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0. INTRODUCTION

Let me be clear about the development in school algebra I wish to track here. It is exemplified by the "solution of equations," as I learned it in my childhood around 1938. Before the era of The New Math, the setting up of linear and quadratic equations to model stories of perimeters and areas, and ages of fathers and sons,was generally done in the 9th grade. Having got an equation with x the unknown — this having been the hardest part — we would apply some rules such as that "Equals added to equals are equal", or maybe some rituals called "transposition" and "dividing through," to obtain one or more numbers we called the "solution," which we then "checked" by substitution. If the answer didn't check, one would look back for some miscalculation; otherwise we were done.

Most books — and many teachers, including my own — made little effort to put into English what we were doing. Algebra, it appeared, was a language and literature of its own, unconnected with words like "if," "then," or "but." Its pronouncements did not begin with capital letters or end in periods. It was no wonder that routine calculations like factoring were easy for us and "story problems" very hard. What can stories have to do with algebra?

It will be the purpose of most of what follows to work through a rather simple problem such as should be understandable to any beginner in high school algebra, in order to show how putting it into English makes all the difference between a ritual and an epiphany. Not that I advocate a *lesson* along these lines (it would take some weeks, I should think, and in part would have to stretch over years), but that I advocate a *curriculum* along these lines, or, if not a curriculum, a continuing conversation in algebra classes that conveys the lesson I hope to illustrate with this example.

The last few sections concern more sophisticated interpretations of this problem and a very similar one, which illustrate how mathematics of an unreal sort can get used in a real world, and how understanding of such usage would be impossible without full appreciation of the logic of the simpler versions.

1. A SIMPLE PROBLEM IN 9TH GRADE ALGEBRA

Here is a typical "story problem" such as might have been found in high school algebra in 1840 as easily as in 1997. Very likely this problem was known (and solved) in 1997 B.C. as well, in ancient Babylonia. No calculators are needed, and only the simplest arithmetic and algebraic notation enter.

<u>Problem</u>: A rectangular garden is to have an area of 600 square yards, and its length is to be fifty yards greater than its width. What are its dimensions?

<u>Solution</u>: Let x be the length; then x-50 is the width, and x(x-50) therefore the area. So:

$$x(x-50) = 600$$
$$x^2-50x-600 = 0$$
$$(x-60)(x+10) = 0$$
$$x = 60$$
$$x = -10$$

Now what? Well, -10 can't be the length of a garden, so the answer must be 60. We put a circle around the '60' and wrote, if we were meticulous, and the year was 1938, something like this:

"<u>Check</u>: Length x = 60
w = x-50 = 10
60•10 = 600, check."

In my day we got 10 points for this. What more is there to say?

A thoughtful student might wonder where that -10 came from and where it went so suddenly. "Length x = 60" we wrote; why not "Length x = -10?" If asked, the teacher might reply, "Well, that's not a length, is it?" Or, "The length can't be negative." Somewhere else in the book (fifty years ago; today it is no longer so) there might have turned up "extraneous roots," i.e. apparent answers to algebraic equations that for some reason didn't work; maybe this was such a case. Perhaps that was why we had to go through "checking the answer." We will come back to this later on. For the moment, let us consider the language in which the above solution was written. The standard format appears to be a string of equations without punctuation. Let us look again at the model "solution" as it typically appears (*verbatim*) in the student's notebook, and even as printed in many typical textbooks:

Solution: Let x be the length, so w = x-50, A = x(x-50).

$$x(x-50) = 600$$
$$x^2-50x-600 = 0$$
$$(x-60)(x+10) = 0$$
$$x = 60$$
$$x = -10$$

After the setting-up, "Let x be the length, so...", there are no commas, no periods, no words. The meaning of "x" was established at the beginning, and the rest seems to be equations, i.e. sentences so simple that periods aren't needed. But actually it is *not* clear that the meaning of x has been established, for all that it was said to be "the" length, since we seem to end with a sudden appearance of two answers, one of which (the negative number) gave little pleasure to either the textbook or the teacher. One might ask the teacher, of these last two lines in the student's notebook, does this mean "x = -10 OR x = 60" — or does it mean to say "x = -10 AND x = 60?" Neither interpretation seems to fit the idea that "x" had a *definition*, that is, a single meaning.

