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Review: Lie Structure in Semiprime Superalgebras with Superinvolution

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Lie structure in semiprime superalgebras with superinvolution. (English summary)

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The Lie structure in the set of all skew elements of a given associative algebra has been studied since the 1958 article of W. E. Baxter [Trans. Amer. Math. Soc. **87** (1958), 63–75; MR0095866 (20 #2364)]; also see [I. N. Herstein, J. Algebra **14** (1970), 561–571; MR0255610 (41 #270)]. A natural extension of these results to the graded case of a superalgebra has been initiated by C. Gómez-Ambrosi and I. P. Shestakov [J. Algebra **208** (1998), no. 1, 43–71; MR1643975 (99j:17032)] and Gómez-Ambrosi, Laliena Clemente and Shestakov [Comm. Algebra **28** (2000), no. 7, 3277–3291; MR1765316 (2001g:17007)]. The paper under review further extends these results.

Let A be a semiprime associative superalgebra with superinvolution, over a commutative unital ring where 2 is invertible. Let K be the set of all skew elements of A with respect to this superinvolution. The main result of the paper under review is a description of the ideals of K. In particular, if U is any Lie ideal of K, then either there is an ideal J of A such that the Lie ideal $[J \cap K, K]$ is nontrivial and lies completely in U, or the superalgebra A can be written as the subdirect sum of A' and A'' such that the image of U in A' is central, and the second component A'' is a subdirect product of orders in simple superalgebras, each of which is at most 16-dimensional over its center.

The significant contributions of this particular paper to its predecessors cited above are: the change of the (super)algebra base from a field of characteristic not equal to 2 to a general commutative unital ring with an inverse for 2; and the extension of the results to a semiprime superalgebra in place of a prime superalgebra. The paper is well written and easy to follow.

Reviewed by *Gizem Karaali*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.