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Using Guided Inquiry in Teaching Mathematical Subjects

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The guided inquiry method of teaching promotes students' active participation in the learning process. It increases students' ability to analyze, synthesize, evaluate, and relate the intended learning concepts to multiple disciplines and everyday life, thereby making the material more relevant to students. In this paper, we introduce the guided inquiry method in teaching mathematical concepts. This method is used here to teach the golden ratio and the golden rectangle concepts.

INTRODUCTION AND RATIONALE

In order to capture the students' attention and interest, a teacher must actively engage the students in discovery activities that demonstrate the mathematical concept. After students have had opportunities to explore real life phenomena surrounding the concept and to understand the concept's correlation with other disciplines, the teacher provides students with a formal presentation of the concept. Finally, discussion of its application in multiple environments, including professional and non-professional settings, reinforces the understanding of the concept.

Although there are many proposed forms of inquiry development (e.g. Dewey, 1910; Schwab, 1962), all contain basic similarities that pertain to teachers and students alike. In Suchnian's method (1966), which parallels other proposed methods, the steps in the process of inquiry are to (1) present discrepant events or specific problematic situations, (2) encourage observation for developing a statement of research objectives, (3) ask students for observations and explanations, (4) encourage the testing of those hypotheses, (5) develop tentative conclusions and generalizations, and (6) debrief the process. In order for this process to work, the teacher must create an appropriate classroom climate where asking questions and hypothesizing about the given problem are encouraged. Teachers must also create an environment where the students do not just passively take notes and/or regur-

gitate factual information, but where they actively participate in the learning process.

There are many mathematical concepts that lend themselves to the guided inquiry method. First, we will explain what we mean by "mathematics," then we will introduce the general technique for using the guided inquiry method in the teaching of mathematics, and finally we will demonstrate the use of this method in teaching one specific mathematics lesson.

WHAT IS MATHEMATICS?

There is a common belief that mathematics is the study of numbers. In this oversimplified perspective, mathematics seems straightforward enough not to need an approach to teaching like guided inquiry. Sometimes, more comprehensively, it is believed that mathematics is the science of numbers. Still this perspective is not quite accurate: it is both simpler and more complicated than that. If we consider mathematics historically, we can trace an evolution of the understanding of this subject, and begin to understand why a more comprehensive approach to the subject is required.

Up to 500 B.C. (Egyptian-Babylonian understanding of mathematics), mathematics was indeed the study of numbers.

From 500 B.C. to 300 A.D. (in Greece), geometry became the foundation for mathematics. The study at this time involved forging a relationship between shapes and numbers. Mathematics was now regarded as an intellectual pursuit, having both aesthetic and religious elements. It was at this time that axioms and theorems were born. Because of this development, precisely stated assertions of mathematics could be logically proven by a formal argument.

There was no major change in this conception of mathematics until the middle of the 17th century, with the conception of calculus. Newton (English) and Leibniz

(German) studied the relationships among previously disparate elements.

At the end of the 19th century, the study of numbers became augmented by its more complex relationship with motion, change, space, and, most importantly to the issues raised in this essay, the utilization of specific mathematical tools.

Today, mathematics is not just the science of numbers, but of patterns. These patterns can be real or imaginary, visual or mental, static or dynamic, qualitative or quantitative. They can arise from the world around us, from the depths of space and time, or from the inner workings of the human mind. They are, then, both more complicated than the first historical understandings of mathematics, for they involve much more than just numbers, and more simple, for they are the foundations of much of what we already observe in the world and in our own schema of thought and discovery (Gialamas, 1997).

To introduce the mathematical concept of concern to this study, then, let us initially consider a more elementary mathematical topic: proportion. A proportion is a relationship between two ratios and is expressed as $a : b :: c : d$, or as $\frac{a}{b} = \frac{c}{d}$. A ratio, in turn, is a comparison of two different sizes, quantities, qualities, or ideas, and is expressed by the formula $a : b$ or $\frac{a}{b}$.

With this in mind, there is a unique geometric proportion of terms that has been called the golden ratio. It is this ratio which is the focus of this mathematical investigation. To make it truly an investigation, students must be able to discover, understand, and relate the learning concept to real life situations. Using the guided inquiry method with the steps outlined below, they learn to do so.