Yet that was the way it began: "x" was supposed to represent "length," a very definite length, a very definite number: the length of the garden wall, perhaps, or a prescription for the purchase of lumber. But isn't x also a "variable?" Maybe it is an "unknown." Is a variable or an unknown different from a number? This isn't funny. The amount of nonsense that has been written about "variables" has not only filled volumes, but has confused generations of both students and their teachers. One begins to suspect that the lack of punctuation and connectives such as "or" and "and," in the traditional way of writing the solution to this problem, are not just abbreviations, but evasions.

2. PLAYING WITH FALSE STATEMENTS

Now, what *was* that definition of x? "The length" is what was written above, as if "the length" of a garden with the given description necessarily existed, or was unique. But this is exactly what we are trying to discover: Is there such a length? Maybe there isn't. For example, one might ask for the length of the side of a rectangular garden whose perimeter is 100 yards and whose area is 1000 square yards. We can write equations until blue in the face; we can call its length x as above, so that 50-x is the width and $50x-x^2$ its area, but it should be plain that there is no such rectangle even before trying to solve the equation $50x-x^2=1000$. You simply can't enclose 1000 square yards in a rectangle with only a 100 yard perimeter (try a few guesses). Calling the length of such a rectangle "x" doesn't make x the name of anything real. This impossibility was undoubtedly known in ancient Babylonia, and most elaborately analyzed in geometric language by Euclid.

How can such a problem, as it eventuates in an equation, be understood in the first place, then? What right do we have to say "Let the length of the field be x," before we even know there is such an x? Without a more careful statement of what we are trying to find out and how, no amount of "subtracting the same thing from both sides" and the like will do us a bit of good, except maybe on multiple-choice exams. Both sides of *what*, for goodness sakes? An equality between symbols involving a possibly non-existent number — or maybe variable — named "x?" (In my second example here, 1000 for area and 100 for perimeter, x is a *certainly* non-existent length, or maybe variable, or place-holder, or unknown, yet it still seems to have a name, "x," and an equation to describe its properties. Are we permitted to debate the physiology of unicorns?

One reason for a more careful statement of the problem is that it will explain some of the wordless, comma-less, period-less "algebra" that appears in the middle of the typical solution. Consider: the textbook says we have an "axiom" stating that if A and B are numbers, and if A = B, and if C is some other number, than A-C = B-C, i.e. "subtracting the same number from equals yields equals." In the solution to the original rectangular garden problem above this fact was used in the following way:

From x(x-50)=600 we derived x(x-50)-600 = 0 by "subtracting 600 from both sides." Both sides of what?

An equation? Yes, the equation x(x-5) = 600. We call it an equation because it has an equals sign in the middle, but does that make it true? And, if it isn't true, does our axiom still hold? Indeed, the equation in question is usually false. When x = 14.7 it is false; when x = 435 it is false. What right have we to subtract 600 from two sides of an "equation" that is usually false, and then call the result a consequence of some axiom cribbed from Euclid?

(How that "axiom" got from Euclid into 19th century algebra books is a story of its own. Euclid in his axioms did not mean "equals" in the algebraic sense at all, but was talking about geometric figures, where by "equal" he meant "congruent" in the first instance, and then decomposable into pieces congruent piece by piece, and ultimately even more sophisticated equivalences than that. There is also the equality of *ratios* to be found in Euclid (Book V), with a definition of "ratio" hardly anyone remembers today. Furthermore, the "added to" and "subtracted from" phrases used by Euclid in his Postulates did not refer to anything numerical at all. Modern algebra textbooks tend to forget the origin of these axioms, and they list them along with corresponding rules for division and multiplication, too; something that would have been quite meaningless to Euclid, and which cannot be made to have meaning in his geometric context. [Footnote: See, e.g., Dressler and Keenan's *Integrated Mathematics*, Course 1, New York, Amsco School Publications 1980, p.108: "Postulate 7: Multiplication Property of Equality".])

In their present-day 9th Grade use, these statements may be *called* axioms, because of a long tradition culminating in the author's ignorance, but they are no more axioms than any other properties of the arithmetic operations construed as functions or operators. One might as well call an "axiom" the statement that if x(x-50) = 600 then log[x(x-50)] = log(600), or cos[x(x-50)] = cos(600). These statements are, as applied to "equality of numbers," nothing more than the recognition that taking cosines, subtracting 600, etc. are well-defined operations with unambiguous results. It isn't that two *numbers* are equal, in these applications, as that the two algebraic expressions are intended to be different names for a single number. In Euclid, "equality" denoted not a mere renaming of a number, but an equivalence between two genuinely different geometric entities.)

