overidentifying restrictions are not rejected, suggesting that allowing for consumption smoothing and changes in the tax elasticity of output can explain sources of rejection for the uniform tax model that are due to the business cycle and war, although not rejections due to the political business cycle.

II. The Model

The government chooses the sequence of tax rates and path for public debt to maximize its intertemporal utility function, subject to a sequence of budget and resource constraints, and initial and transversality conditions on outstanding debt. The government's intertemporal utility function is considered to reflect the public's preferences.

$$\left\{\max_{\tau_t, b_{t+1}}\right\}^{E_0} \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{1}$$

subject to

$$R_t^{-1} b_{t+1} + \tau_t y_t = b_t + g_t \tag{2}$$

$$y_t = c_t + g_t \tag{3}$$

$$b_0 > 0 \tag{4}$$

$$\lim_{t\to\infty} \prod_{j=0}^{t} R_j^{-1} b_{t+1} \to 0 \tag{5}$$

[1] This implies that real interest rates are not affected by taxes or the amount of public debt outstanding. Evans (1985) examines U.S data and rejects the hypothesis that large budget deficits raise interest rates.

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* Federal Reserve Board

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where

β = The government's time discount factor

c_t = Real consumption in period t

$U(\cdot)$ = CRRA utility function, $U(c_t) = c_t^{\gamma}/\gamma$, $\gamma \in (0, 1]$, where γ is the risk aversion parameter

R_t = The real interest rate factor

b_{t+1} = The number of one period discount bonds outstanding at the beginning of time period $t+1$

y_t = Real output

$\{g_t\}$ = The exogenous sequence of real government expenditures, net of seignorage.

Output is assumed to depend negatively on the current tax rate and positively on the current realization of the positive, exogenous shock, x_t, namely,

$$y_t = y\Big(\underset{(-)}{\tau_t}\,,\ \underset{(+)}{x_t}\Big) \tag{6}$$

where the shock, x_t, could be correlated with g_t due to wars.

At time t the government knows x_t, R_t and g_t, and has expectations over the future course of these sequences. By restricting these shocks so that a feasible tax plan exists, the Euler equation is

$$\frac{U'(c_t)(\partial y_t/\partial \tau_t)}{(y_t + \tau_t(\partial y_t/\partial \tau_t))} = E_t \frac{\beta U'(c_{t+1})(\partial y_{t+1}/\partial \tau_{t+1})R_t}{(y_{t+1} + \tau_{t+1}(\partial y_{t+1}/\partial \tau_{t+1}))}. \tag{7}$$

The left-hand side of equation (7) is the loss in utility at time t from raising an additional of tax revenue at time t. The right-hand side is the expected discounted loss in utility from raising an additional unit of revenue at time $t+1$, and paying a premium for not having raised the unit of revenue at time t, equal to R_t.

Rearranging (7) and imposing the CRRA specification for utility yields

$$E_t\beta(c_{t+1}/c_t)^{\gamma-1}(\tau_t/\tau_{t+1})(\eta_{t+1}/\eta_t)((1-\eta_{t+1})/(1-\eta_t))R_t - 1.0 = 0. \tag{8}$$

Consider the perfect foresight case. If the tax elasticity of output is constant, the real interest rate factor is constant and equal to β^{-1}, and the government is risk neutral, then tax rates should be constant across time periods. This is similar to the simplification made by Kingston (1984).[2]

[2] In a model with endogenous labor-leisure choice and gross real returns on government bonds, Kingston (1991) shows that the result that tax rates should be constant depends on CRRA utility and the constant elasticity assumption. However, this result depends critically on his assumption that the net real returns to public debt are both state contingent and under direct government control.

Trehan and Walsh (1990) also incorporate fluctuating elasticity and marginal revenue effects on optimal taxation, but do not allow for the consumption smoothing effect. Instead, they analyze the smoothing of a tax distortion function with fluctuating preferences, similar to Poterba and Rotemberg (1990). To empirically identify their "revenue/preference" shocks, however, they take a logarithmic, first-order approximation of their Euler equation and move the shock into the error term as an unobserved moving average process. Rather than make a similar approximation to equation (8), I directly estimate the effects of wars on movements in the tax elasticity of output

III. Empirical Results

A. *The Data*

To estimate the tax Euler equation, data for real per-capita consumption, excluding purchases of consumer durables, the ex-post real interest rate factor on government debt, and the average tax rate was needed. The tax rate series, total federal government receipts less transfers from the Federal Reserve divided by nominal GNP, was obtained from Kremers (1985) and Barro (1981) The real interest rate factor was calculated using inflation data for consumer non-durables and the nominal annual interest rate factor on marketable government debt. The latter is the computed annual interest charge, defined as the average interest rate "that would be paid if each interest bearing issue outstanding at the end of the year should remain outstanding for a year at the applicable annual rate of interest" Prior to 1938 these data are available from the Department of Commerce (1975), and thereafter is published by the Treasury Department.