USING THE GUIDED INQUIRY TECHNIQUE TO TEACH MATHEMATICAL CONCEPTS FOR CONCEPTUAL CHANGE

For this investigation seven main questions have been modified from Cherif's (1988) proposed guided inquiry method for teaching science, to be used as general guided inquiry questions for the teaching of mathematical concepts (Appendix I).

With these questions, the teacher engages the students in discovery activities that will eventually demon-

strate the mathematical concept for the students. The questions have been grouped into "Before the Activity" and "After the Activity":

BEFORE THE ACTIVITY

- 1) *What do you think will happen, given an initial set of conditions and a specific set of procedures to follow?(Here, students should make conjectures for what they believe will happen.)*
- 2) *Which conjectures seem the most mathematically viable?*

AFTER THE ACTIVITY

- 3) *What is the result of completing the procedures?*
- 4) *Which initial conjectures were most reasonable?*
- 5) *To arrive at a conclusion, what steps were needed in order to complete these procedures?*
- 6) *Where can you identify correlations between introduced mathematical concepts and activities in daily lives?*
- 7) *Can you generalize the results by completing these procedures?*

Using these steps drawn from the principles of guided inquiry, mathematics may be taught in a manner that engages students' intellectual curiosity.

USING GUIDED INQUIRY QUESTIONS TO TEACH THE GOLDEN RATIO

One might define that two quantities a and b satisfy the golden ratio property if the ratio $\frac{a}{b}$ is approximately 1.618. The questions that follow are derived from the general guided inquiry approach and suited for use in a lesson on the golden ratio. These questions were used in a seventh grade classroom, and those students' conjectures and answers are given below.

BEFORE THE ACTIVITY

- 1) *What do you think will happen when you compare the length of your hand and the length of your arm?*

According to Cherif (1988), a question such as this belongs to the Synthesis Level of Bloom's educational objectives. Its aim is to arouse interest, to stimulate thinking, and to produce educated conjectures. It deals with expectations. When we ask "What will happen if...?" we set the stage for the students to recognize that there is a problem, and therefore capture their immediate interest. Furthermore, this question promotes the ability to use mathematical tools in order

to express a hypothesis, an assumption, or possible conclusion clearly. Here are some examples of students' initial hypotheses:

1. The ratio of measurement among male students will be twice as great as the ratio of measurement among the female students.
2. The ratio of measurement among female students will be 1 and 1/2 times greater than among the male students.
3. The ratio of measurement among taller students will be greater than the ratio of measurement among the shorter students.
4. The ratio of measurement among taller girls and shorter boys will be almost the same.
5. The ratio of measurement among male and female students most of the time will be the same.

2) *Which of the above conjectures seems like the best answer?*

To promote educated conjectures, learners must have enough time to discuss their conjectures amongst themselves. Moreover, they must be able to justify their conjectures and also to change their conjectures in the case that someone else has a better point of view. In this situation, the student might discuss the idea that two sets of numbers might be different but might have the same ratio (e.g. 8/16 and 4/8). The educated conjectures then go up on the blackboard for further use.

At this point, students complete the activity and record the results. Students are given the opportunity to test their own conjectures by performing the measurements and calculating the corresponding ratio of the length of arms to hands among male and female students in the classroom. Therefore it promotes the integration of students' understanding and the manifestation of their understanding on the investigated problem. In addition, the students must observe carefully, measure and calculate accurately, and describe their findings in writing (the actual final result) in a concise manner.

AFTER THE ACTIVITY

3) *What is the result of completing the procedures?*

This question offers students the opportunity to actually plan and carry out experiments on their own to

determine whether their conjecture is reasonable. As a result, they will have the opportunity to gain the skills of designing experiments, testing hypotheses, reasoning and debating results, etc.

4) *After completing the measurements and finding the ratios, what do you think about your initial assumption? Which of the initial assumptions were the most reasonable?*

This is a descriptive-discovery question based on the careful observation that characterizes any scientific process. It is aimed toward building an awareness of what actually happened and encouraging students to willingly change their thinking (conjectures) based on the results of the experiments (Cherif, 1988).