But this is a digression. Axiom or not, it is true that if two symbols represent the same number, subtracting 600 from each will yield two new symbols also representing the same number, i.e. the original number diminished by 600. Now let us return to the equation "x(x-5) = 600," which is almost always a false statement, and see why we have a right to subtract 600 from both sides of it and somehow use the result for a good purpose.

3. INDUCTIVE AND DEDUCTIVE REASONING

To understand all this we must return to the origins of algebra, which was brought to Europe in the Middle Ages by Arabs who themselves had been influenced by Indians, Babylonians and Greeks many centuries before that. The Greeks in the three hundred years between Socrates and Appollonius of Perga, and mainly in the unparalleled age of Plato's Academy, 2400 years ago, had perfected what is now called the synthetic method in geometry (and a bit of number theory as well), showing the world how to proceed from axioms and other known truths to more complicated statements by means of a sequence of airtight deductions, going from each known truth to the next by a step whose validity can no more be denied than the plain evidence of our senses — and even more so, in that Plato had some doubts about our senses that he did not entertain about geometry.

Most of human life goes in the other direction: we humans use experience more than logic. This use of experience we call inductive, as opposed to the deductive, or synthetic method. We see a thing happen and we look for its cause; if its apparent cause is consistent with what we see, we call that connection a *theory*. And *then* we use the connection, the theory, *as if* deductively (for we can never be as certain of our scientifically postulated causes as we are of the axioms of geometry) until or unless we find out it was wrong or not useful.

This method is certainly not Euclidean mathematics, but it is natural to mankind, and while it has led to many mistakes it has also given us science. The use of experience has been most fruitful of all, as Galileo explained, when the hypothetical "cause" is linked to observation, both past and future, and both real and imagined, by a deductive mathematical argument. Hence Galileo's insistence that experience be reducible to quantities amenable to mathematical method, to number and figure, as in Euclid.

The inventors of algebra were faced with problems that had no counterpart in Euclid's scheme. We want a rectangle whose sides do this and that; how do we find it? Can we begin with the 'known,' as in Euclid, whose assertions *begin* with a given circle, a given length, a given point, or *some* hypothesis, and go on from there? In the problem of the rectangular garden we haven't been *given* anything! We don't know the longer side, the shorter side, or even if there can be such sides! It *looks* as if we have been given an area; hmm, some "gift!" Area of what? Can there even *be* an area such as we hope to have been 'given?' Where do we begin?

Instead of starting with the known, as in Euclidean geometry, let us start with the unknown, BUT PRETEND IT IS KNOWN!

4. THE ANALYTIC METHOD: INDUCTION FOLLOWED BY DEDUCTION

It begins by guessing. Nobody can stop us from guessing, after all, and if we guess right we can easily show the answer is right, by "checking." If by some miracle I could think, "Eureka! 60 by 10 will do it!" I then could convince any child that this is surely a rectangle of the desired type. Of course, I couldn't convince anyone immediately that this is the *only* rectangle that would do it, and it is hard to see at first how one could show such a thing, but in fact algebra formalizes the "method of guessing" in such a way as to answer both sorts of questions: (1) Find a number or numbers that answer the problem, and (2) Show that these are the only answers there are.

The Arabic method of "algebra" (a word itself of Arabic origin, having to do with taking apart and putting together) is entirely systematic and convincing, but accomplished the goals (1) and (2) in the opposite order. It first finds out the only (possible) answers there are (or, rather, can be), and *then* shows them, or it, to answer the problem in fact.

Instead of starting with the known, as in Euclidean geometry, let us start with the unknown, BUT PRETEND IT IS KNOWN! What is unknown? The length of the rectangle, for one thing — and, for that matter, the very existence of such a rectangle. O.K., we pretend there is such a rectangle and that we know its length: we give it a name, "x." But remember now, x is really the *pretend* length of the *pretend* rectangle that we are *pretending* to know all about, that solves the problem, if the problem can be solved. Maybe it can't. We are not entitled yet to guarantee the problem can be solved — we have earlier, above, seen an apparently similar one, that can't be solved — but we can pretend this one has a solution.

