1-1-2009

Review: Dynamical Yang-Baxter Maps with an Invariance Condition

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MR2389797 (2009e:16078) 16W35 (20F36 20N05)

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Dynamical Yang-Baxter maps with an invariance condition. (English summary)

Publ. Res. Inst. Math. Sci. **43** (2007), *no. 4,* 1157–1182.

Let H and X be nonempty sets, and φ be a map from $H \times X$ into H. A map $R(\lambda)\colon X \times X \to X \times X$ as λ varies in H is a dynamical Yang-Baxter map associated with H,X and φ if for every $\lambda \in H$, $R(\lambda)$ satisfies the equation

$$R_{23}(\lambda)R_{13}(\varphi(\lambda, X^{(2)}))R_{12}(\lambda) = R_{12}(\varphi(\lambda, X^{(3)}))R_{13}(\lambda)R_{23}(\varphi(\lambda, X^{(1)}))$$

in $X \times X \times X$, where, for $u, v, w \in X$, we define

$$R_{12}(\lambda)(u, v, w) = (R(\lambda)(u, v), w),$$

$$R_{12}(\varphi(\lambda, X^{(3)}))(u, v, w) = R_{12}(\varphi(\lambda, w))(u, v, w),$$

and

$$R_{23}(\varphi(\lambda, X^{(1)})(u, v, w) = (u, R(\varphi(\lambda, u))(v, w)).$$

The remaining terms $R_{23}(\lambda)$, $R_{13}(\varphi(\lambda, X^{(2)}))$, and $R_{13}(\lambda)$ are defined analogously.

The dynamical Yang-Baxter map was defined by the author [Int. Math. Res. Not. **2005**, no. 36, 2199–2221; MR2181454 (2006f:16058)] as a way to generalize the notion of the Yang-Baxter map, which has been extensively studied in the literature; see for instance [P. I. Etingof, T. Schedler and A. V. Solov′ev, Duke Math. J. **100** (1999), no. 2, 169–209; MR1722951 (2001c:16076); J.-H. Lu, M. Yan and Y. Zhu, Duke Math. J. **104** (2000), no. 1, 1–18; MR1769723 (2001f:16076); A. V. Solov′ev, Math. Res. Lett. **7** (2000), no. 5-6, 577–596; MR1809284 (2001h:16048); A. D. Weinstein and P. Xu, Comm. Math. Phys. **148** (1992), no. 2, 309–343; MR1178147 (93k:58102)]. In that paper [Y. Shibukawa, op. cit.], it was shown that one can construct dynamical Yang-Baxter maps using left quasigroups and ternary systems satisfying a particular invariance condition.

In the paper under review, this invariance condition is studied more comprehensively. A better characterization of the maps which satisfy it is provided. The reviewer suggests that one read the two papers in tandem to get the most out of this discussion.

Reviewed by *Gizem Karaali*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.