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A cursory look at the proceedings of the last four ICME's international conferences reveals an increasing interest in the role of cultural factors in mathematics education. This paper attempts to enrich these discussions in two directions. First, to apply the contrastive analysis approach in bringing out the role of values in mathematics education; and, second, to go in some details into the role of values in the various aspects of mathematics education. The paper thus consists of three main parts. The first part presents episodes from two distinct cultures to be used as examples for illustrative and contrastive purposes. The second part contrasts the episodes from different perspectives. The third part discusses and analyses in some details the potential role of values in mathematics education as well as computer education.

EPISODES

Episode One

"And unto you belongeth a half of that which
your wives leave, if they have no child;
but if they have a child then unto you
the fourth of that which they leave, after
any legacy they may have bequeathed, or debt
(they may have contracted), hath been paid.
And unto them belongeth the fourth of that
which ye leave if ye have no child, but if
ye have a child then the eighth of that which
ye leave, after any legacy ye may have
bequeathed, or debt (ye may have contracted),
hath been paid. And if a man or a woman
have a distant heir (having left neither
parent nor child), and he (or she) have a
brother or a sister (only on the mother's
side) then to each of them twain (the
brother and the sister) the sixth, and if
they be more than two, then they shall be
sharers in the third, after any legacy that
may have been bequeathed or debt (contracted)
not injuring (the heirs by willing away more
than a third of the heritage) hath been paid.
A commandment from Allah. Allah is Knower,
Indulgent." (Surrah IV, 12). (Pickthall, 1970).

Were it not for the archaic English language, one would have thought that the above quotation is from a tax code of a modern Western country. Surprisingly, the quotation is from a religious book more than 1350 years old: the *Glorious Qur'an*. For Muslims the *Qur'an* is not simply a revelation, but the very words of God, embodied in an immutable text in Arabic. The above quotation from the *Qur'an* is intended here to illustrate three points relevant to the issues at hand. First, the level of details and the degree of precision illustrate the extent to which the *Qur'an* establishes a complete system for civil laws and social and political institutions. In fact, until now, the inheritance laws in effect in Islamic courts adhere fully to the precise and explicit commandments in the *Qur'an*. Second, the quotation illustrates the sophisticated use of numbers in communicating precise quantitative concepts in such a systematic way that they can be easily transformed into a flow-chart with almost all possible options covered. Third, the quotation illustrates the view of the *Qur'an* and Islam of the function of mathematics as an important knowledge in so far as it contributes to utilitarian purposes. However, the mode of thinking in mathematics, like all other forms of knowledge, has to remain subservient to the Islamic mode of thinking whose ultimate purpose is to know God through His Book.

Episode Two

Early in my career in the late sixties and the early seventies, I had the experience of extensively observing mathematics instruction in the classrooms in the Sudan, Saudi Arabia, and Palestinian camps in Lebanon, Syria, Jordan, West Bank and Gaza. Invariably, the teacher, the students, and the textbooks were engaged in what seemed to me a "well-rehearsed act" in which the actors and the set changed but the roles and characters remained essentially intact. The teacher presents the lesson to the class by talking and writing on the board while the students are silent and perhaps listening. Occasionally, the teacher asks a question to which the students respond by raising their hands to indicate their ability to answer. The teacher picks one student, listens to the answer, determines its correctness and moves on with his presentation. Towards the end of the class, the students are asked to solve problems from their textbooks (most often on the board). The textbook is the determinant of what is to be taught, and the teacher is the interpreter of the textbook and the judge of the correctness of what is learned.

Episode Three

In his dialogue "Meno" (Jowett,1937) Plato presents his ideas about knowledge, teaching, and learning using an example from mathematics. The persons in the dialogue are Meno, Socrates, and a slave of Meno (referred to as Boy). The illiterate "Boy" learns certain mathematical conclusions through the answers elicited by Socrates' questions. The dialogue proceeds as follows:

(1) The "Boy" learns that the area (size) of a square of side two feet is four (square) feet
(2) To the question about a square of double area (8 square feet), The "Boy" conjectures that it should have double the side i.e. four feet
(3) Socrates make him recollect or "discover" that such a square actually has a side of less than 3 and greater than 2.
(4) The dialogue then proceeds as follows:

