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Analogic Thinking in Science and Math

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One of the great pleasures of my position in a nationally known writing-across-the curriculum program is discovering in many scientists a deep appreciation for humanistic thinking. The science wars wage on in the background, and I do find plenty of evidence of rifts between the "two cultures" that C. P. Snow described a half century ago. Nonetheless, there are on our campus many mathematicians and scientists who not only harbor all sorts of artistic talents, but also call upon their students to use language and to think imaginatively. Long before Professor Dennis Sentilles' calculus course was formally designated "writing intensive," he had asked his students to write. That is, Sentilles recognized the power of language to help students conceptualize the mathematical procedures they were working through. His most noteworthy assignment, now a staple in his writing intensive sections, asks students to compare differential calculus to a videotaped tennis game. Students use the extended metaphor (see "A Leitmotif for Differential Calculus," facing page) to explain the nature and measurement of time and motion and their representations from practical, cognitive, scientific, and mathematical points of view.

Intrigued by this professor's assignment, I wanted to review other scientists' use of analogic thinking and to investigate, however informally, some students' responses to analogic thinking. Following is a brief tour through the history of analogic thinking in science as well as a discussion of analogic thinking as reported by six students, three from Sentilles' calculus classes and three from a writing-intensive genetics class that also foregrounds language and imaginative thinking.

SCIENTISTS AND LANGUAGE

Scientists have typically defined scientific writing in terms of the other: It is not literary. It is not ambiguous, expressive, personal, or persuasive. It certainly does not favor metaphor. This prejudice against "literary language" was strong in 1660, when members of the first British society of scientists denounced "all amplifications, digressions, and swellings of style" and called for a return to a "primitive purity" of language. Instead of the "superfluity of talking" that has "overwhelm'd most other <u>Arts</u> and <u>Professions</u>," the new sciences demanded a "naked, natural way of speaking; positive expressions; clear senses; a native easiness" (as quoted by Locke 4 and Bizzell 642).

Three hundred years after the founding of the British Royal Society, many scientific style manuals still pan any use of metaphor or figurative language. Sentilles' use of an extended analogy in calculus might be suspect except for its heuristic or pedagogical value. Analogies might be useful for communicating something to a broad or popular audience, but many scientists would still argue that analogies have little place in discovery or in communicating to a specialized audience. A look at the history of science suggests otherwise, though: analogic thinking has been important both in the discovery and the communication of knowledge, as well as in the more obvious role of teaching. An informal protocol/interview analysis of six students suggests that analogic thinking may be valuable, not so much because it bridges old and new concepts and expedites learning, but because in many cases it disrupts and slows down learning.

ANALOGIC THINKING IN SCIENCE

Investigations into the role of analogy in science have not been limited to pedagogy. Philosophers and historians of science have also studied the role of analogy at the points both of scientific discovery and scientific argument or justification. In the nineteenth century, physicist and mathematician Henri Poincaré as-

A Leitmotif for Differential Calculus

Imagine being out on the tennis court with the ball rising toward you. Differential calculus is the mathematics that describes and measures change in such an ever changing "time-ball" system. One can use this easily imagined setting to elucidate a leitmotif for differential calculus along the following conceptual theme line, where f(t) is the height of the tennis ball at time t:

Computation < ——-— Abstraction < ———— Life/Reality —-— > Cognition

Geometry of Graph	Math Model	Videotape	External Reality	Cognition
Graph of *f*	The function *f*	The whole videotape	The path of the ball	The time-ball "system"
One point on the graph: *(t, f(t))*	*f(t)*	Image on frame taken at time *t*	Where the ball is at time *t*	State at time *t*
Horizontal axis	Domain of *f*	Length of time spent taping (# frames)	How long the ball is in flight	Duration of events
Vertical axis	Range of *f*	All frames on the videotape	All the different positions of the ball	All individual states of the system
Vertical increment in graph	Difference in function values: *f(t+h)-f(t)*	Change between images	How much the ball rises between two moments	Change-of-state
Horizontal shift or increment	Difference in underlying variable: *(t+h)-t=h*	Time between frames	How much time changes	Time span
Slope of secant (two-point) line to the graph between *(t, f(t))* and *(t+h, f(t+h))*	Average rate of change: $\frac{f(t+h)-f(t)}{h}$	Sense of change between images	How quickly the ball appears to rise between two separated moments	Average rate of change (rise): one's sense of motion over a span of time
Slope of tangent line at the point *(t, f(t))*	$f'(t)=\lim_{h \to 0} \frac{f(t+h)-f(t)}{h}$	Video motion seen at time *t*	How fast the ball is rising at time *t*	Rate, or change-of-state, at the moment *t*
All tangents to the graph of *f*	The derivative of *f*: *f'* as a new function	Viewing the video	The flight of the ball	The motion, or flow, of the system

serted that "logic, which alone can give certainty, is the instrument of demonstration; intuition is the instrument of invention," and he credited analogy with being the guide to mathematical invention. The Italian rhetorician Giambattista Vico made similar claims two hundred years earlier. Many scientists, including Humphry Davy, Robert Hooke, Johannes Kepler, Antoine Lavoisier, and Robert Oppenheimer, have also acknowledged the role of analogy in discovery or in intuition (Leatherdale). Perhaps the most famous scientific analogy is Friedrich August Kekule's account of dreaming about a serpent biting its own tail just prior to his discovering the structure of the benzene ring:

> During my stay in Ghent, Belgium, I lived in a fine room on the main street. I sat in this room and wrote on my textbook, but could make no progress—my mind was on other things. I turned my chair to the fire and sank into a doze. Again the atoms were gamboling before my eyes. Little groups kept modestly in the background. My mind's eye, trained by the observation of similar forms, could now distinguish more complex structures of various kinds. Long chains here and there were firmly joined; all winding and turning with snake-like motion. Suddenly, one of the serpents caught its own tail and the ring thus formed whirled exasperatingly before my eyes. I woke as by lightning and spent the rest of the night working out the logical consequences of my hypothesis (qtd. by Leatherdale 20).

Astonishing as some accounts of analogic thinking are for scientific discovery, they are less controversial than the accounts of analogy in scientific argument, particularly in scientific induction. While Aristotle cautioned against argument by analogy (as many logicians have since), Francis Bacon recognized the importance of analogy to scientific argument. John Maynard Keynes further credits Bacon with distinguishing between positive and false analogies. Twentieth-century philosopher of science Mary Hesse modifies Bacon's distinction between positive and false analogies by examining the positive and false elements within any given analogy. Within any one analogy are both positive and negative components. The predictive power of analogic thinking comes, according to Hesse, from a third element, the part of the analogy about which scientists are still undecided. This distinction is similar to one made by Mike in the discussion below: students might be irritated by the false elements of an analogy, but constructively troubled by a "third element," the part that slows them down and causes them to mull over the concept. Few scientists or philosophers of science deny that analogies offer a heuristic value—in the classroom or in the profession, but there is less consensus about the necessity of analogy for scientific explanations. Hesse, among others, argues that analogy is necessary for scientific argument.

Philosophers have asked parallel questions about the role of analogy in language. Friedrich Nietzsche's radical assertion that all language is metaphoric (and, therefore, analogic) has become commonplace in the twentieth century. Postmodernists have largely dismissed the cautionary hedge in I. A. Richards' comment, "Even in the rigid language of the settled sciences we do not eliminate or prevent [metaphor] without great difficulty" (92). I contend that we in the late 1990s need to revisit Richards who, on one hand, denounced "the one and only one meaning superstition" and boldly asserted that "metaphor is the omnipresent principle of language" and, on the other hand, recognized greater rigidity and stability in the language of science.

