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Review: Intertwining Symmetry Algebras of Quantum Superintegrable Systems

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Calzada, Juan A. (E-VALD-APM); **Negro, Javier** (E-VALD-TP);
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Intertwining symmetry algebras of quantum superintegrable systems. (English summary)

SIGMA Symmetry Integrability Geom. Methods Appl. **5** (2009), *Paper* 039, 23 *pp*.

This paper presents an algebraic study of some quantum systems associated to a family of superintegrable Hamiltonian systems in terms of shape-invariant intertwining operators. The eigenstates of the associated Hamiltonian hierarchies belong to unitary representations of these algebras. Also included is a study of the corresponding superintegrable classical systems. The paper contains explicit constructions as well as several helpful figures.

Reviewed by *Gizem Karaali*

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