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Review: Nearly Invariant Subspaces for Backwards Shifts on Vector-Valued Hardy Spaces

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Nearly invariant subspaces for backwards shifts on vector-valued Hardy spaces. (English summary)

J. Operator Theory **63** (2010), no. 2, 403–415.

Recall that a subspace \mathcal{F} of $H^2(\mathbb{D}, \mathbb{C}^m)$ is *nearly invariant* under the backward shift S^* if \mathcal{F} is closed and if every element $f \in \mathcal{F}$ with $f(0) = 0$ satisfies $S^* f \in \mathcal{F}$. In the scalar case these spaces were introduced by D. Hitt [Pacific J. Math. **134** (1988), no. 1, 101–120; [MR0953502 \(90a:46059\)](#)] in a study of the simply S -invariant subspaces of the Hardy space on an annulus. They were later studied by D. E. Sarason [Mem. Amer. Math. Soc. No. **56** (1965), 78 pp.; [MR0188824 \(32 #6256\)](#)].

In this article, the authors characterize the subspaces of a vector-valued Hardy space which are nearly S^* -invariant. This provides a vectorial generalization of a result of Hitt [op. cit.]. The authors also study compact perturbations of shifts which are pure isometries. Extending a result of D. N. Clark [J. Analyse Math. **25** (1972), 169–191; [MR0301534 \(46 #692\)](#)], the unitary operators which are compact perturbations of restricted shifts are described. The authors' classification of nearly invariant subspaces yields information on the simply shift-invariant subspaces of the vector-valued Hardy space $H^2(A, \mathbb{C}^m)$ of an annulus A . In particular, the authors generalize a result of A. Aleman and S. Richter [Integral Equations Operator Theory **24** (1996), no. 2, 127–155; [MR1371943 \(99b:47010a\)](#); erratum, Integral Equations Operator Theory **29** (1997), no. 4, 501–504; [MR1484863 \(99b:47010b\)](#)] by proving that any such subspace has index bounded by m .

Reviewed by *Stephan R. Garcia*

References

1. J. Agler, N.J. Young, Functions which are almost multipliers of Hilbert function spaces, *Proc. London Math. Soc.* (3) **76**(1998), 453–475. [MR1490245 \(99g:46027\)](#)
2. A. Aleman, S. Richter, Simply invariant subspaces of H^2 of some multiply connected regions, *Integral Equations Operator Theory* **24**(1996), 127–155; Erratum, *Ibid*, **29**(1997), 501–504. [MR1371943 \(99b:47010a\)](#)
3. C. Benhida, D. Timotin, Functional models and finite-dimensional perturbations of the shift, *Integral Equations Operator Theory* **29**(1997), 187–196. [MR1472098 \(98d:47032\)](#)
4. C. Benhida, D. Timotin, Finite rank perturbations of contractions, *Integral Equations Operator Theory* **36**(2000), 253–268. [MR1753419 \(2001f:47011\)](#)
5. I. Chalendar, N. Chevrot, J.R. Partington, Invariant subspaces for the shift on the vector-valued L^2 space of an annulus, *J. Operator Theory* **61**(2009), 101–118. [MR2501008 \(2010b:46053\)](#)
6. J.A. Cima, A.L. Matheson, W.T. Ross, *The Cauchy Transform*, Math. Surveys Monographs, vol. 125, Amer. Math. Soc., Providence, RI 2006. [MR2215991 \(2006m:30003\)](#)

7. D.N. Clark, One dimensional perturbations of restricted shifts, *J. Anal. Math.* **25**(1972), 169–191. [MR0301534 \(46 #692\)](#)
8. S.R. Garcia, Conjugation and Clark operators, in *Recent Advances in Operator-related Function Theory*, Contemp. Math., vol. 393, Amer. Math. Soc., Providence, RI 2006, pp. 67–111. [MR2198373 \(2007b:47073\)](#)
9. E. Hayashi, The kernel of a Toeplitz operator, *Integral Equations Operator Theory* **9**(1986), 588–591. [MR0853630 \(87m:47068\)](#)
10. D. Hitt, Invariant subspaces of \mathcal{H}^2 of an annulus, *Pacific J. Math.* **134**(1988), 101–120. [MR0953502 \(90a:46059\)](#)
11. Y. Nakamura, One-dimensional perturbations of the shift, *Integral Equations Operator Theory* **17**(1993), 373–403,. [MR1237960 \(94h:47019\)](#)
12. N.K. Nikol'skii, *Treatise on the Shift Operator*, Grundlehren Math. Wiss., vol. 273, Springer-Verlag, Berlin 1986. [MR0827223 \(87i:47042\)](#)
13. J.R. Partington, *Linear Operators and Linear Systems*, London Math. Soc. Stud. Texts, vol. 60, Cambridge Univ. Press, Cambridge 2004. [MR2158502 \(2006d:93001\)](#)
14. A. Poltoratski, D. Sarason, Aleksandrov–Clark measures, in *Recent Advances in Operator-related Function Theory*, Contemp. Math., vol. 393, Amer. Math. Soc., Providence, RI 2006, pp. 1–14. [MR2198367 \(2006i:30048\)](#)
15. H.L. Royden, Invariant subspaces of H^p for multiply connected regions, *Pacific J. Math.* **134**(1988), 151–172. [MR0953505 \(90a:46056\)](#)
16. D. Sarason, The H^p spaces of an annulus, *Mem. Amer. Math. Soc.* **56**(1965). [MR0188824 \(32 #6256\)](#)
17. D. Sarason, Nearly invariant subspaces of the backward shift, in *Contributions to Operator Theory and its Applications (Mesa, AZ, 1987)*, Oper. Theory Adv. Appl., vol. 35, Birkhäuser, Basel 1988, pp. 481–493. [MR1017680 \(90m:47012\)](#)
18. I. Serban, F. Turcu, Compact perturbations of isometries, *Proc. Amer. Math. Soc.* **135**(2007), 1175–1180. [MR2262923 \(2007j:47023\)](#)
19. D.V. Yakubovich, Invariant subspaces of the operator of multiplication by z in the space E^p in a multiply connected domain, *J. Soviet Math.* **61**(1992), 2046–2056. [MR1037771 \(91c:47061\)](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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