In this case, an example of an accepted answer is: "There is no significant difference in the ratio of the length of hands to arms on male and female students." When all the students have agreed about the actual findings and the conclusions pertaining to the compared ratio, they are asked to compare their own initial conjectures with the actual finding. Then, they are asked to come forward and erase from the blackboard any matched predictions.

This is an exciting stage of self-correcting where the students, while engaged in the whole process independently, are actually learning by thinking and doing. Since students are devoted to conducting experiments to test them, the analysis of experimental results will allow for some hypotheses to be rejected and some to be retained (Cherif, 1988).

At this stage, Cherif has warned teachers from falling into the "right answer syndrome," where many teachers feel they must give the right answers to students' questions. In the spirit of inquiry, the students should be allowed to make discoveries for themselves. To use Popp's words (1981), teachers should help students develop or enhance a frame of mind "which can allow familiar and perhaps pet beliefs to be released in favor of alternative better supported ones."

The following are examples of how seventh grade students tested their hypotheses that were listed in question number two:

1. They measured the ratios for their sisters and brothers and used those results to justify their con-

- jectures pertaining to hypothesis (3).
- They measured the ratios for their pets and drew conclusions in terms of their conjectures in hypothesis (5).
 - One student used charts from his father's medical office in an attempt to determine patterns of growth, as they relate to hypotheses (1) and (2).
 - A student used her footprints from the hospital certificate created on her date of birth to compare with the current ratio of segments of her foot length, to test hypothesis (1).

$$\frac{\text{The length of the foot}}{\text{The length of the ankle to toe}} : \frac{\text{The length of the ankle to toe}}{\text{The length of the middle toe}}$$

- What steps have you taken in order to conclude that there is no significant difference in the ratio of the hands to the arms of different students?*

Students need to describe precisely and in detail all the previous steps they and/or the teacher have taken before reaching the final conclusion. In other words, with this question, students need to be able to describe the experimental pattern that led to the final results. Cherif (1988, 1993) has stated that the objectives of asking this question are:

- to keep students up-to-date with the inquiry processes,
- to establish in their minds the cause and effect relationship and that the final results could not have been determined without all the previous steps, and
- to encourage students to think of everything that took place not as a separate or isolated event, but as a total and integrated whole.

Most teachers go directly to the question "Why?" after they ask the question "What happened?" Teachers should be cautioned not to pass over the process too lightly, simply because the students have gained some skills and information and have developed an awareness of the problem. It is necessary for the students to reflect on the experience of having discovered the final result, in order to help them deepen their understanding and appreciation of the gained knowledge and processes (Cherif, 1988 and 1998).

In answering this question, a seventh grade student wrote:

- We measured the lengths of several parts of our bodies. We calculated the ratios of two of the measurements.
 - We compared all of the ratios.
 - We drew conclusions about the ratios and our conjectures.
 - We discovered that not all of our conjectures were wrong.
 - After taking all the measurements, we compared our findings with our conjectures.
- Can you identify a correlation between the demonstrated mathematical concept and real life?*

Cherif (1988) calls this question an idea-application or testing-understanding. He argues that its aim is to help students generalize from the ideas at hand and to encourage them to think of the investigated concept as a part of their lives. This question is asked in order to confirm the following:

- to make sure that students understand the idea or the concept under investigation,
 - to make sure that they master the inquiry processes,
 - to help them develop the ability to apply the reasoning pattern in other situations,
 - to see mathematics as a part not only of society, but also of themselves, and
 - to accept mathematics as a way of knowing and understanding. Once the students have undergone the process of guided inquiry in order to understand a specific mathematical concept, it is important to reinforce their understanding with applications in other disciplines and in daily life.
- Can you generalize the results of completing these procedures? How can you show mathematically that there is no significant difference in the ratio of the length of hands to arms on a variety of students?*

Here, students must provide enough evidence in their attempt to prove their conjectures in general. This is the causal question or the reasoning explanation. The point of this question is that students are asked to generate a reasoned and testable hypothesis. At this stage, it is the generation of a hypothesis and not the testing of the hypothesis that is of concern. Teachers must remember that it is "the theory and not the experiment [that] opens up the way to new knowledge"

(Karl Popper; cited in Hurd, 1969, p 17).

