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SUMMARY
In this paper I discuss some results got in 1997/98 with Brazilian mathematics school teachers. The research was done to investigate their mathematical language as related to the concept of function. A dichotomy was detected between "formal" and "practical" language they used to express their own conceptions of function, as well as to teach their students this subject. Also, I found teachers' conceptual images "shrinking" as soon as they were far from their colleges or universities programs.

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In the 17th issue of this Journal, Wenstrom, Martin & King (1998) wrote about the necessity of re-examining programs for college and university mathematics departments as concerns the preparation of mathematics school teachers.

Those authors emphasize that "high school mathematics teachers are the products of these programs. They not only teach what they learned to their students but also how they learned it" (Wenstrom, Martin & King (1998), p. 12).

This paper is intended to resume the subject above, reporting an investigation done in Brazil about the mathematical language used by high school teachers. Despite the differences one can observe in the educational policies in many countries, I have reasons to believe that the problem of preparation of mathematics teachers is essentially the same everywhere, and it is much more complex than one can suppose.

The results of my research show that these teachers (at least in Brazil) seem to use "models" of mathematical language in their classes unlike those they learned at their college or university. Instead, these "models" are much closer to those ones they had in their own experiences as students in high school.

Thus a new question takes place: what kind of influence do college or university programs have on the preparation of high school teachers today? If these programs seem not to interfere significantly in their mathematical language, to teach or to express their own conceptions about mathematical notions, what are their effective contributions to the professional development of these teachers?

As quoted in Wenstrom, Martin & King (1998, p. 12), "unfortunately, few university mathematics departments maintain meaningful links with mathematics in school or with the mathematical preparation of school teachers... Only when college faculty begin to recognize by deed as well as word that preparing school teachers is of vital national importance can we expect to see significant improvement in the continuity of learning between school and college" (Moving Beyond Myths, 1991, p. 28).

THE INVESTIGATION
In 1997/98 I conducted a qualitative study (Ande, 1995; Rockwell, 1985) of how secondary [or high] school mathematics teachers used mathematical language to treat ideas about the concept of function (Zuffi, 1999). My purpose was to investigate the ways these teachers—being mediators and 'catalysts' of the developing processes of their students (Vygotsky, 1962, 1989)—deal with their own conceptions about functions as well as how they explore them in their classrooms. Also, I was interested in knowing how conscious these teachers were about their use of mathematical language.

Seven high school math teachers were interviewed and answered a collection of twenty written questions related to the subject "function." These questions were proposed to give them the opportunity to express their own conceptions about that notion through mathematical language, and were freely answered by the teachers, in such a way that they could write *everything* they knew about functions, beyond the facts they teach in high school.

Even thus, it was very surprising to see, after the analyses of data, that the investigated teachers expressed themselves through mathematical language essentially in the same way they teach, and not in the way they had learned in their college or university courses. Even after telling them to go beyond the ideas they teach, they kept pointing to exactly the same topics and patterns they teach. None of those individuals went far from the language they used in the classrooms with their students.

However, the teachers' formal mathematical expressions tended to approach present day definitions for function (as the ones by Bourbaki or Dirichlet), although they had a very small formal repertoire to communicate it safely and correctly. Many mistakes were made, especially when they insisted on using symbolic notation.

I observe here that the preparation they received in their undergraduate mathematics courses seems to be insufficient to develop self-confidence and awareness of the use of mathematical language, mainly with respect to its formal aspects. On the other hand, my results reveal that there seems to exist a real dichotomy between the teachers' mathematical language dealing with theoretical frameworks and the expression of "practical" questions and situations.

For instance, while Dirichlet or Bourbaki are invoked in formal definitions, in dealing with examples and problem solving these teachers are restricted to classical conceptions for functions, such as Euler's definition. That is, they pointed out only "patterns" given by analytic formulas in very simple algebraic expressions, similar to the ones they often teach in their classrooms (e.g. $f(x)=3x+5$, or $f(x)=5x^2-7x+3$). In the "practical" situations, for the investigated teachers, the ordinary examples they present to their students seem to be enough to "encapsulate" (Dubinsky & Harel, 1992) all the meanings involved in the concept of function. This may be contributing to building narrower conceptual images (Vinner, 1991, 1992) in the high school teachers' expressions for the idea of function, and I don't believe they are really conscious of this fact.

Their conceptual images tend to be limited to the ones they use to teach in high school, and the images seem to "shrink" as these teachers become more and more distant from their undergraduate courses.

In a second part of my research, observing three high school teachers in their classrooms, I got similar results to those obtained with the questionaire and interviews. In their classrooms these teachers use formal mathematical language in such a way that definitions seem to be of much less importance than the "practice" for functions. What really should "count" for the students is the way the teacher deals with algorithms, examples, and techniques for solving mathematics problems. Definitions are in a second plane, which it is not necessary for students to reach.

The mathematical language pointed out in the observed classrooms was static, with purposes in itself, and syntactic aspects were much more emphasized than the meanings of the language. The concepts related to the notions of functions, as I saw in the high school classrooms, do not emerge from a context which has to do with the students' lives. Nor have they to do with the construction of a powerful way of communication, such as the ideal of mathematics. On the contrary, these notions are associated with abstract symbols and algorithms, and these symbols, in turn, become objects for themselves, in a fragmented and truncated language.