"***Soc***. Do you see, Meno, what advances he has made in his power of recollection? He did not know at first, and he does not know now, what is the side of a figure of eight feet: but then he thought that he knew, and answered confidently as if he knew, and had no difficulty; now he has a difficulty, and neither knows nor fancies that he knows.
Men. True.
Soc. Is he not better off in knowing his ignorance?
Men. I think that he is.
Soc. If we have made him doubt and given him the 'torpedo's shock' have we done him any harm?
Men. I think not.
Soc. We have certainly, as would seem, assisted him in some degree to the discovery of the truth; and now he will wish to remedy his ignorance, but then he would have been ready to tell all the world again and again that the double space should have a double side.
Men. True
Soc. But do you suppose that he would ever have enquired into or learned what he fancied that he knew, though he was really ignorant of it, until he had fallen into perplexity under the idea that he did not know, and had desired to know?
Men. I think not, Socrates.
Soc. Then he was the better for the torpedo's touch?
Men. I think so.
Soc. Mark now the farther development. I shall only ask him, and not teach him, and do you watch and see if you find me telling or explaining anything to him, instead of eliciting his opinion. Tell me, boy, is not this a square of four feet which I have drawn?
Boy. Yes.
Soc. And now I add another square equal to the former one?
Boy. Yes.
Soc. And a third, which is equal to either of them?
Boy. Yes.
Soc. Suppose that we fill up the vacant corner?
Boy. Very good.
Soc. Here, then, there are four equal spaces?
Boy. Yes.
Soc. And how many times larger is this space than this other?
Boy. Four times.
Soc. But it ought to have been twice only, as you will remember.
Boy. True.
Soc. And does not this line, reaching from corner to corner, bisect each of these spaces?
Boy. Yes.
Soc. And are there not here four equal lines which contain this space?
Boy. There are.
Soc. Look and see how much this space is.
Boy. I do not understand.
Soc. Has not each interior line cut off half of the four spaces?
Boy. Yes.
Soc. And how many spaces are there in this section?
Boy. Four.
Soc. And how many in this?
Boy. Two.
Soc. And four is how many times two?
Boy. Twice.
Soc. And this space is of how many feet?
Boy. Of eight feet.
Soc. And from what line do you get this figure?

Boy. From this.
Soc. That is, from the line which extends from corner to corner of the figure of four feet?
Boy. Yes.
Soc. And that is the line which the learned call the diagonal. And if this is the proper name, then you, Meno's slave, are prepared to affirm that the double space is the square of the diagonal?
Boy. Certainly, Socrates." (pp. 363-365)

Episode Four

Between 1970 and 1973 I was in the United States working for my Ph.D. at the University of Wisconsin, Madison. During this period I had a chance to visit elementary school classes in Madison. What struck me most in the mathematics classes, which were using the then experimental instructional materials of the Developing Mathematical Processes Project (DMP), the "chaos" in contrast to the "law and order" that I experienced in Lebanon and Palestinian schools. Children roamed around, talked to each other, had fun, and occasionally engaged in some mathematical activities. The teacher seemed to have assumed the role of an organizer whose main responsibility was to structure the environment for the children to learn and occasionally engage them in some mathematical activities. From my perspective at that time, whatever mathematics learning was taking place in the American Schools was different from whatever learning I had experienced.

CONTRASTS

Values as Psychological Constructs

By contrasting the two classroom episodes (episodes two and four) one observes (at least from the perspective of the author) that the type and sequence of actions and interactions performed by the players (students, teachers, instructional materials) is quite different. In episode two the teacher is the major determinant of instruction, the students are the recipient audience, and the textbook is a concrete definition of the tasks to be explained by the teacher or performed by the students. The tacit assumption, on the part of both the teacher and the class, is that mathematics is knowledge that is possessed by the teacher and is to be transmitted to the students who thus are expected to possess it i.e. learn it.

In episode four, the teacher in contrast determines the setting of learning but not instruction. The students select what to do with the learning environment organized by the teacher and hopefully learn the mathematics injected in the environment and as intended by the teacher. Mathematics seems to be some interesting tasks we do because they are there around us and because they are fun to do.