Furthermore, in this stage of the inquiry, Cherif has argued, the tentative explanations (testable hypotheses) offered by students should reflect their ideas, experiences, and understanding, and thus present teachers with the opportunity to find out how and what their students think about the given instance. Based on such findings, teachers should make the decision to continue the session of inquiry without further assistance, with more guided assistance, or to give the students more time to look for related information needed for generating testable hypotheses related to the investigated problem(s). Teachers should have a set of follow-up questions ready for use to stimulate the students should there still persist many ill-founded and unsettled hypotheses.

The following are examples of students' testable hypotheses in seventh grade:

1. Humans grow symmetrically.
2. Human body parts grow proportionally.
3. The growth pattern is the same for the human body in males and females.
4. The growth pattern is the same within all living organisms (plants and animals).
5. The growth pattern is constant within each species in mammals.

As Cherif has argued, only those conjectures that have provided enough evidence of how they might be proven must be considered. The students who generate conjectures, but fail to provide enough evidence of how they can prove them, should have their conjectures rejected by the teacher for consideration.

THE FORMAL INTRODUCTION OF THE MATHEMATICAL CONCEPT

At this stage in the guided inquiry method, the teacher formally introduces the mathematical concept that was previously intuitively presented to the students. The first mathematics concept under investigation is the golden ratio.

THE GOLDEN RATIO

Given a line segment AB and a point C between A

and B:



the ratio $\frac{AB}{AC} = \frac{AC}{BC}$ is denoted by ϕ and is called the golden ratio. One can compute the value of ϕ as follows. Let $AB = x$ and $AC = m$. Then $BC = x - m$.

The ratio becomes

$$\begin{aligned}\phi &= \frac{x}{m} = \frac{x}{x-m} \\ \Rightarrow x(x-m) &= m^2 \\ \Rightarrow x^2 - mx &= m^2 \\ \Rightarrow x^2 - mx - m^2 &= 0\end{aligned}$$

Let us consider x as the variable and m as the constant. Then we have a quadratic equation with the solution as follows:

$$\begin{aligned}x_1 &= \frac{m + \sqrt{m^2 + 4(1)(-m^2)}}{2(1)} = \frac{m + \sqrt{5m^2}}{2} \\ x_2 &= \frac{m - \sqrt{m^2 - 4(1)(-m^2)}}{2(1)} = \frac{m - \sqrt{5m^2}}{2}\end{aligned}$$

Then $x_1 = \frac{m + m\sqrt{5}}{2} \Rightarrow x_1 = m \frac{1 + \sqrt{5}}{2}$

or $x_2 = \frac{m - m\sqrt{5}}{2} \Rightarrow x_2 = m \frac{1 - \sqrt{5}}{2}$,

a negative number.

Therefore, there is only one positive solution, and the ratio that is accepted is

$$\frac{x}{m} = \frac{1 + \sqrt{5}}{2} \cong 1.618033989\dots$$

or $\phi = \frac{1 + \sqrt{5}}{2} \cong 1.618033989\dots$,

which is an irrational number.

For our computations, we will be using a 3-digit approximation for the value of ϕ , which will be 1.618.

THE GOLDEN RECTANGLE

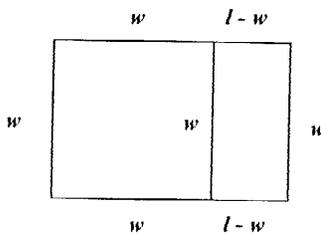
A rectangle with length l and width w is a golden

rectangle if the ratio, $\frac{l}{w} = \frac{w}{l-w} = \phi$.

Alternatively, a rectangle with length l and width w is a golden rectangle if, when we remove a square with side w from the original rectangle, the remaining rectangle is similar to the original—that is to say, two rectangles ABCD and KLMN are similar if the following condition is satisfied:

$$\frac{AB}{KL} = \frac{DC}{NM} = \frac{AD}{KN} = \frac{BC}{LM}$$

So that golden rectangle would look like this:



where $\frac{l}{w} = \frac{w}{l-w}$

and the original rectangle with sides l and w is similar to the remaining rectangle with sides $l-w$ and w .

