Humanistic Mathematics Network Journal

Issue 21 Article 13

12-1-1999

# Calculus for the Liberal Arts: A Humanistic Approach

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**BRIEF SUMMARY**
At the Humanistic Mathematics Sessions at the Mathematics Association of America meetings in Orlando in January 1996 I presented an outline of the approach I planned to take in introducing Calculus into a Liberal Arts Mathematics course. I followed that plan of action in four classes in spring and summer 1996. This paper is a report on the success of this endeavor containing some of the essays written by students in the spring courses.

**INTRODUCTION**
The Calculus has played an enormous role in the development of modern science and technology. It still constitutes the primary college mathematics course for those continuing in mathematics and other technical fields. However, we rarely touch on the Calculus in the courses offered for general education purposes, the so called Mathematics Appreciation or Liberal Arts Mathematics Courses. Liberal arts students should be exposed to this important branch of mathematics in a positive way, one which they will find relevant and interesting. It has had an enormous impact on their lives and is the cornerstone of modern mathematics and science. Nevertheless, the Calculus is usually ignored in general education courses. Some argue that it is impossible to do Calculus without sufficient algebra skills, and many or most liberal arts students are lacking those skills. It has been deemed unproductive to force these students into yet another attempt to learn algebra, and therefore calculus is not available to them. This argument, while it once had some merit, is really not compelling today. With computer algebra systems readily available, there is no reason liberal arts students with weak algebra backgrounds must be denied an opportunity to understand calculus concepts or to use calculus concepts to solve problems. Also, if the goal is that students appreciate the Calculus, rather than master it, then it is not necessary that they, themselves work the problems or go through the algebra. This is one place where perhaps it might be appropriate for mathematics to be a "spectator sport."

There still, however, remains the question of interest and a conceptual framework. Why should students who do not plan to use the Calculus in their fields have any interest in it? Algebra and Calculus are rarely presented in a social or historical context. This makes it difficult for liberal arts students to find them relevant. Calculus can be presented in terms of its role in the Enlightenment and its historical and cultural significance. Then students from all disciplines would have a framework of interest in which to work. They should see the Calculus as one of the most profound intellectual achievements of the modern world. They can and will appreciate its significance. They should no longer have to ask, "What is Calculus?"

This paper is a progress report on my attempt to introduce the Calculus in our Liberal Arts course, Mathematics and Culture. Using an essay on the Calculus which I prepared while on sabbatical last fall, I led three sections of students through a three week tour of the Calculus this Spring and one section this summer. I do not claim to have perfected the approach yet. I would like to incorporate more use of technology so that students can solve Calculus problems themselves, and I expect to be able to do that this fall. However, in the three sections I taught last spring, access to a computer lab was limited. Nevertheless, I feel that the overall experience was successful.

**THE APPROACH**
The first thing we discussed were four problems that are central to the motivation for the Calculus, Zeno's

Paradox (infinite sums), the area problem, the tangent problem and optimization problems. Zeno's paradox in particular is an interesting problem to debate with liberal arts students. After that we digressed into the foundations, or "preliminaries." Interestingly enough, the "preliminaries," or preCalculus topics: Functions, the real number system, etc. were developed after Newton and Leibnitz had published the Fundamental Theorem of Calculus. This fact and the reasons for developing the real number system, functions, and limits are discussed as well. At this point the students are asked to write down three questions about what we've covered so far. I put these questions on an overhead and we go over them in class. The questions they had at this point include (some have been paraphrased as they were essentially repeated by more than one student):

