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# Review: Lusztig Symmetries and Automorphisms of Quantum Superalgebras

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**Lusztig symmetries and automorphisms of quantum superalgebras  $U_q(\mathfrak{osp}(1|2r))$ . (English summary)**

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Lusztig symmetries (as defined in 37.1 of [G. Lusztig, *Introduction to quantum groups*, Progr. Math., 110, Birkhäuser Boston, Boston, MA, 1993; [MR1227098 \(94m:17016\)](#)]) have been proven to exist for various quantized algebras (see the references in the paper). Here the authors investigate whether the notion can be extended to the realm of quantized algebras obtained from Lie superalgebras. In particular they focus on the quantized enveloping algebras  $U_q(\mathfrak{osp}(1|2r))$  due to the nice properties of the representation theory of the latter. They show that these super versions of Lusztig symmetries also satisfy braid relations. The proofs are mainly combinatorial.

Reviewed by *Gizem Karaali*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*