To the degree that philosophers and scientists agree that metaphor and analogy play a vital role in science, they aren't entirely celebratory. Turbayne cautions that victims of metaphor are trapped unwittingly by prevailing metaphors, much as Thomas Kuhn argues that the prevailing metaphors in a given paradigm both shape and limit scientists' thinking.

However, the "problem" areas of analogies might be prime sites for "disequilibrium," Jean Piaget's term for the tension between the known and unknown that motivates leaming—in this case, learning about science and learning about language. This is the concept that Robert Mayer builds upon in "The Instructive Metaphor: Metaphoric Aids to Students' Understanding of Science" (1993).

ANALOGIC THINKING IN NOVICE SCIENTISTS

Mayer is not the first to think about the role of a particular kind of analogy, metaphor, in learning. The 1970's marked the "cognitive turn" in psychology and in "metaphorology," a time in which psychologists, cognitive linguists, anthropologists, and literary theorists widely accepted the premise that metaphor is not just a marker of deviance (genius), as Aristotle believed, but is common (by degree) to all thought. Cognitive linguists explored not only linguistic structures in a text, but also the ways in which a reader processed them. In the following quarter of a century, sociolinguists have called attention to the importance of context and social relations in discourse.

In a 1994 study of student processing of metaphor, *Understanding Metaphor in Literature*, Gerard Steen analyzes the ways in which students of literature process both potential metaphors (linguistic structures identified by experts as metaphor) and realized metaphors (cognitive reconstructions of potential metaphors). As a discourse analyst working in the realm of pragmatics, Steen assumes that the reader, the text, and the context are all constituents in the study, but that the reader is at the center of the investigation. He assumes that a reader's goals are partly socially determined and that discourse communities share certain regularities and conventions. In this study Steen attempts to move reception theory from the text to the reader as a locus of discourse analysis. Steen first asked students to underline metaphors in a text to see if students recognized as potential metaphors the same linguistic structures as those identified by expert readers, in this case a panel of literature professors.

Sharing scholarly debts to pragmatics and discourse analysis, I wish to explore the relevance of Steen's inquiry to science literacy studies. I broadened and altered the scope from a study of literature students' processing of metaphor to a study of science students' processing of analogy. Over a dozen students participated in this study, but I focused on six, three from the calculus course described above and three from a writing-intensive genetics course. Slightly modifying Steen's methodology, I asked students first to underline analogies in each of three texts and explain them in a taped interview afterwards, and secondly to "think aloud" as they orally read three different passages. In each of the two sessions, the underlining/explanation session and the think-aloud/explanation session, students responded to an excerpt from a work of popular science written for a general audience, an excerpt from a science text (*Human Genetics*) written for college students, and an excerpt from a science journal written for experts in neurophysiology. My initial question, to what degree do student readers of science think analogically, developed into the following six questions as the interviews took place:

- Do these six students recognize as potential analogies the same linguistic structures identified by experts?
- Do these six students process potential analogies analogically? (That is, are potential analogies read literally or figuratively, and, if figuratively, in what ways?)
- Do these six students find analogies helpful in understanding the content?
- What happens when an analogy breaks down, as most analogies eventually do?
- To the degree some analogies bridge new and old information, how does the bridging work?
- Does analogic thinking lead to greater insight into the nature of language?

In general, there was a wide discrepancy between potential and realized analogies. Most of the students only realized or reconstructed the potential analogies when they talked or wrote about them. These students found some analogies much more helpful than others, but all of the students interviewed affirmed the potential instructional value of using analogic thinking in the sciences and of having qualitative learning precede quantitative learning. And, for most students, the interview project led them to think about the language of science in ways that had never before occurred to them. The not-so-literal dimension of language is more pervasive than most students had realized. This awareness, in turn, did lead a few students to think about the ways in which science is "made" in new ways, but it did not cause them to question the value of science. These conclusions, along with the success of the written assignments foregrounding analogy in the calculus course, point to the value of making deliberate use of carefully selected analogies in the sciences.