If the pretend length is x then the pretend width is x-50; that's what the problem demands. Some children have trouble with a number like x-50, which looks more like a 'problem' than like a 'number.' We can explain, though, that this is because we don't actually know what number x is. If x were 258 then the width would be 208, which also could be written 258-50; if the length were 111 the width would be 61, which also could be written 111-50. So, if L is the length, the width is L-50. In our case we called the length x; so... "x-50" is the width. The pretend width. Then the pretend area of this pretend rectangle is the product x(x-50), which can be written in the 'expanded' form x^2-50x if we like, because that's what the distributive law says we can do with numbers — and remember, we are pretending that x and x-50 are numbers, maybe not known to us, but, we hope, known to God at least. Notice that x does not have to be called a "variable," or anything else with mystical import. It is a number — well — a pretend number.

Now if this pretend rectangle is to be a real one as demanded in the problem, it must be that its area is 600 square yards, or, to put our pretenses into an English sentence:

IF x is the length of a rectangle that can solve our problem, THEN $x^2-50x = 600$.

This is the key to the whole analytic method, and it is meaningless if it is not written (or understood) as a whole sentence, with a very strong "if" at the beginning and a very strong "then" in the middle. The mere equation, $x^2-50x = 600$, is not the statement of the problem. It is not even a restatement of the problem; it is only a part of a longer statement, the one that begins with "if" and ends with "then." In the language of English grammar, the equation "$x^2-50x = 600$" is but a clause in a complex sentence.

A clause, in English grammar, is a statement that sounds a bit like a sentence itself, since it has a subject and predicate of its own, but within a sentence it doesn't actually say what it sounds as if it is saying. In a true sentence a clause can nonetheless be false. "If pigs could fly, then they would have wings." This sentence is true, even though both its clauses happen to be false. Such is often the case with sentences of the "if... then ..." form, which is what most mathematical sentences sound like. Of course, some of the clauses might be true, too. But we must not confuse the truth of the sentence with the truth of the clauses. We can even know certain sentences to be true while we have no idea whatever whether the clauses in it are true or not. We don't even need to *care* if the clauses are true (when taken as if they were sentences of their own) or not. Try this one: "If John is 6 feet tall and Jim is 5.9 feet tall, then John is taller than Jim." Who John? Who Jim? Doesn't matter; the *sentence* is true, even though it says *nothing at all* about John or Jim, or even whether they exist.

Thus in our restatement of our problem one need not ask whether "x^2-50x = 600" is true or false. It is an *equation*, to be sure, a statement that a couple of things are equal, but, like "John is six feet tall" and "John is taller than Jim," it is just part of a true sentence, having no truth value of its own, except the knowledge that IF the opening clause or clauses are true, this one is, too.

Despite these uncertainties, we have got somewhere; we have narrowed down the problem. IF the problem can be solved, THEN x will have to satisfy the equation x^2-50x = 600. Very well; next question: Are there any numbers x which do in fact satisfy "x^2- 50x = 600?" To answer this, we go on with "if... then..." sentences.

If x^2-50x = 600 then x^2-50x-600 = 0. Why? Because x^2-50x really and truly = 600? NO! Don't let a student believe this for a minute! We don't know if that equation is true (it usually isn't, remember), or that there exists even one value of x which would make it true. What we know is that IF it were true, THEN the second statement would also be true. Subtracting 600 from a certain number, whether it is called x^2-50x or is called 600, can produce only one result, and since we happen to know the result is 0 when the "certain number" is called 600, so we also know the result is 0 when that "certain number" is called x^2-50x, *provided* x^2-50x is another name for 600.

One can say here that "the same quantity subtracted from equals produce equals," and that is a common way to remember the drill, but in logic it doesn't say very much, for x^2-50x and 600 are not just "equals" in the sense of Euclid. x^2-50x and 600 are here assumed to be the same THING, a supposedly "certain number," except that one of the descriptions of that number is more complicated than the other. OF COURSE subtracting 600 from a thing is the same as subtracting 600 from that thing! Only the names are different. And don't forget, it is only a pretend equality to begin with, in that we are *assuming* we are dealing with a number x, for the moment, that *does* make x^2-50x that real thing, 600. Who knows but that we might not someday find out that there really cannot be any such number x?

5. A CHAIN OF IMPLICATIONS WITHOUT TRUTH

Now we can apply a rule of logic called "the transitivity of implication." There was a time when textbooks made much of this idea, which is really only common sense which we use every day. The rule is this: If A implies B and if B implies C, then A implies C. What are A, B, and C here? They are not numbers, they are statements. The clause "A implies B" is mathematical shorthand for the statement "If A, then B," and it is sometimes more convenient to use the word "imply" and its allies than to go through the entire "if...then..." routine.