If this account of the two episodes is reasonably valid, the differences in the scenarios may be partly accounted for by the beliefs and values of the players in each episode. These values and belief are contributors to, if not determinants of, the actions and interactions in classroom instruction. The two episodes reflect different values related to the nature of learning and teaching, nature of mathematics, objectives of learning/teaching mathematics, role of instructional materials and learning environment, and above all, "who" determines the legitimacy of truth and validity of mathematical knowledge.

In these two episodes, values may be looked at as psychological constructs that students and teachers have formed as a result of cumulative individual and collective contextualized experiences. Thus values may be considered regardless of their historical development. The claim is that, even if values are detached from their cultural history, they do impact mathematics instruction in specific and definite ways.

Values as Cultural Products

By contrasting the two historical episodes (episodes one and three), quite different patterns of discourse emerge. One may attribute the differences in the two discourses to the differences in their contexts, the discourse in episode one being from a religious book (the *Qur'an*) after Christ and in episode two from a philosophy book (the *Dialogues of Plato*) before Christ. Nevertheless, each of the two discourses is in its own context a value-capturing sample of the greatest books in the Islamic and Greek cultures. After all, Islam for the Arabs was what philosophy had been for the Greeks.

A close analysis of these two samples of discourses reveals differences in the value-systems in which they are embedded. Specifically, the values encompass differences regarding the nature of learning, knowledge, and mathematics. In episode one, knowledge is fixed and final whereas, in episode three, it is a continual dialectical process of thinking. In episode one, the truth has the finality and authority of the Divine whereas, in episode three, it has the tentativeness and fragility of human reasoning. In episode one, learning is the act of receiving knowledge as expressed by the words of God, whereas, learning is a continual testing of hypotheses in episode three. Mathematics comes as concepts and techniques which are useful in life and in executing the commandments of God in episode one. In episode three, however, mathematics comes out as a medium and vehicle for questioning and reasoning.

In these two episodes, values may be regarded as shared meanings which had captured in certain periods in history the collective experience of a culture. Thus values may be considered as cultural products of the past regardless of their subsequent impact on the value-systems of the present. The question arises as to the extent and form of this impact in mathematics education.

Relation of Values as Cultural Products to Values as Psychological Constructs

In addition to contrasting values as psychological constructs or cultural products, one may also focus on the relationship between values as cultural products and values as psychological constructs. My claim is that the impact of values as cultural products on values as psychological constructs is strong enough to be observable. The few episodes provided earlier do not warrant any inference but may be used to illustrate this relationship.

There is a close affinity, I suggest, between the values reflected in episode one and those reflected in episode two. The values of the finality of truth as determined by the Divine in the form of an immutable text and of the function of mathematics as an instrumental knowledge to utilitarian purposes in episode one are echoed in episode two in the form of valuing the textbook as the repository of mathematical knowledge, the teacher as a determinant of knowledge, and mathematics as a body of knowledge to be transmitted by those who possess it to those who do not. In the same manner, in episode three, the values of tentativeness of truth, of the dialectical nature of learning, and of mathematics as a medium for coping with reality resonate in episode four, perhaps in an exaggerated and confused ways, in valuing "chaos" in the American classroom, the role of the teacher as an organizer, and mathematics as a vehicle to cope with reality. These should not imply, however, that the values operating in mathematics classrooms can be accounted for solely in terms of values as cultural products. However, their impact cannot be neglected with the understanding that values are hybrids which have resulted from complex interactions of the value-systems of different cultures. The metaphor is that of a wave (cultural values) which, as it travels in space and time, interact with other waves (other cultures) and produce new waves having some from each source but more of the primary source.

EXAMPLES OF VALUES IN MATHEMATICS EDUCATION

Many attempts have been recently made to identify values that impact mathematics education. Bishop (1988), using the four components of culture as defined by White (1995), has identified three pairs of complementary values relating to Western mathematics corresponding White's sentimental, ideological, and sociological components. The first pair relates to the two values of control (power of mathematics to offer feelings of security and control) and progress (development of knowledge through mathematics). The second complementary pair of values which belong to the ideological component is rationalism (logic as a criterion of mathematics knowledge) and objectism (power of mathematics in using symbols to deal with abstracts entities as if they were objects). Openness-mystery is the third complementary pair of values which belong to the sociological component.