- How can one master calculus without a strong mathematics background? [The answer is of course that you can't "master" calculus without the background, but you can "appreciate" it.]
- How can I use Calculus in my daily life? Why should it be important to the average person? What real life questions will Zeno's paradox help me solve?
- Why is anyone interested in things like Zeno's paradox? Especially if it is a non-issue due to the definition of limit & infinite sums? Why is it a paradox? We figured out when Achilles would catch the tortoise; Zeno was wrong; why are we even discussing it?
- What is the significance of learning sin, cos and tan...?
- What is the significance of Calculus to this course?
- Why are these "problems" so crucial to Calculus? They don't seem life-threatening if they weren't solved.
- Something that occurred to me about the problems we did was that they were more interesting than I believed math problems could be. Math is not too bad when you incorporate more than just numbers. Are we going to examine more problems like these?"
- Can we do some "hands on" exercises to illustrate the exercises? [This is why I hope to be able to incorporate some lab exercises in the future. There is one other "hands on" optimization exercise I'd like to use but didn't in the interest of time]
- Is Calculus a product of mathematics, or is it a discovery of mathematicians? Is it a part of the world, or did mathematicians develop it as a means of problem solving?
- Now that we have real numbers, have we become lazy and started to take them for granted?
- I do not understand how we can add measurements that do not exist into problems, only to take them out for the purpose of being unable to divide by 0. They were not taken out mathematically, but simply disregarded. Mathematics is a subject that requires great precision, and this invalidates the entire finding. How can you be sure if an answer is mathematically correct when some of the problem was not only unknown but unreliable? [Bravo! That's exactly why we need the preliminaries.]
- What are some real world applications of Calculus?
- Zeno was a philosopher; are any of the other problems philosophical in nature?
- Explain why the diameter and the circumference of a circle could not be compared. What are the difficulties in assigning numbers to geometric quantities? What does incommensurate mean? How can you compare two areas if you don't know one?
- Why does the area of a rectangle have a curved graph?
- Just how small is infinitely small? How do you visualize an infinitesimal?
- What is meant by a rigorous definition?
- Are there any practical uses of logarithms?
- How exactly are limits applied by using the definition?
- What did the Greeks think pi was if it wasn't a number? What is B? How is it calculated?
- Would Zeno's problem still be a paradox if the endurance of the competitors changed midway through the race? Depends on how it is set up...What if the tortoise gets a 20m head start?...Why could everyone deal with the time travel paradoxes but have so much trouble with Zeno's paradox? Is there a shortage of non-mathematical paradoxes?
- Why is there no real number such as an infinitesimal? What is an infinitesimal?
- Please prove that 1-1+1-1+1.... = 1/2. Doesn't everything in mathematics always have a right answer? If you put this into a computer you would always get 0 or 1; why is it not possible to refute this? Is this actually equal to anything? Was Euler on crack?
- What is the difference between natural and whole

numbers other than the fact that natural numbers start with 1 and whole numbers start with 0? What's so special about 0?

- Why did it take so long to begin using irrational numbers in the real number system?
- What does it mean to approach 0, and does approaching 0 relate with the notion of infinity?
- If a function is not continuous, can you make it continuous mathematically?
- What exactly is the method of exhaustion? How do you find the area between the circle & the triangle?
- On the tangent problem, how do we pick the point? Won't we get a different answer if we use a different point?
- In the area problem, why can't you just find the area using $A = \pi d^2/2$?
- How do we know the sequences 2, 2.1, 2.14,... and 3, 3.1, 3.14, 3.141,... do not have rational limits?
- What are the properties of a logarithm function?
- What are sequences which converge within themselves? Why did Cauchy define real numbers this way?

What was most interesting in the discussions of these questions was the way that students would start to answer each other's questions. In one class a particularly lively discussion ensued about the "why do I have to know this?" questions. In any event, the questions clearly show that the level of understanding varies from student to student but that all of them could ask questions, and most of the questions were about the material and reasonable. Taking time to address all these questions increased the comfort level of the class at a time when some students were beginning to get desperate about what might be expected of them.

After the discussion on their questions, we go over the Derivative and the Integral including a discussion of how they help solve three of the problems (Zeno was dealt with in the preliminaries). During the course of this discussion we "prove" a few theorems, including Rolle's theorem (then the Mean Value theorem is stated and used) and the Fundamental Theorem of Calculus.

### RESULTS

At the conclusion of the Calculus section of the course students are given the following assignment:

*It is now the year 2002. You are hosting a party for a mixed group of neighbors, colleagues, friends and some of their adult or almost adult children. One of those present is an older gentleman, Ben, who is well respected and whose good opinion you value. An 18-year-old college freshman, Jean, is also present. Ben asks Jean what (s)he is studying that semester, and (s)he replies that (s)he is taking English, History, Psychology, Computer Programming and Calculus. Ben then says, "You know, I've known a lot of people who say they are studying Calculus, but I've never quite figured out what Calculus is; what is it?" You notice that Jean is rather uncomfortable and doesn't seem to be able to answer. As the Host(ess) you want to jump in and help out by answering. So, How do you respond to "What is Calculus?" Remember that you want to make a good impression on Ben and that he will not appreciate your interruption if your answer is flip or doesn't really satisfy his curiosity.*

Responses pretty much fell into three categories. The first group says something about Calculus being an important branch of mathematics or something engineers use or something else along those lines. They address the importance of the Calculus to our technology or our society but don't really get into what it is. They give reasonable answers, but their responses could just as easily apply to almost any branch of mathematics. The second, and largest, group have very good responses, showing that they realize that Calculus involves change and/or going through the four problems. They are pretty much parroting back what they've read or been told, but they are doing so in a reasonable manner. Their essays clearly show that they have picked up an appreciation for what Calculus is all about. The third group surprised me. They really were able to describe Calculus, or some aspect of it. in their own words. I asked some of them if I could share their work with you.

