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Prologue: The following vignette intends to provoke thought and discussion on the question of how to introduce non-mathematicians to the joys of mathematics. In the future, when my dissertation ceases to distract me, I hope to write a longer article elaborating on the assertions contained herein.

Meals provide a unique opportunity to enjoy company while satisfying a basic appetite. As lively animals, we sit around the table as if it were a trough, and as high-minded animals, we take the opportunity to converse. Our conversations often carry us to realms far away from the dining room, and when many of us are mathematicians, our conversations sometimes lead us to abstract planes of exotic spaces defined by abstruse axioms.

A few months ago, a dinner party provided an opportunity for myself and another mathematician to try to persuade an undergraduate liberal arts major to explore our discipline....

The meal for which we had gathered was dinner. It was hosted by a classmate of mine from the PhD program in math. As we were being served, an undergraduate at the party asked for opinions on what he should take during his last semester. My classmate suggested that he take abstract algebra. The undergraduate expressed some reservations. A lively exchange ensued. I could not help myself from joining in the fray, and in the end we managed to win him over.

The undergraduate gave us many reasons why the prospect of an algebra class did not immediately excite his interest. First, he expressed some scepticism about the value of taking a math course because he did not anticipate needing mathematical methods in his future academic work in literature. (He came just short of expressing aversion of a "Math, ugh" variety.)

We argued that mathematics should be an integral part of the liberal arts education because it provides a forum to explore the power of abstraction and formal logic. Though there is more to mathematics than abstraction and formal logic, these elements are of primary concern to mathematicians in a way that is unparalleled even in the most mathematical of the sciences. Exposure to rigorous mathematics raises awareness of the role of formal logic in other fields. In the process, students sharpen their logical skills, skills which are important to all areas of intellectual endeavor.

Convinced that he should take some math course, the undergraduate told us that his friends' experiences in their calculus courses had not been as enlightening as our advertisements promised. Calculus claims a unique position in college mathematics departments as the most commonly taught course. It gained this stature because of its usefulness in the physical sciences. The useful concepts and methods of calculus include functions, continuity and differentiation, which are as complicated as they are applicable. This makes calculus a difficult course to teach effectively; often the methods are taught while the concepts sacrificed. Even in the most theory-oriented calculus class, the complexity of the concepts obscures the formal logic of the underlying theory. All this said, we added that the interplay between method and theory in calculus represents another part of the discipline of mathematics, a very rich and exciting part, but this aspect of the discipline is much less accessible at an introductory level.

Our undergraduate interlocutor was acutely aware of the math department's reputation for inaccessible material. He doubted that he had the necessary prerequisites to enter an abstract algebra course. Of course, there are prerequisites for algebra addition, multiplication, whole numbers, fractions and polynomials, basics which our undergraduate had encountered many times in primary and secondary school. Abstract algebra introduces students to a formal system of definitions that characterize these familiar objects. This formal system provides a remarkable structure to what begin as collections of facts. Furthermore, students see how a small set of axioms can allow them to prove rigorously familiar properties of their old mathematical friends, leading them to reevaluate why they accepted certain facts (like prime factorization) without rigorous proof. Students also meet new mathematical objects that share the formal properties of objects that they already know well. This process shows students something of the heart of mathematics, how it moves from familiar country to new frontiers along a path of logic and abstraction.

And so our once reluctant undergraduate friend was persuaded to follow our suggestion. I was pleased that he agreed and even more pleased to have had an opportunity to remind myself of how inspiring I find the discipline to which I hope to dedicate much of my efforts.

Epilogue: Indeed, the undergraduate did take our advice and signed up for Abstract Algebra. He enjoyed the material for the first half of the semester, which included the Sylow Theorem, but after that point he found himself overwhelmed by the very fast pace of the syllabus that was designed to challenge the advanced math majors in the class. He would certainly have remained an enthusiastic participant in a course designed for non-majors or a course for majors without such impressive backgrounds.

Thanks to the host of the dinner, Dani Wise, my classmate in the Princeton math Ph.D program and fellow mathematics missionary.