Dramathizing Functions: Building Connections between Mathematics and Arts

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Synopsis

This article focuses on connections between mathematics and performance arts (drama). More specifically we offer an exposition of a segment of college algebra mathematics (an introduction to functions), with an approach primarily emphasizing the aesthetic aspects of mathematical learning, teaching, and performing.

Keywords: *connections; representations; functions; dramatization; mathematics education; technology; dynamic geometry software*

In this article, I share some of my classroom teaching experiences using drama as a method for delivering mathematical ideas, which for me and my students has been an effective way of exploring mathematics. Dramatizing is one of many powerful ways of artistically expressing mathematical ideas, hence bringing mathematics to life. For this I use the term *dramathization*, as a metaphor that stands for the dramatization of mathematical practices. The scene of the mathematics classroom itself is a theatre where different characters exhibit their roles. The teacher and his/her students are all actors on the same stage. Could drama be used in the teaching and learning of mathematics? In this article, I introduce a mathematical scenario in an attempt to respond to this question.

The specific experiment described in this article involves an exposition of a segment of college algebra mathematics (an introduction to functions). The suggested dramatization-based mini-curriculum aims to support the teaching of the foundational structures of a function. My approach primarily emphasizes the aesthetic aspects of mathematical learning, teaching, and

performing. Composed of two acts, which may take about 20 minutes of a mathematics lesson, this scenario dramatizes some crucial elements of the function concept such as *ordered pair*, *set of ordered pairs*, *image*, *pre-image*; and suggests the use of "mirror" to represent a function. The mathematical experience and hence the supporting curriculum necessitate active participation of the teacher and the students — namely the actors — as well.

Dramathization can be based on a script, or an on-stage improvisation by students. If based on a script, the teacher could give students some time (a week or so) to practice before the drama takes place. If based on an improvisation, on the other hand, the teacher would rather first introduce the concepts (e.g., ordered pair, relation, set of ordered pairs, relation, function, squaring function, cubing function, etc.) to the students before an on-stage improvisation takes place. By experience, I know that students need scaffolding and guidance when they are asked to perform an on-stage improvisation.

I wrote the following scenario based on what I remember from my students' on-stage-improvisations. They were introduced to the concepts of ordered pair, set of ordered pairs, relation, function, squaring function, cubing function, before they were asked to perform dramathematically.

INSTRUCTIONS FOR THE ON-STAGE-IMPROVISATION

- The student M representing the mirror takes the stage.
- Student M represents the squaring function.
- Student A represents number 2 and Student B represents number 4.
- A and S take the stage in such a way that M resides in between them.
- A will be the one to look in the mirror. M faces Student A's face.
- A, S, and M perform their act.
- A and S come forward; show themselves to audience, conclude their part.
- Other students then in pairs come to stage and perform their act (similar to A and S)

ACT ONE: YOU ARE MY SQUARE, YOU ARE MY IMAGE...

Student A: [*to Student B*] You are my square ... when I look in the mirror, I don't see myself at all. All I see is you; you are my image in the mirror.

Student B: [*to Student A*] I am your square and I belong to you. You are my pre-image in this mirror.

[The mirror (Student M) comes forward facing the audience]

Student M: 2 cannot see itself through me because I am the one who creates all images. 2's image is 4 through me, I am the squaring function...

[Student M then takes her original position in between Student B and Student A, facing Student A]

[Hand in hand, Student A and Student B walk forward and show themselves to the audience. They are paired in a particular order. The student facing the mirror, Student A, is on the left, and his "image," Student B, on the right. The mirror then claims possession]

Student M: Now you two belong to me ... stay where you are ...

Student C and Student D, representing the numbers 3 and 9, respectively, take the stage. Student M is in between Student C and Student D, and Student C's face is facing Student M's face

Student C: [*facing the mirror*] I am 3 and I am looking into this mirror now and my image is not myself at all! I see you Student D! You are my image through this mirror ...

Student D: [*to Student C*] I am your square ... I belong to you, I am yours ... I am your image and you are my pre-image!

Student C: [*to Student D*] You are my square, and you are my image ... You are mine. I am your pre-image!