Lilly McCready wrote:

#### WHAT IS THE CALCULUS?

There is not one concise definition of calculus that everyone agrees on. I think that calculus is a series of functions and relationships which are defined in terms of one another. These functions or relationships are generally changing, too. This is why Newton first called calculus "fluxions" because the variables are constantly changing.

Calculus is unlike basic elementary mathematics in that it does not usually entail a lot of actual numbers or simple arithmetic. Instead, symbols are used to represent numbers. One reason for this is that the numbers are always changing. By using a variable, it can represent any number that we choose. Another reason that symbols are used instead of numbers is because often times in calculus, the numbers one is dealing with are extremely large or small. One other reason that symbols might be used is because one might not know what number is wanted. The purpose of the function may be to find out the number that you need. Other times a graph might be drawn to explain a certain function or relationship between two variables instead of doing computations.

The calculus may not appear in your daily life, but it is in life. In a world where things are constantly changing, the calculus is useful. Since calculus offers a different way to deal with relationships that are either independent or interdependent of one another, it can help with the value of the dollar, exports going up and imports becoming more expensive and even the space shuttle taking off.

Amy Bauersfeld wrote:

#### WHAT IS CALCULUS: A SIMPLE DEFINITION WHILE EATING COCKTAIL WEENIES

"Sorry to interrupt you, Jean, but I couldn't help but overhear your discussion about calculus. It got me nostalgic for this class I once took in college at good old Salisbury State. When I was a sophomore, Ben, my teacher asked me that very same question: What is calculus? At the beginning of the class I had not the slightest clue. But as the class progressed I discovered that calculus was a system that made solving complex mathematical problems simpler. For the most part I learned that calculus is a blending of geometry, algebra, and arithmetic. It involves a method of using symbols to represent numbers in an equation to solve for unknown factors.

As far as the history of calculus goes, it's been around for something like two thousand years. If I remember correctly Sir Isaac Newton and this other guy Wilhelm von Leibnitz were the mathematicians who simplified the process of the Calculus. You have to remember that this was before the advent of the modern real number system or the use of infinity or infinitesimals which I'm sure you're studying now, Jean, in your class.

Newton and Leibnitz, in my opinion, must have been geniuses to develop a system with such utility and simplicity. I don't know where we would be today if calculus had not been developed. If you think about it, calculus has been used to discover some of the greatest technologies of the twentieth century, nuclear power being just one of them. Without the usage of calculus equations, the physics behind the power would never have been discovered. That's only one thing off the top of my head where I know calculus is put to use, but I'm positive that there must be a million more applications out there.

Rebecca Hudson concentrated on the problems:

#### WHAT IS CALCULUS?

Calculus is the study of four problems and the questions that these problems bring to mind. The first of these problems is called Zeno's Paradox. It deals with the fact that if you want to move from one point to another point, you must first travel half the distance between the points, then half the remaining distance, and so on. If this is true, then you can never reach your destination. This is called Zeno's Paradox.

The second problem is the Area Problem. The area of any region with straight line borders can easily be found by dividing the region into rectangular and triangular divisions. The problem that Calculus confronts is how to find the area of a region with curved borders.

The third problem is the Tangent or Velocity Problem. A tangent is a line that intersects a circle at only one point. If an object was moving along the edge of a circle and suddenly broke loose, the object would run along a tangent. Another property of the tangent of a circle is that it is perpendicular to the radius. Calculus helps us determine what to do when we have curves instead of circles.

The last problem that Calculus deals with is called the Optimization Problem. These are problems in which it is necessary to find the largest or smallest possible value of something. For example, suppose you were given a certain amount of material and asked to construct the largest structure possible. Calculus allows us to find the many possible dimensions that such a structure might have.

Jennifer Taylor had seen some Calculus before this class. Nevertheless, her response showed that she had gained from our discussion as well.

#### WHAT IS CALCULUS?

If I were hosting this party, I would pleasantly ask if I could join the interesting conversation. Once accepted, I would give my opinion of calculus to get Jean off the spot like any good hostess would do. I would explain to Ben that my experience with calculus has led me to the belief that calculus is a type of mathematics that helps you solve complex real life problems. For example, calculus is divided into two sections, differential calculus and integral calculus. These sections help solve problems a step beyond what algebra and other types of mathematics allow us to do.