In the present case our statements A, B and C are as follows:

A. "x is the length of a 600 square yard rectangular field whose width is 50 yards less than its length;"
B. "x^2-50x = 600;"
C. "x^2-50x-600 = 0."

Remember, these are merely statements, clauses, things that look like assertions but are really only parts of assertions we intend to make seriously. We have already established that A implies B, though we wrote it down in the "If A, then B." format. "Subtracting 600 from both sides" is the most usual language we use to justify, in this problem, "B implies C." So the transitivity of implication, combining the two assertions, tells us "A implies C," or "If A, then C;" that is,

IF there is a rectangle answering the conditions of the problem and x is its length, THEN x^2-50x-600 = 0.

Well, now that the idea is plain, that at each step we are faced with a hypothetical statement and not an absolute statement, we can speed things up a little, making our explanations briefer. We continually use the transitivity of implication to permit us to "forget" the intermediate stages of our argument. Knowing A implies C permits us to forget all about B from now on. B has served its purpose. Similarly we will soon be able to forget C, as follows; consider the two clauses:

D. "For any number x whatsoever,
x^2-50x-600 = (x-60)(x+10);" and
E. "(x-60)(x+10) = 0."

Statement D is simply a true statement, as everyone knows and anyone can check using the elementary rules of arithmetic (the distributive law, etc.). That D is true for all real numbers x is not trivial, of course, and it demands a careful definition of "real number" before it can be asserted.

(Actually, D is true not only for real numbers, but also for complex numbers and for many things that are not numbers at all, provided addition and multiplication are suitably defined for these things among themselves and between these things and ordinary numbers. Square matrices of size 7X7 are an example, but this is by the way.)

Is E a true statement, like D? Of course not. For most values of x it is false. What is true is this: If C is true (for a certain x), then E is true. Why? Because D assures us that the left hand side of C is the same as the left-hand side of E *even though we do not know what x is*, and so if C is true, then E, known to be the same statement, is also true.

Here is where we stand now: A implies E. If there is a length x that does our job, x satisfies the equation in E. From here it is easy. The product of two numbers can be zero only if one or both of the numbers is zero. So, if E is true, then so is F:

F. "x-60 is 0 or x+10 is zero, or both."

Finally, if x-60 is true, then x = 60 (I won't repeat the details about doing the same thing to both sides), and if x+10 is true then x = -10. We can discard the "or both" because we know a single number named x cannot be *both* 60 and -10. But we do have to pay attention to the "or." In other words, F implies G, the statement

G. "x = 60 or x = -10."

Combining all the implications in a sort of chain, A implies B implies C implies E implies F implies G (remembering D was merely a truth we used along the way) we end up with the statement "A implies G" worded as follows:

If x is the length of a 600 square yard rectangular field whose width is 50 yards less than x, then x = 60 or x = -10."

We see from this statement that we do not yet have the solution, if any, of the problem; all we know is that any number which is not 60, and is not -10, will not solve the problem. This is rather a limited result, but it does clear away the underbrush. (Notice that we have now answered one of the questions about the original way I quoted a typical solution of this problem: The word is "or," not "and.") And the actual solution is now not far away. With only two possible answers, we don't have to have a flash of inspiration and shout "Eureka!" We can systematically try out the two possible solutions. Try 60: Then the width is 10, and since 60•10 is indeed 600 we have a solution. Put a circle around it. Ten points? Not yet; there might be another answer, since we haven't yet excluded -10 by all those implications. But any fool can see that -10 can't be the width of a rectangle of area 600, so we reject -10, as the book said. There is one answer, and the answer is 60.

6. CHECKING THE "SOLUTION"

This last part of the argument, the actual multiplying out of our candidate answer (x = 60) by the number fifty less than x, to see if it indeed gives us our area of 600, is called "checking the answer" in most schoolbooks, and students and sometimes teachers tend to consider this part a check on whether or not one has made a numerical error somewhere along the way. [Footnote: The Dressler and Keenan *Integrated Mathematics* mentioned earlier is but one among many texts containing no logical explanation of why one has to check an answer. Their typical instruction is "solve, and check, ..." and they make it appear that if the "solve" part contains no errors the "check" is supererogatory.] It is true, of course, that if one has made a numerical error the "checking" step will very likely uncover it, and this already makes the step valuable, but the logical function of the "check" is not often mentioned.