THE FIBONACCI SEQUENCE OF NUMBERS

The golden ratio is related to a special sequence of numbers, discovered by Leonardo Fibonacci of Pisa, an Italian mathematician in the 11th century. A sequence of numbers $a_0, a_1, \dots, a_{m-1}, a_m, a_{m+1}, \dots$ is called a Fibonacci sequence when the following relationship between its terms is satisfied

$$a_{n+1} = a_n + a_{n-1}, \text{ for } n \geq 1, \text{ and } a_0 = 0, a_1 = 1$$

By replacing n with $1, 2, 3, \dots, 21$, we obtain the first 21 terms of the sequence. Therefore

$$\begin{aligned} a_0 &= 0, & a_1 &= 1, \\ a_2 &= 1, & a_3 &= 2, \\ a_4 &= 3, & a_5 &= 5, \\ a_6 &= 8, & a_7 &= 13, \\ a_8 &= 21, & a_9 &= 34, \\ a_{10} &= 55, & a_{11} &= 89, \\ a_{12} &= 144, & a_{13} &= 233, \end{aligned}$$

$$\begin{aligned} a_{14} &= 377, & a_{15} &= 610, \\ a_{16} &= 987, & a_{17} &= 1597, \\ a_{18} &= 2,584, & a_{19} &= 4,181, \\ a_{20} &= 6,765, & a_{21} &= 10,946. \end{aligned}$$

SEQUENCE OF "ALMOST" GOLDEN RECTANGLES

Let us apply the principle of the golden rectangle using certain selected numbers as sides of a rectangle.

We create a sequence of "almost-golden" rectangles as follows:

1. Choose as the first rectangle the one which has sides $a = 10,946$ and $b = 6,765$ which are consecutive terms in the Fibonacci sequence). We see that the ratio a/b is approximately 1.618.
2. Removing a square with side b (6,765) from the first rectangle we create the second rectangle. The length and width of the new rectangle are respectively 6,765 and 4,181, and their ratio is approximately 1.618.

If we continue this process of removing squares with a side length equal to the shorter sides' length of the rectangle, we obtain a sequence of rectangles with corresponding lengths and widths as indicated in the following table.

COMPARISON	STAGES	RECTANGLE LENGTH	RECTANGLE WIDTH	RATIO
Ratio $< \phi$	1	10,946	6,765	1.618033963
Ratio $< \phi$	2	6,765	4,181	1.618033963
Ratio $> \phi$	3	4,181	2,584	1.618034056
Ratio $< \phi$	4	2,584	1,597	1.618033813
Ratio $> \phi$	5	1,597	987	1.618034448
Ratio $< \phi$	6	987	610	1.618032787
Ratio $> \phi$	7	610	377	1.618037135
Ratio $< \phi$	8	377	233	1.618025751
Ratio $> \phi$	9	233	144	1.618055556
Ratio $< \phi$	10	144	89	1.617977528
Ratio $> \phi$	11	89	55	1.618181818
Ratio $< \phi$	12	55	34	1.617647059
Ratio $> \phi$	13	34	21	1.619047619
Ratio $< \phi$	14	21	13	1.615384615
Ratio $> \phi$	15	13	8	1.625
Ratio $< \phi$	16	8	5	1.6
Ratio $> \phi$	17	5	3	1.666666667
Ratio $< \phi$	18	3	2	1.5
Ratio $> \phi$	19	2	1	2
Ratio $< \phi$	20	1	1	1

One realizes that there is a pattern involving the ratios of the dimensions of the rectangle at each stage. In particular when we compare these ratios with ϕ at

each stage we observe that these ratios alternating from being less than ϕ to being greater than ϕ . Finally it is clear that after Stage 12, the differences are increasingly divergent from the golden ratio.

One might conclude that the Fibonacci number sequence, which appears in many cases in nature, is closely related to the golden ratio. The visual presentation of the first six stages of the table and the curve associated with the sequence of the rectangles is presented in figure 1.

THE SPIRAL CURVE ASSOCIATED WITH THE SEQUENCE OF "ALMOST" GOLDEN RECTANGLES

To begin, we take the first "removed" square (from the golden rectangle), with its center as one of the vertices of the square and radius the l of one side, and draw an arc from one adjacent vertex to the other. We continue the same process for each removed square at each stage in the sequence of "almost" golden rectangles. The resulting continuous curve is called the equiangular spiral curve. In looking at the chambered nautilus seashell in relation to this curve, one can see that the spiral curve appears on the boundary of the shell.