Student M: [*to Student C and Student D*] Okay now you now what you got to do. [Student C and Student D, hand in hand, walk forward the audience to show themselves in a particular order where Student C is on the left, and Student D on the right]

Student M: You two belong to me, too. Not you Student C alone, and not you Student D alone; but you together as a pair belong to me, in that specific order. Stay where you are and don't go anywhere!

[In a similar manner, various students (Student E, Student F), (Student G, Student H),

(Student J, Student K) representing the pairs $(4, 16)$, $(5, 25)$, and $(6, 36)$, respectively, perform their dramatization, and walk forward the audience to show themselves in the same particular order: the student facing the mirror on the left, and the "image" student on the right]

ACT TWO: MIRROR CLAIMS POSSESSION

[Pairs talking to each other]

Student A: [*to Student B*] This is not something new. You have always been my square. You are my square. And you will be my square forever. Because I am 2, and what I am claiming possession is 4, namely my square. It's in the name: my square, you are mine.

Student A and Student B: [*hand in hand*] We stay where we are and we don't go anywhere.

Student F: [*to Student E*] I am your image through that particular mirror. I am your square; I am yours. I belong to you ... through that mirror. You are my pre-image.

Student E and Student F: [*hand in hand*] We stay where we are and we don't go anywhere.

Student H: [*to Student G*] I am 25, Student G. I am the square of 5, I am the square of you ... I am your square ... I am yours ... I belong to you ... through that mirror.

Student G and Student H: [*hand in hand*] We stay where we are and we don't go anywhere.

Student M: You all belong to me [embracing each pair with her arms, one by one, respectively] you two, and you two, and you two, and you two, and you two ... you all belong to me in pairs ... in that particular order. Pre-images on the left! Images on the right! Stay where you are and don't go anywhere ...

All Ordered Pairs Together: [*hand in hand*] We stay where we are and we don't go anywhere. [*to Mirror*] We all belong to you ...

CONNECTIONS

When implementing this drama, teachers could share a "connection chart," such as the one below, with their students.

Students	Mathematical Connection
Student A and Student B (Student A's image)	Ordered Pair $(2, 4)$
Student C and Student D (Student C's image)	Ordered Pair $(3, 9)$
Student E and Student F (Student E's image)	Ordered Pair $(4, 16)$
Student G and Student H (Student G's image)	Ordered Pair $(5, 25)$
Student J and Student K (Student J's image)	Ordered Pair $(6, 36)$
Student M (the mirror)	The function $f(x) = x^2$ as a set of ordered pairs

Table 1: Connection Chart.

My students performed a similar dramatization to represent the cubing function $f(x) = x^3$ as well. In another example, they also represented a function that assigns "number of sides" to various polygons, in which case the pre-images were polygons (e.g., triangle, square, rectangle, pentagon, etc.) and the images were the numbers of sides of these polygons. For each example, I had to emphasize that we had to "replace" the mirror accordingly. In this way, the students grasped the meaning of a function. This understanding provides a basis for the next part. And that next part could be anything the instructor desires; for instance, introducing a function graph on the coordinate system and helping students figure out its analogy to their drama performance.