POTENTIAL ANALOGIES

With one exception, students preferred the popular science genre to either the text excerpts or to the technical academic articles, and they attributed their preference to the abundant images and comparisons in the popular science writing. Students made comments such as "It got you interested" and "That was helpful; I probably wouldn't've understood it [the article] without them [the analogies]" — or "It gave me something concrete to hold in my mind." In the first popular science excerpt, entropy was compared both to an engine running out of gas (a conventional analogy used in most physics courses) and to a casino closing down (a novel and productive analogy for all six readers). Even when students claimed to enjoy excerpts with many "potential analogies," though, they didn't always identify the analogies as such. Five of the students rarely identified as analogies anything other than similes or phrases that were announced by tags such as "...is like." None of the students, for example, identified "cDNA library," "transcription," "editing," or "palindrome" as part of an extended linguistic analogy, even though they could readily identify more terms in the same group once the extended analogy had been pointed out to them. The distinction made here between "potential" and "constructed" analogies is affirmed by the students' "monovalent" reading of many conventional analogies (by a literal reading of a conventional analogy). With the exception of the same student (Steve), the undergraduates disliked the technical article, which made little blatant use of analogy.

> "
> ***...if you force people to make analogies or have an analogy set up for them, the fundamental parts of calculus won't be glossed over so much, but will be used and understood. --Steve***

Steve, the most advanced of the calculus students, expressed decided appreciation of analogies, but constructed his own analogies with or without the prompt of the "potential analogy." As he put it, "I'd almost say that any time I see something that I'm familiar with, the whole index [of mind and memory] is opened up, and I can pull out my file card and say, ah, here's one!" The lack of "potential analogies" in the technical article did not bother him because he was rifling his own mental files, including many "received" analogies from other texts and lectures. He liked the technical article precisely because it was the most foreign to him, because it challenged him the most to construct his own analogies or to recall analogies from memory. In the genetics text excerpts, also, one phrase after another would elicit an analogy not present in the text structure but present in Steve's memory from a drawing on the blackboard in a previous course or from a picture in an old textbook.

CONSTRUCTED ANALOGIES

Although Steve was the most active reader—the most ranging in his connections beyond those presented in the text—all six students constructed analogies when given an opportunity to write or talk about them. Some analogies were grounded in "potential analogies," those text structures that expert readers would identify as analogies; other analogies were comparisons between seemingly-literal information in the text and something in the students' experience. For example, few students read "blind watchmaker" as much other than a placeholder for an unfamiliar idea in the think-aloud interviews, but the more they talked, the more they began to make sense of "blind" and to sort through similarities and differences in the sonar capacity of bats (a result of chance and evolution) and the sonar capacity of machines (purposefully designed by engineers).

The three calculus students had also just completed the course in which they were asked to explain in writing an extended analogy of a videotape of a tennis game. As indicated earlier, the professor of this course, Dennis Sentilles, compares calculus to an ever-changing "time-ball system." A function is compared to the whole videotape of the path of a tennis ball, and the domain of f is compared to the length of time the ball is in flight (the number of frames on the videotape) and the range of f is compared to all the different positions of the ball (or all the frames on the videotape). All three students found this approach to calculus revolutionary and constructive. Steve, who claimed to have an intuitive understanding of the equations, found himself re-defining and clarifying ideas that had already made some sense to him. He found the videotape analogy indispensable and, when asked if he would teach in the manner of Sentilles, responded, "most definitely...I think if you force people to make analogies or have an analogy set up for them, the fundamental parts of calculus won't be glossed over so much, but will be used and understood." That, unfortunately, is what Tom described as being the case in previous courses: "Before I went in there, I just got pushed through calculus, and I didn't really learn to conceptualize it." Learning to think qualitatively and not just quantitatively proved to be revolutionary for him in other courses as well. "After this course, after I started in this course, my grades shot up because I would sit back and look at something and say, 'Okay, I can't get this, why? What are we doing?' And after a while, I'd say, 'ah, that's why!'" Mike, the third calculus student, also said that "for me, it was the analogies that made my understanding. I couldn't just throw those out." All three of them, though, identified the writing process as the place where the received analogy started to make full sense, where the "potential analogy" created by Sentilles became a "constructed" or "reconstructed" analogy in their own minds. Two tentative conclusions might be drawn from this: first, potential analogies might not offer much if students aren't asked to play with them, if students aren't given the time and resources to reconstruct them. Second, all analogies are not equally valuable: Sentilles had experimented with many other potential analogies before settling on the time-ball analogy that proved to be powerful for a wide variety of math students.