The post-1929 data for population, real personal consumption of non-durables and its deflator, and real government defense expenditures is from the National Income and Product Accounts data base. The pre-1929 data for these variables was obtained as follows. the data for real consumption of non-durable goods and its deflator is from Kuznets (1961), and Kendrick (1961) The data for defense spending and population is from the Department of Commerce (1975).

B. *Results with Constant Elasticities*

Since the majority of the empirical literature assumes that the tax elasticity of output is constant, imposing this assumption is a first step in testing the "naive" optimal tax hypothesis. Expression (9) simplifies to

$$\beta(c_{t+1}/c_t)^{\gamma-1}(\tau_t/\tau_{t+1})R_t - 1.0 = \epsilon_{t+1}. \tag{9}$$

FIGURE 1.—RESIDUALS FROM EQUATION (9)
Estimation Range: 1869–1989

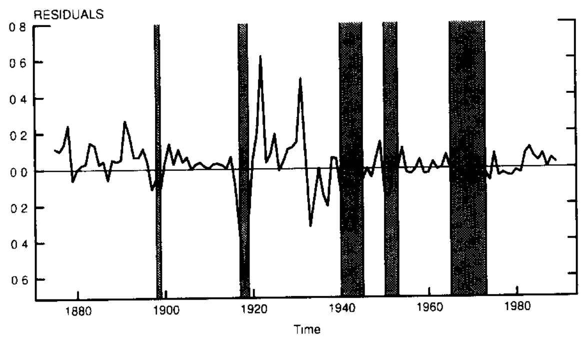
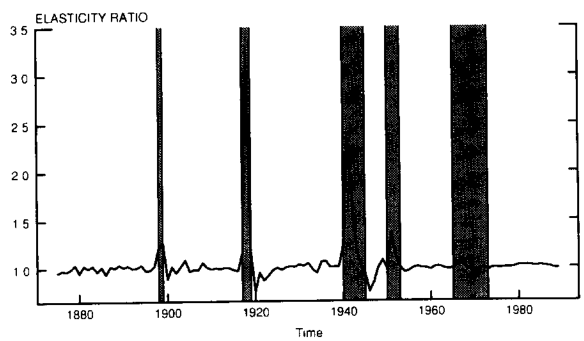


FIGURE 2 —ESTIMATED ELASTICITY RATIO FROM EQUATION (10)
Estimation Range. 1869–1989



Equation (9) is estimated using generalized instrumental variables (see Hansen and Singleton (1982)). Because the data are averaged over times that are large relative to decision periods for consuming, a first order moving average is likely induced onto the error term (see Working (1960)). To accommodate the error term's moving average structure, the instruments are lagged sufficiently and the Newey-West (1987) correction is incorporated into the calculation of the optimal weighting matrix.[3] This technique is also robust to heteroskedasticity of unknown form, which is relevant since the size and magnitude of the government expenditure forcing variable varies greatly over time. Finally, the J-statistic, which tests for violations in the orthogonality conditions implied by the Euler equation, is reported

[3] In the results presented below, the estimates are essentially unchanged if we allow for an MA(2) error Auxiliary diagnostic tests reveal that an MA(1) is a good statistical representation for the residuals However, if we assume the error term is white noise, for the full sample the estimate of the risk aversion parameter is significantly greater than one and a Lagrange multiplier test for no serial correlation is rejected at the 1% level

TABLE 1.—ESTIMATES OF THE TAX EQUATION—CONSTANT ELASTICITIES

	NLAGS	β	γ	σ	*J*-statistic
1869–1989	2–4	972[c]	780[c]	.144	952
		(022)	(.271)		
1869–1947	2–4	.960[c,d]	776[c]	174	954
		(012)	(287)		
1948–1989	2–3	994[c,d]	657[c,d]	051	329
		(.002)	(184)		