Others have attempted broader social-cultural values that may impact mathematics education. Swadener and Saedjadi (1988) illustrate how mathematics education may promote the values implied in the five fundamental principles of the *Panca Sila* which is the foundation of the national values in Indonisia. Jurdak (1989) identified some of the values of Arab-Islamic culture which may act as cultural carriers or barriers in mathematic education.

CONJECTURES

In recent years there has been a tendency towards more microscopic description of mathematics education. The macro descriptions of each of the goals, content, methods of instruction, and evaluation of mathematics education are giving way to more analytic descriptions in such a way that not only larger "blocks" are being broken down into smaller and finer "units" but also new relationships among these units are being identified and refined.

The micro description of mathematics education presents a more explicit and powerful method for analyzing and investigating the role of values in mathematics education. The impact forces which are not apparent at the macro level become so at the micro level. Thus, the role of values, for example, seems marginal if mathematical content is defined as terms, concepts, and skills but would be greatly enhanced if content is defined as points in multidimensional taxonomy consisting of subject matter, teacher intention, time allotment, and order of presentation, all of which are value-loaded.

An attempt will be made therefore to identify and analyze the role of values on mathematics education using the micro descriptions which exist in the literature. A similar attempt will be made regarding computer education.

Goals and Values

The goals of mathematics education reflect values regardless of their macro or micro description. After all, goals are primarily value-judgements as to what is important in learning/teaching mathematics. I advance a speculation that values not only determine, to some extent, the goals of mathematics education but also play a major role in prioritizing these goals. This speculation is almost a truism, yet we tend to ignore it because, perhaps, of the predominant belief almost everywhere, that mathematics education is value-free. The goals in the public perception are determined by the experts (mathematicians and teachers) who are knowledgeable about their field and know why and what to teach in mathematics.

If the goals of mathematics education are not affected by values, how could one account for differences, say, between the goals of mathematics education in the NCTM Standards (NCTM,1989) and Yemen except to say that these goals are reflections of the needs of two different societies whose valuing of certain "purposes" of mathematics education (purpose is used here in the sense of Niss (1981)? How could one explain why reading in the U.S. takes priority over mathematics, and the priority is reversed in Japan and Taiwan (Stigler, Lee, and Stevenson,1987) whereas religion takes priority over mathematics in Saudi Arabia? One plausible explanation is that the needs and beliefs of these different cultures result in different degrees of valuing mathematics in relation to other areas.

Content and Values

The multidimensional definition of mathematical content in the context of instruction has helped to clarify the role of values in content decisions. A three-dimensional taxonomy (general intent, nature of material, operation) which had been suggested to describe the content of elementary school mathematics was used to investigate the existence of national elementary school mathematics curriculum in the U.S. (Freeman *et al*, 1988). The large variability in the content (as defined) has challenged the commonly held belief about the existence of a national curriculum. A refinement and a generalization of this definition to all school mathematics will most likely produce a tool powerful enough to pick up even smaller differences. This in turn discredits the commonly held conviction about the universality of school mathematics content across different cultures.

Much of the variability of content in school mathematics across cultures can be explained by the value-systems of the latter. Content decisions that involve not only the selection of mathematical skills and concepts are bound to be value-mediated decisions. For example, a content decision to teach fractions for skill building vs. problem solving (general intent) is embedded in a value-system about what mathematics and its teaching are.

Cross-cultural research, meager as it is, provides support to the speculation about the role of values in content decisions in mathematics. Grade-placement of addition and subtraction topics in the U.S. elementary textbooks was found to be different than in Japanese, Chinese, or Soviet textbooks (Fuson, Stigler, & Bartsch, 1988). This is essentially a content decision which reflects differences in beliefs and values about what is possible to learn at certain ages. Likewise, the content decision to embed school mathematics in out-of-school setting or de-contextualize it from real life applications reflects a difference in the purpose of teaching mathematics and this in turn is highly value-mediated.

Teaching Methods and Values

Teaching methodology is bound to be affected significantly by values. Because teaching methodology is in essence a complex interaction among teachers, students, and materials, it requires decision making which is value-mediated.