For another example, the book *The Teaching of Junior High School Algebra*, by David Eugene Smith and William David Reeve (Ginn & Co. 1927) was written by two of the most prominent mathematics educators of the time, both professors of the teaching of mathematics and each the author of numerous books on the subject. On page 191, a paragraph headed *The Value of Checking* contains this instruction for future teachers of algebra:

...what they fail to mention, and probably don't even have in mind, is that "checking" is in fact the only genuine proof of the "result" they think was already in hand.

> On the whole, however, it is usually better for a pupil to solve one problem and check the result than to solve two and not check at all... (1) he does a piece of work that is ordinarily quite as good an exercise as the original solution; and (2) he has the pleasure of being certain of his result and of his mastery of the whole situation.

Smith and Reeve thus consider checking to be good for the student; what they fail to mention, and probably don't even have in mind, is that "checking" is in fact the only genuine proof of the "result" they think was already in hand.

For in truth, the "result" they refer to (or the results, in the present case 60 and -10) is only *hypothetical* until the checking, the real proof, is done. Otherwise, -10 is just as good a "result," having been obtained by the same means as the 60. But 60 "checks" in the problem, while -10 — which solves the *equation*, to be sure — fails any check imaginable concerning the area of a rectangle with such a side length.

The so-called check, simple as it might appear, is really the *deductive* proof in the sense of the ancient Greeks, that our answer is right. What is a deduction? It is an argument that proceeds from something *given* to something else we then deduce from it.

In the present case we are now (after all that analysis) given a length 60 yards to study. We can actually build parallel garden walls 60 yards long and the other walls 10 yards long, i.e. fifty less than the length, and compute the area. Behold! (The word "Theorem" is ancient Greek for the English word "Behold.") Behold, the area *is* 600. No ifs or buts here. This particular "theorem" is pretty trivial, but it is a theorem nonetheless: What is this theorem? It says that if a field is 60 yards long, and 50 yards less than that in width, then its area is 600. That's all the problem asked us to show, isn't it? And in truth, we didn't really know that before we got to the so-called "check" of the answer; all we knew earlier was that IF a field did this and that, its length had to be — *if anything*! — either 60 or -10. The "check" is in fact the solution, while what is usually called the "solution" is nothing but the narrowing-down of possibilities.

Yet the traditional "solution" did tell us something else, perhaps equally valuable. It told us that the number we did check out by multiplication was the *only* one. Or the only positive one, anyhow, and since we didn't want a negative one we now know our solution is unique. There is only one set of dimensions for a garden with the properties demanded. The "theorem" given us by our *check* tells us that 60•10 worked; the other "theorem", given us by the preceding *analysis*, told us that *only* 60•10 could work, unless we wanted to get into negative "lengths," whatever that might mean.

For there does remain a nagging question about that -10. Where did it come from? Of course it can't be the solution to the problem, but it *was* a solution to the *equation* that somehow got into the problem. If you diminish -10 by 50 you get -60, and (-10)•(-60)= 600. If -10 checks in the equation, and the equation expresses the conditions of the problem, maybe there is some reason for its having turned up there. Why do we reject it? Because we know something about gardens? What have gardening facts to do with mathematics?

Suppose we hadn't been talking about gardens, but about something we didn't have so much advance information about? How would we have known to reject the "wrong" solution? How wrong is it? It checks in the equation, doesn't it?

Well, there was a slippery phrase two paragraphs back: "...the equation expresses the conditions of the problem..." That isn't quite true. The conditions of the problem were two: First that x be a positive number, since we are looking for the length of the side of a real garden, built of real fencing in a real city; and second, that the equation be satisfied. This is how we know to reject the -10. Had we been more careful in stating the problem, we might have put it thus at the very outset: "Find the (positive) *length* of the side of a garden... Then at each step of the narrowing down part of the solution, well before the "check," we would repeat "positive number" before the symbol "x," e.g. "Let x be the positive number of the pretend length of ..." and so on. We would end, "Then the positive number x must be either 60 or -10," and it is clear that our final statement would be "Then x must be 60" (if such an x exists). There would be no need to worry further about the -10, but the check that 60 works would still be needed as before.

Part II of this article will be published in the next issue of the Humanistic Mathematics Network Journal.