The main purpose of differential calculus is to find the rate at which a known, but varying, quantity changes. For example, if we wanted to know the speed at which a plane travels over a distance of 500 miles in one hour, we could use simple algebra to find the answer (R = D/T). Differential calculus comes into play when the rate, or in this case, the speed of the plane does not remain constant over the 500 miles or one hour. Differential calculus allows us to determine the speed of the plane at any moment in time.

Integral calculus, on the other hand, is used to find a quantity, knowing the rate at which it is changing. One example is when you want to determine the distance a bullet has gone at a given time, knowing the rate at which the speed is decreasing. Integral calculus is often used in geometry in finding areas of curved objects.

The history of calculus is also helpful in understanding its function. Some of the first ideas of calculus began with Archimedes, a Greek mathematician, who formulated methods of finding the volume and surface area of spheres.

Calculus is the answer to questions that other types of mathematics cannot answer. The concepts of calculus allow us to perform many important problem solving skills, and is used in many scientific and engineering fields.

Lauren Michener, a particularly articulate communication arts major, had a unique and very creative response to the question.

#### CALCULUS

"My dear friend Ben, let me interject and try to answer this question for Jean. When I was younger my school teacher taught us a poem, which although may be slightly childish, it effectively and pleasantly answers the question, 'What is calculus.' I happen to have a copy of it here in my apron, for I was tutoring my niece just before the guests began to arrive. Let me read it to you:
'What is calculus?,' a teacher might say
The student replies, 'It will ruin my day
It doesn't light up and it doesn't talk back
It doesn't play ball, not good for a snack
It's not on T.V. and it's not found in stores
All that it is is a terrible bore!'
The teacher looked grim and to this she replied
'Yes, but it's in your book, just look inside.
Calculus has been used for quite a long while
Since the 17th century it's been the style
To solve computations it was used first
Or to calculate problems that were the worst
This method of solving was a fabulous creation
So grand it was done in special notation
Such as logic or symbolic that also was seen
And for people who knew it this all was real keen
Other methods of calc were created by von Leibniz
These were differential and integral, boy what a whiz!
Then while trying to find a "universal characteristic"
That could unify all thoughts that were mathematic
For symbolic logic he laid the foundation
Next calculus grew and with it exploration
Parts of math and physics got new application
And theoretical basis went under examination
It all was accepted but not 'till centuries passed
And four basic problems were the start to calc class
These inspired the need for this powerful math tool
That today every student can learn in their school
For technicians, engineers, and teachers the same
Calculus is more cool than any light-up game
In the words of Dr. Shannon I'll repeat this dedication
"It is a beautiful monument to human imagination."'"

"Bravo! Bravo! That was an absolutely exquisite explanation of calculus! I understand completely now. It was created in the seventeenth century as a means of creating or solving computations or calculations in a special notation such as that of logic or symbolic. Those must be the numbers and letters I see scribbled across mathematicians' notebooks today. That symbolic logic was not created until von Leibniz, that clever fellow, attempted to find one "universal" way of expressing mathematical thought. All of these systems and styles were then lumped together to get what we know today as calculus. And there are four problems which tormented the minds of mathematicians, and because calculus was the only man fit for the job, he was created and called upon. My, calculus is a glorious thing! It is used everyday, isn't it? Even in places and ways I'm sure I was not previously aware of. Where is this Dr. Shannon; I want to learn more about calculus."

"She is over by the Diet Cokes. I'm glad I could help, and I'm sure you'll love the study of calculus."

### CONCLUSION

In conclusion, it is both possible and rewarding to include a discussion of the Calculus in a liberal arts mathematics course. In the course of our Department's review of the Calculus sequence, while we were discussing the importance of the Calculus in the context of the course description it struck me that if we really do believe that Calculus is one of the greatest intellectual achievements, then we should not reserve an appreciation of its development to only a select few. If we avoid Calculus in our Mathematics Appreciation courses, then we do our students and our discipline a disservice.

# Ode to Mathematics

*Sandra Z. Keith*
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*A revised version of a poem by Serge J. Zaroodny*

The world has far too many boring books;
For books, par excellence, are very static;
But of this statics, by its very looks,
Most boring is the book that's mathematic.

Long laughed at human logic human tongue;
With logic tends it to be causalistic
So to untangle simple truths from wrong
One must resort to methods symbolistic.

Oh, why is't so that language must conceal
A simple thought in cloudy definition?
Why can't we lucid truth at once reveal
Without disguising it by erudition?

Cheer up! That quantum jump, sweet understanding's jerk--
It only comes to those who do the work.