ANALOGIC BREAKDOWN

Steve identified "negative analogy" or inaccurate comparisons in both the genetic text and popular science excerpts. He conceded that these limitations could lead to misunderstanding. Instead of ending the analogy, though, he felt readers should keep extending the analogy: "Keep re-defining, keep talking." Here, I would like to distinguish between those analogies that fail because the student already possesses a more refined understanding of the concept and those analogies that are troubling, provocative. Most students who found some analogies too simplistic simply skipped over them. In the *Human Genetics* text, numerous similes were used, and some simply failed for students who had considerable coursework in biology. For example, most students found useful a comparison of Vitamin D and a faulty receptor to a ferry unable to dock, but advanced biology students found other similes limited. However, if an analogy was troubling (not simplistic, but troubling), most students found even the troubling part stimulating. Mike and Steve both objected to some of the emotional implications of the casino analogy for entropy; neither wanted to view entropy as something "bad" or something to lament. Mike didn't fault the writer, though, for an imperfect analogy is thought-provoking, slows a reader down: "Because if he didn't have words in here like that you'd just read it and go on...but then he used words that have a lot of different meanings, or could have...and you have to think...and it makes you mull it over." In other words, the "meaningful calculus" sought by professors such as Sentilles or the "instructive metaphor" sought by learning theorists such as Richard Mayer comes about by gaps that motivate new learning.

BRIDGING OLD AND NEW INFORMATION

All six students were quick to credit analogies with getting them interested in new material. Only one of them expressed much interest in entropy, but all of them found the article about entropy quite interesting. It appears, though, that analogies function even more effectively by breaking up a bridge, by creating a hurdle, or slowing down a train of thought. In both cases, the analogy often functioned as a placeholder, a space for a concept that would become better understood in time. Several of Mike's comments pointed to still another function of analogy: a bridge not from the unfamiliar to the familiar, but from the now-thoroughly-understood to memory. In other words, analogies can function as a way of compressing and repackaging already-understood concepts for long-term storage.

INSIGHT INTO LANGUAGE

The more I gave these six students an opportunity to explain themselves, the more they realized that they were dependent by degree on analogies. Put another way, the more they tried to remove themselves from analogic thinking, the more they realized they couldn't do it. They began to realize that words and concepts are born and grow and change, and most found it impossible to express scientific concepts in absolutely value-free language. Many had never before thought about the etymology of conventional vocabulary such as "bacteria" (little staffs). Several returned to the second interview with examples of analogies they had found in other science texts. Although students gained insight into language, their insight was a byproduct of their learning about something—entropy or natural selection or transcription or neural transplants. They weren't bashing science, but were gaining insight into it.

CONCLUSION

Writing across the curriculum (WAC) programs, such as the one in which I work, have largely substituted a belief in linguistic positivism (which treats language as if it were a transparent medium and writing skills as if they were generalizable across all contexts) with a belief that language can never be completely "clear," can never be completely rid of analogy, and, even if it could, it shouldn't. As scientists and humanists work together to better understand the languages and conventions that do characterize our disciplines, we may also better understand each other.

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Between Childhood and Mathematics: Word Problems in Mathematical Education

continued from page 32

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