At this point therefore, the teacher's next task could be to show the relation between the idea of "mirror" and graphs of functions (Figure 1). For example, considering number 2 (Student A), which was in front of the mirror, its place indeed corresponds to its actual location on the x-axis. Ditto for number 3 (Student C). All numbers are seeing their images through the mirror. And their images, therefore, must be somewhere on the y-axis. The curve in Figure 1 can be interpreted as a mirror indeed. The mirror on the coordinate system is nothing but the function's graph through which the numbers on the x-axis see their images. Students also may be helped to visualize that the curve (Student M) represents a set of infinitely many ordered pairs (points).
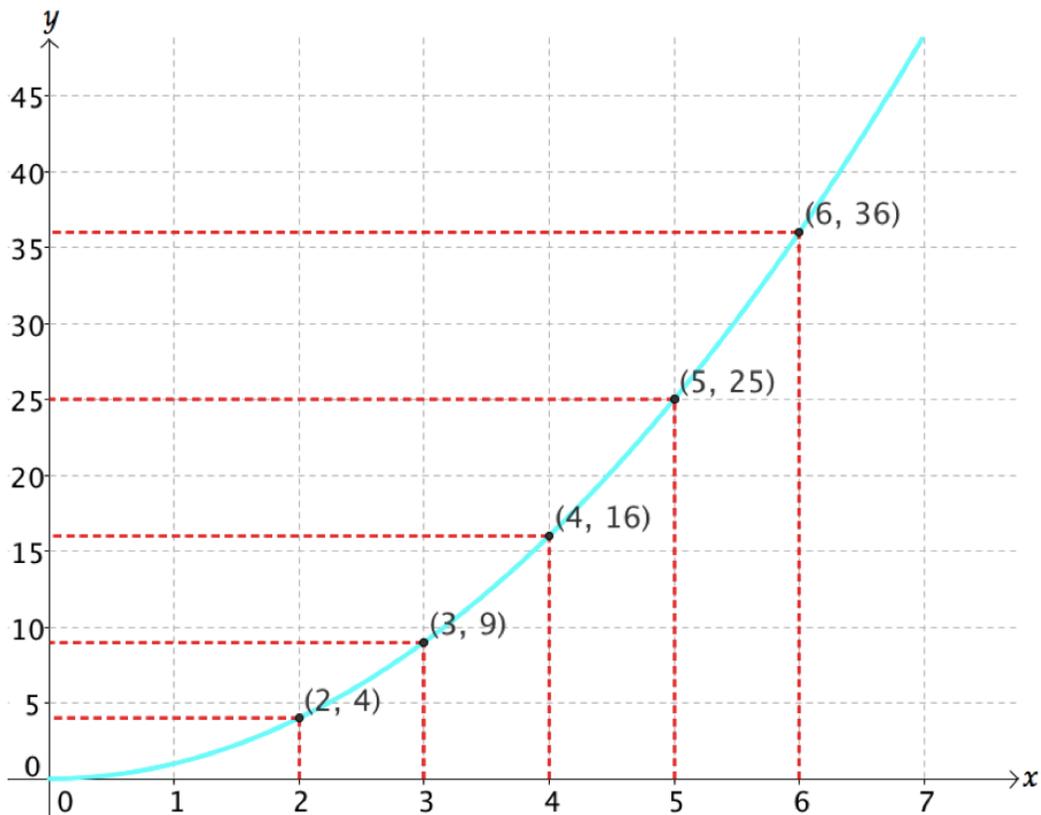
Figure 1: Squaring function graph as a mirror.

REFLECTIONS

I used these dramatized mathematical scenarios with college algebra students when introducing functions. Students enjoyed dramatizing functions, something they had never experienced before, and the activity helped them build strong foundations for the concept. It was also quite "sticky" and was much easier to remember. This was something different from simply being able to calculate the square of numbers. Student A (number 2), one of the main actors, claimed possession of Student B (number 4), because that was his image and his square. Using the idea of ownership, students might see numbers as mathematical objects. Moreover, they claimed possession of each other as actors, while becoming those numbers, hence getting themselves more involved: an immense mathematical experience.

These actors were describing each other using phrases like "my image," "my pre-image," "your image," which involve possessive adjectives. I received positive feedback from them, explicitly noting that the activity was really helping them learn about the subject and make sense of what was going on. An image was something corresponding to a pre-image through a mirror: an ordered pair was being formed based on this mirror, which is the essence of what makes a function.

Students liked these demonstrations, not (only) because it was different from the teaching styles they were used to. They liked it also because numbers had never been so concrete. They touched the numbers, they held them, and they became those numbers. In all these scenes, they claimed some kind of possession. It seems to me that once they claimed possession, it was a proof of their understanding of the subject. With the help of these dramatizations, through the mirror analogy, they seemed to have a solid understanding of the essence of what a function is.

Functions are a part of the mathematics curriculum at both the middle school and the high school level. I believe that dramatization offers a verbal context for representing relations and functions, with words in action. It seems very likely that the foundational structures (ordered pairs, image, pre-image, function as a set of ordered pairs) presented through dramatization will stick with the students for much longer than typical presentations. Dramathization could provide an opportunity for students, teachers, parents, community members, and curriculum writers to experience, understand, and represent mathematics in a new, refreshing manner.