Research which has focussed on the lesson structure has provided us with a more detailed picture of what goes in classroom instruction. The reconstruction of lessons in terms of segments, routines, scripts, and agendas (Lienhardt & Greeno, 1986; Putman, 1987; Yinger, 1980) has enriched our understanding of instruction in the context of expert/novice contrasts. The basic assumption in these studies is that teaching is a complex cognitive skill that rests on lesson structure knowledge and subject matter knowledge. I believe the picture is not complete without considering the complex value-system associated with the lesson structure (cultural and social values) and knowledge structure (values associated with subject matter i.e. mathematics in this case). Any curriculum or class script cannot be fully understood without reactivating the value-system which mediated the many and complex decisions that take place in a class script. The teaching episodes I described earlier are examples of how cultural values (for example, the finality or tentativeness of knowledge) and subject matter values (nature of mathematics, utilitarian or way of thinking) help us understand the dynamics of decision-making in classroom instruction.

Cross-cultural research supports the hypothesized critical role of values in explaining variations in methods of teaching mathematics. The Michigan Studies (Stigler, Lee, and Stevenson, 1987) report, for example, that classroom organization (whole class, group, or individual) and teacher leadership in the U.S. differ significantly from those in Japan or Taiwan. These differences may be accounted for partly in terms of the values attached to leardership and team solidarity in the three different cultures.

Evaluation and Values

As the term indicates, values play a major role in all aspects of evaluation: who, what, and how. Cultural-social values influence the roles of teachers, parents, school, and state in evaluation. What is evaluated is contingent on what is intended (goals) and this in turn is significantly dependent on mathematical as well as cultural-social values. Social values also affect how evaluation is conducted. The teaching episodes described earlier illustrate how social-cultural values (locus of authority) mediate the "who" in the evaluation process. One should note also that evaluation is closely related to goals, methodology, and content decisions. As such evaluation in mathematics education is subject to be influenced by values in the same manner.

Computer Education and Values

Bishop (1988) had the following to say about the role of values in computer education:

> "There is even more of a pressing need today to consider values because of the increasing presence of the computer and the calculator in our societies. These devices can perform many mathematics techniques for us, even now, and the arguments in favor of a purely mathematical training for our future citizens are surely weakened. Society will only be able to harness the mathematics power of these devices for appropriate use if its citizens have been made to consider values as part of their education." (p. 181).

The role of values in computer education is readily apparent in the goals-component, and consequently it manifests itself in the other components: content, methodology, and evaluation. Ralston (1992) identifies two distinct ways in which the computer can be used in classroom instruction: an "electronic blackboard" by the teacher or an interactive tool by the students. These two functions are closely related to social-cultural values such as control-autonomy, passivity-activity, and imitation-exploration (Burkhardt & Fraser, 1992; Noss, 1988).

A value-system which regards knowledge as a construction of human beings interacting in a social context are likely to embrace the objectives of autonomy of learners to take initiatives and explore possibilities. Such a value-system is likely to adopt the function of the computer as an interactive tool. Consequently, the remaining components will be affected accordingly. Content decision (selecting, using and organizing software) will be done in such a way to capitalize on the initiative and activity of the individual students to explore alternatives on their own. Teaching methodology is likely to reduce the control of the teacher over instruction and will change his role from a model to be imitated to a facilitator of learning. Evaluation is likely to be more intrinsic and personal, and less extrinsic and judgemental.

On the other hand, a value-system which regards knowledge as final as determined by "authorities" are likely to embrace the objectives of transmitting knowledge by proper control of the environment in order to maximize efficiency. The function of the computer as an "electronic blackboard" is likely to be favored by such a value-system. Consequently, content decisions, teaching methodology, and evaluation will exhibit less autonomy and exploratory activities on the part of the students as they use the computer.

CONCLUDING REMARKS

The examples and conjectures I have provided are not intended to promote culturo-centrism. On the contrary, my intention is to call attention to the critical role of values in mathematics education in bringing about cross-cultural dialogue. It is only by better understanding of the role of values in mathematics education that we can capitalize on differences in values as keys to open windows for interaction among different culture. Contrasts and conjectures will hopefully provide the impetus for further research in this area.

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