Note *J*-statistic = Significance level for Hansen's test for the null hypothesis that the residuals and instruments are orthogonal $\hat{\sigma}$ = Estimated standard error of the regression *NLAGS* = The number of instrument lags employed Instruments include a constant, and lags and squared lags of the variables (c_{t+1}/c_t), (τ_t/τ_{t+1}), R_t and (D_{t+1}/D_t)
[a] Significantly different from zero at the 1% level
[b] Significantly different from zero at the 5% level
[c] Significantly different from zero at the 10% level
[d] Significantly less than one at the 5% level

The estimates for the full sample, 1869–1989, are presented in the upper panel of table 1. The estimate for β is 0.972 and is not significantly less than one at the 5% level. The estimate of the elasticity of intertemporal substitution/risk aversion parameter is .780, and is significantly different from zero at the 1% level but is insignificantly less than one. The over-identifying restrictions are rejected only at over the 90% significance level.

To test the stability of the specification, the model is estimated over the sub-samples 1869–1947 and 1948–1989 These estimates are presented in the bottom two panels of table 1. For the 1869–1947 sub-sample, the estimated value of β is 0.960, which is significantly less than one, and that for γ is 0.772, which is significantly different from zero but insignificantly less than one. Again, the over-identifying restrictions are not rejected. The estimates of the parameters for the 1948–1989 sub-sample differ from above in three crucial respect. The parameter estimate for γ is equal to .657 which is statistically different from zero at the 1% level, as well as statistically less than one at the 5% level. Also, the estimated value for the discount factor rises to 0.994, although it is still significantly less than one. The increase in the estimate for β is perhaps due to the negative *ex-post* real interest rates during the 1970s Finally, the estimated standard error of the error drops dramatically in the post-WWII sample to 0.051, due in part to smaller fluctuations in the aggregate economy and defense expenditures during wars in this time period.[4]

The residuals from the full sample are presented in figure 1 An examination of the residuals in figure 1 demonstrates their tremendous volatility during the

[4] For example, changes in real defense spending were much more dramatic for World War I than for the Vietnam War See the discussion below and the estimated impact of changes in real defense expenditure on the tax elasticity of output presented in figure 2.

TABLE 2 —ESTIMATES OF THE TAX EQUATION— NON-CONSTANT ELASTICITIES

	NLAGS	β	γ	ζ	σ	*J*-statistic
1870–1989	2–4	966[c,d]	.570[b]	312[c]	132	.646
		(010)	(290)	(054)		
1870–1947	2–4	.946[c,d]	766[b]	239[c]	146	728
		(012)	(303)	(050)		
1948–1989	2–3	995[c,d]	479[c,d]	249[a]	049	835
		(001)	(.184)	(146)		

Note See notes to table 1

World Wars. In particular, the residuals display a similar hump shaped pattern that initially descends during the early war years (around 1917 for WWI and 1940 for WWII) and tends to reverse itself at the war's conclusion. To a lesser extent, the Spanish-American War and the Korean War display similar dynamics

C Results with Non-Constant Elasticities

The behavior in the residuals from the optimal tax model with constant elasticities suggests that it may be too restrictive. Consider the following pattern of elasticity fluctuations during war periods that would account for the observed pattern of residuals in figure 1 Suppose the tax elasticity of output in early war years is extremely inelastic in comparison to non-war periods, due to the economy's need to more fully employ the resources of production for war The effects of this initial positive shock to the ratio of elasticities may persist until the completion of the war, when the factors of production are re-deployed away from the war effort At this point, output might be relatively elastic until the re-deployment of resources is complete Therefore, in response to a major war, a plausible path for the ratio of the elasticities is for them to be greater than one and decreasing in magnitude early in the war, followed by values less than one following the conflict's termination.

To test the hypothesis that elasticity movements are due to changes in resources due to major wars, I use defense spending to proxy these resource movements.[5] The elasticity portion of equation (8) is modeled as follows:

$$(\eta_{t+1}/\eta_t)((1-\eta_{t+1})/(1-\eta_t)) = 1 + \zeta \cdot ((D_{t+1}/D_t) - 1) \qquad (10)$$

where D_t is real per-capita defense spending and $\zeta > 0$.