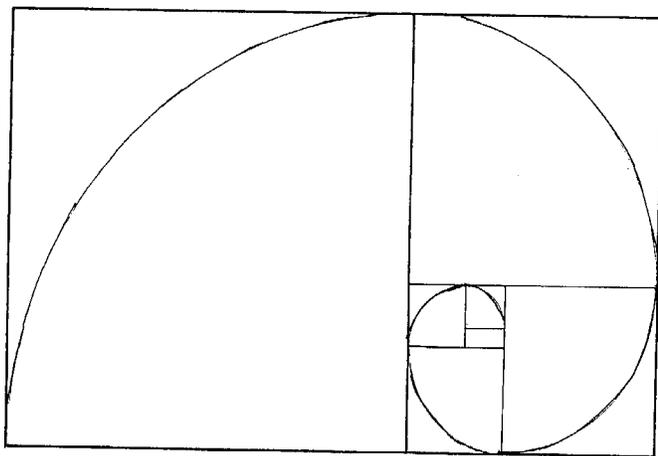


Figure 1

A visual presentation of the first six golden rectangles from the sequence of the rectangles represented by the table and the spiral curve associated with them.

ACTIVITIES TO REINFORCE THE MATHEMATICAL CONCEPT

After a student has studied the mathematical concept through the process of guided inquiry, there are several ways in which that concept can be reinforced to

ensure a more complete understanding of the mathematics. First, as the guided inquiry approach is meant to be a process-based form of learning, for both the instructor and the student, it would make sense for the instructor to embark on the teaching of related concepts in subsequent lessons, so that the student might build upon the information he or she has recently learned through guided inquiry.

Also, in the ending stages of the guided inquiry approach, an instructor might also use other disciplines to reinforce the mathematical concept. The students might draw a representation of the process by which the concept was learned, or the student might write a creative piece demonstrating his/her understanding of the concept in new terms altogether. Both methods would meliorate the students' initial interaction with the mathematical concept. In addition, these forays into other disciplines provide the instructor of the class an opportunity to assess students' understanding of the concept in alternative ways. If a student excels, for example, in the arts and has had a general disdain for mathematics before this lesson, he can demonstrate his understanding of the topic in his own terms, according to his strengths, and his grade would be decided based on a broader range of activities. Any sort of project relating this mathematical concept to other disciplines (science, art, history, etc.) is a fine way to continue the active learning process initiated by the guided inquiry approach.

CONCLUSION

The guided inquiry approach promotes active learning: not just hands-on learning, but minds-on learning. Activities in any discipline that capitalize on the guided inquiry approach will help students and teachers alike make academic material more meaningful, for guided inquiry inspires intellectual curiosity rather than defensiveness. For students who ask, "why do I need mathematics, again?" and for insouciant students who'd rather stare out the window than engage in listening to a teacher lecture on fundamental mathematical principles, the guided inquiry approach offers a reason to become participants in the learning process.

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APPENDIX 1

No.	Guided Inquiry Question in Teaching Science Concepts	Nature of the Question	Aim and Objective of the Question
1.	What do you think will happen given this set of conditions? (If, for example, X is added to Y?)	Predicted Question	To arouse interest, stimulate thinking, and provide predictions.
2.	What actually happened?	Descriptive-Discovery Question	To build an awareness of what actually happened.
3.	How did it happen?	Holistic-Descriptive Question	To establish in students' minds the cause and effect relationship; to think of all the processes that took place as a total integrated whole; to provide general understanding of the process that took place and resulted in what actually happened.
4.	Why did this happen?	Casual Question or Reasoning Explanation	To develop and apply some kind of mental analysis that enables students to generate a reasoned and testable hypothesis (tentative explanations) using their ideas, experiences, and understanding.
5.	How can we find out which of these hypotheses is the most reasonable?	Experimental Question	To provide the opportunity to actually plan and carry out experiments of their own; to gain skills of designing experiments, testing hypotheses, reasoning, and debating results.
6.	How can you relate the investigated ideas, concept, or principle to your daily lives?	Idea-Application or Understanding-Testing Question	To understand the idea or the concept under investigation; to master the inquiry processes; to apply reasoning patterns in other situations; to accept science as a way of knowing and understanding.