All this can be supported by the following evidence:

i) The observed teachers used the term "dependency" as a synonym of "function," as if that word had clearly encapsulated all the mathematical subtleties the ultimate definition for function presents;

ii) The relation in a functional correspondence was always given by an explicit and very simple "rule" or "law" (algebraic expression);

iii) The symbolic notations "x," "y," "a," "b," "c" are always in straight association with the ideas of "independent variable," "dependent variable," and "constants," respectively. This leads the students (and very often, even teachers themselves) to think about these notations always in a limited meaning. (When the roles of "x" and "y" were interchanged, these teachers had difficulty identifying independent variables and constants);

iv) During observed classes, two of the teachers refered to "x" sometimes as "the variable," other times as "the domain," and finally, as one specific element of the domain which should be determined by the students. Since these teachers did not make clear the contexts in which they used each of the terms, I concluded that their own comprehension about these notions were limited. Even more, the sets of domain and image, in the teachers' expressions, seemed to be determined only by the sequence in which they appeared—the first one is the domain, and the second one is the set where the image lies. Therefore, these teachers seemed not to realize that both sets don't have symmetric roles (Sierpinska, 1992);

v) Although the observed teachers worked with real functions of real variables, the variation of elements they proposed for the domain—to build graphs, mainly—had "models" always in the set of integer numbers. They rarely "picked up" rational numbers, and never selected the irrationals to plot the graphs;

vi) The graphic forms were previously presented to students by the teachers, so that these same students only had to locate the graphs. To do that, three or five coordinates seemed to be enough. Hence, continuity was not discussed, and there were many difficulties (with high school students and teachers) dealing with discontinuous graphs of functions.

vii) The interviewed teachers' conceptual images (Vinner, 1991, 1992) for functions are restricted to the facts they teach in high school, even when I asked for broader answers.

SOME REFLECTION

Of course most of the results of my research are not really new. The important fact revealed was that many of the problems we see with high school students are still the difficulties of their teachers. The distance between high school mathematics teachers' conceptions about functions, and the knowledge they received in college seems to be wider and wider as they become more and more involved with their classrooms, and as they move further away from their undergraduate courses.

Here are some reasons I identify for this fact:

1. High school teachers depend almost exclusively on mathematics textbooks to prepare their classes and compose their mathematical language.

As Dancis (1999) reported:

> "It is standard for math textbooks and K-8th grade teachers to provide students with cookbook type directions of what to do in math. It is rare for students to be assigned problems that they have not been programmed to do. It is rare for textbooks and K-8th grade teachers to provide the students with understanding-based explanations which tell the whys and the wherefores of mathematics" (Dancis, 1999, p. 3).

I am sure the same is valid for high school textbooks and teachers in Brazil. And, since these textbooks very often propose a limited and static mathematical language, so is the teachers' language. "Providing students with understanding-based explanations of mathematics is not a common teaching technique" (Dancis, 1999, p. 3). Therefore, syntactic aspects are emphasized, while the construction of meanings of mathematical language is still underestimated.

2. A social fact is involved in the question. There exists a school mathematical culture, at least in Brazil, where teachers must cover a great deal of content, even when the students are not able to reach comprehension for everything. In this case, the mathematical language proposed by these teachers becomes as reduced as possible, to promote very rapid memorization of technical procedures by students.

As Dancis (1999) asserted for middle school, and as we can also read for high school:

> "The natural result [of this situation] is that while the students may develop some proficiency in math skills, they do not gain any understanding of the mathematics. This results in students collecting all sort of misconceptions about mathematics and making a wide range of mistakes while doing calculations. This, in turn, results in less success in high school math classes. Remedying these misconceptions is difficult" (Dancis, 1999, p. 3).

> "The overemphasis on testing, skill development and fact content, etc. [in schools] seems to have inhibited [student] interest in learning, motivation, ability to work with and enjoy ideas, use creativity, and attain satisfaction from an educational experience" (Dancis, 1999, p. 4).

3. There is a great gap between pedagogical disciplines and those related to advanced mathematical content in college and university programs.

These programs generally have the conception that in the first terms the student must learn a lot of advanced math, to be able to apply this content to pedagogical situations. However, most of the program in those courses has nothing to do with the real situations of teaching. Specific math disciplines are isolated from high school programs, and the pedagogical ones are frequently too general to be connected to secondary school or to advanced math.

In the case of functions, many advanced disciplines, such as Algebra, Analysis, Topology, etc. deal with them. And there seems to exist a strong belief that, even isolated, those disciplines are enough to produce a full conceptual image of function for the future high school teachers. But our research revealed that they are not sufficient to produce such a result. Even the teachers who had a strong experience in those disciplines lost self-confidence in using symbolic notations for functions and had a limited conceptual image for them.

All this means that high school math teachers are not being properly prepared at college. And we can see that, even having been educated in the best of institutions, high school math teachers retain a strong influence from social scholar facts that are not foreseen by those undergraduate programs.

I believe that some questions, such as the choice and use of textbooks, the interface between advanced math disciplines and pedagogical ones, and continuity of studies for experienced high school teachers, raising them with future teachers, should concern everyone who cares about the preparation of school teachers, and those who are responsible for college and university programs.

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