The results for the estimation period 1869–1989 are presented in the upper panel of table 2. The estimated parameter value for the discount factor is 0.966 and is significantly less than one at below the 1% level. The estimate of the risk aversion parameter is 0.570, and is significantly different from zero but insignificantly less than one at the 5% level. The estimate of ζ is 0.312 and significantly different from zero at the 1% level—a one percentage point increase in real defense spending during times of major wars corresponds to approximately a one-third of a percentage point increase in the tax elasticity of output.[6] Again, the *J*-test for model missspecification is not rejected.

A graph of the estimated ratio of the elasticities is presented in figure 2 The pattern is similar to the expected pattern from innovations in the elasticity of output during periods of major wars. This provides strong evidence for directly incorporating movements in the elasticity of output when testing optimal tax models using historical data for the United States

Similar results are found for both sub-samples. For the 1869–1947 sample, the coefficient estimates for β and γ and ζ are 0 946, 0 766 and 0.239, respectively The time discount factor is significantly less than one, whereas the elasticity of substitution parameter is not. For the 1948–1989 sample, the coefficient estimates for β and γ and ζ are 0.995, 0.479 and 0 249, respectively. The discount factor and coefficient of risk aversion are both significantly less than one, providing evidence for discounting and consumption smoothing. In both subsamples, ζ is significantly different from zero. However, for the post-WWII period, allowing for non-constant elasticities only lowers the standard error of the estimate to 0.049 from 0.051, suggesting that elasticity changes do not account for much of the model's error during this period.

IV. Conclusion

This paper finds empirical support for considering fluctuations in the tax elasticity of output when testing the optimal taxation hypothesis—a one percentage point increase in real defense spending during times of major wars corresponds to approximately a one-fourth to one-third percentage point increase in the tax elasticity of output. This leads to approximately a 20% reduction in the standard error of the Euler equation for the time period 1869–1947 Allowing for these movements, there is evidence that the government discounts the future and attempts to smooth the distorting impact of taxes on the marginal utility of consumption. Finally, the finding that the model does not reject the over-identifying restrictions suggests that the

[5] The ratio of government expenditures and military mobilizations in the labor force were also analyzed. These variables produced similar patterns of results, although often at the expense of increasing the number of instruments

[6] Since the magnitude of the elasticities is small, fluctuations in $(1-\eta_t)/(1-\eta_{t+1})$ are essentially zero, and are dominated by fluctuations in (η_t/η_{t+1}) Setting $(1-\eta_t)/(1-\eta_{t+1})$ equal to zero and re-arranging (10) allows the direct comparison of percentage point changes in elasticities and real defense spending

uniform tax model may be too simplistic with regards to its treatment of business cycle conditions.

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TESTING FOR SERIAL CORRELATION IN REGRESSION MODELS WITH LAGGED DEPENDENT VARIABLES

Robert K. Rayner*

Abstract—Bootstrap methods are investigated for approximating critical points to several widely-used tests of serial correlation in regression models with lagged dependent variables. Simulation results suggest that the bootstrap accurately estimates the null distributions of the tests, in contrast to conventional approximations. Results of some studies on the size-adjusted power of the tests are also reported.

I. Introduction

Since its introduction, there has been an ongoing debate in econometrics on the appropriateness of the Durbin-Watson d test for serial correlation in models with lagged dependent variables, see for example Dezhbakhsh (1990) and the references therein. Partly as a response to this debate, Durbin (1970) developed two large-sample tests, denoted by h and m, as alternatives to the d test for these kinds of models.

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*Anjoy QSP, (TM) Ann Arbor, MI 48108

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The performance of the d, h, and m tests in finite samples has also been the subject of considerable controversy As Inder (1984) noted, however, much of the apparent difference in empirical results is likely due to the fact that authors have used different approximations to critical values for the tests.

Recently, Dezhbakhsh (1990) provided some additional Monte Carlo evidence on the relative performance of the tests. After surveying several leading journals and finding that, in many applications models with more than one lagged dependent variable are used, he conducted the first simulation studies on such models. His results suggest that the upper bound, d_U for the d test can be a very poor approximation to critical points for the test.

Dezhbakhsh's and other empirical studies underscore the importance for practical applications of determining the null distributions of tests for serial correlation in models with lagged dependent variables. Those studies further highlight the difficulty in making power comparisons when the tests have different size.

This paper considers bootstrap methods for estimating critical points for several tests for serial correlatio